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Abstract

Over the past four decades, numerical modeling based on economic principles has become the dominant analytical tool in U. S. energy policy. Energy models are now used extensively by public agencies, private entities, and academic researchers, and in recent years have also formed the core of "integrated assessment" models used to analyze the relationships among the energy system, the economy, and the global climate. However, the widespread application of these models in policy analysis poses challenges to decision-makers. In addition to the "black-box" problem – that the workings of complex models may be simply unintelligible to non-specialists – fundamental uncertainties are intrinsic in what has become the typical circumstance of multiple, "co-existing" models embodying different representations of the energy-economy, and producing different policy-relevant outputs that model users are compelled to interpret as equally plausible and/or valid. Because the policy implications of these outputs can diverge substantially, policy-makers are confronted with a significant degree of model-based uncertainty and little or no guidance as to how it should be addressed. Decision-makers may reasonably infer that such "ensemble uncertainty" accurately reflects the present-day limits of our ability to predict the consequences of large-scale energy or environmental policy. If so, then the problem of rationally taking account of this form of uncertainty should be analyzed in its own right.

This problem of "model uncertainty" has recently been the focus of groundbreaking work in macroeconomics, where scholars have studied the problem of how a decision-maker should proceed in the face of uncertainty regarding the correct model of an economic system that is the object of policy. A unifying theme in this work is the identification of decision-rules that are *robust* to such uncertainty. This paper describes a first attempt to apply to energy modeling the macroeconomists' insights and methods related to model uncertainty and robust analysis, focusing on the important example of model representations of technical change. Using a well-known model by Goulder and Mathai, we treat contrasting assumptions on technical change – and their implications for CO2 emissions abatement policy – as a phenomenon of model uncertainty. We apply non-Bayesian decision rules – min-max and min-max regret - to this problem and computationally solve the model under the min-max regret criterion, yielding a policy – an emissions abatement path – that reflects a form of robustness to the model uncertainty.

JEL Classification: Q54, D81, C63

Keywords: climate policy; carbon tax; technical change; model uncertainty; min-max regret; robust analysis

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1 Introduction

Over the past four decades, numerical modeling based upon economic principles has become the dominant analytical tool in U. S. energy policy. Models of the energy system or sub-systems, of the national economy with emphasis on energy sectors, and combinations of these two types have both proliferated in number and increased in complexity and detail. They are now used by regulatory agencies, university researchers, private companies and non-profit organizations. Moreover, in recent years numerical energy-economic models have formed the core of "integrated assessment" models that represent the relationships among the economy, the energy system, and the global climate.[1]

Their usefulness notwithstanding, however, the widespread application of numerical energy-economic models in policy analysis poses certain challenges for decision-makers. For example, the "black-box" problem – that the workings of complex models may be simply unintelligible to non-specialists – is well known, albeit not on its way to resolution. From the perspective of model users, such model opacity can be interpreted as a form of uncertainty. An arguably more fundamental uncertainty, however, is intrinsic in what has become the typical circumstance of multiple, "co-existing" models embodying what amount to competing representations of the energy-economy, and producing different policy-relevant outputs. While structured multi-model scenario analyses are a well-established methodology in the energy modeling community, this community does not provide formal or quantitative model rankings. As a consequence, all results from a now sizable group of models must be interpreted by users as equally plausible and/or valid. Given that the policy implications of these results can diverge substantially even in structured comparisons, this state-of-affairs confronts policy-makers with a significant degree of uncertainty and little or no guidance as to how it should be addressed.

The practical implications of such uncertainty were noted by Fischer and Morgenstern (2006) in their study of the divergence of model-based estimates of the potential costs of the Kyoto carbon emissions reduction agreement to the U. S. economy, which varied by a factor of five: "…this variability in cost estimates undermines support for mandatory policies to curb emissions, as policy makers are generally reluctant to adopt a major program without an understanding of its true costs."

Not all multi-model, policy-relevant outputs in energy analysis display this level of variation. Nonetheless, inter-model differences sufficiently large to be policy relevant are the norm rather than the exception. Decision-makers may reasonably infer that such "ensemble uncertainty" accurately reflects the present-day limits of our ability to predict the consequences of large-scale energy or environmental policy. If so, then the problem of rationally using multi-model policy outputs should be addressed in its own right.

In macroeconomics, this problem of "model uncertainty" has been the focus of groundbreaking work by Hansen and Sargent (2005, 2007a,b) and Brock et al. (2007a,b). These

[1] We will use the term "energy model" to refer to each of these types, i.e., numerical economic equilibrium (partial or general) or optimization models, with or without linked environmental components.

scholars have studied the problem of how a decision-maker should proceed in the face of uncertainty regarding the correct model of an economic system that is the object of policy. A unifying theme in this work is identification of decision-rules that are "robust" to such uncertainty. While there are different technical definitions of this concept, colloquially it refers to decisions, or policies, that will yield acceptable although not necessarily optimal outcomes regardless of which model within a certain set is "true."

As noted above, this form of uncertainty very conspicuously characterizes the present state of energy modeling. The goal of this paper is to make an initial application to energy modeling of the macroeconomists' insights and methods related to model uncertainty and robust analysis. We focus on a particularly important dimension of model uncertainty, that of the representation of technical change.

It has long been recognized by both experts and non-specialists that assumptions regarding the determinants and dynamics of technical change are a primary driver of model-based projections of the feasibility, costs, and outcomes of long-run energy policies – especially those aimed at reducing greenhouse gas (GHG) emissions from the energy sector. Among current energy models, quite different sets of such assumptions are maintained – i.e., in different models – and they have divergent policy implications. Broadly speaking, there are two paradigms for representing technical change. In the "autonomous" representation, which can be traced back to Solow's work on aggregate productivity in the 1950s, technical change dynamics are determined exogenously to the market economy; moreover, while these dynamics may be influenced by government policy, the mechanisms of this influence are left unspecified. By contrast, "endogenous" or "induced" technical change refers to theories, and their numerical implementations, in which technical change is explicitly treated, albeit in simplified form, as an outcome of choices by economic agents acting within markets; in certain examples, this paradigm also allows for the representation of government influences such as R&D funding.

As might be expected, these two approaches have quite different theoretical and quantitative implications for energy policy. Yet – even after decades of basic and applied research – there is an absence of consensus within the energy modeling community regarding the appropriate paradigm for representing technical change, reflected in a continuing divergence among different numerical models. The departure point for this paper is the observation that this state-of-affairs is best characterized as one of fundamental model uncertainty, and as such can in principle be addressed by bringing to bear the appropriate concepts and tools developed in macroeconomics. The application we discuss here also both complements and builds upon the pioneering contributions of Lempert et al. (e.g., McInerney, Lempert, and Keller 2012) to the analysis of robust decision-making in integrated assessment modeling.

The paper is organized as follows. In the next section, we sketch the history and key concepts of model uncertainty and validity in the energy analysis and policy field. We then further discuss the representation of technological change in energy models and its policy implications. Against this background, we present a model of Goulder and Mathai (2000) that, while relatively simple, nevertheless allows for analysis of several fundamental issues associated with differing technical change assumptions and how they affect model-derived policy

conclusions. We briefly discuss technical aspects of the model and the key conclusions reached by Goulder and Mathai. Next, we consider the Goulder-Mathai framework from the perspective of model uncertainty, and, following Brock et al. (2007a), introduce two decision rules – min-max and min-max regret – that are applicable in the context of this form of uncertainty. We briefly review previous and recent applications of min-max regret in energy and integrated assessment modeling. We then describe a computational version of the model and a solution for the min-max regret criterion. The paper ends with concluding remarks.

2 Concepts of validity and uncertainty in energy modeling

In this section we provide epistemological context for our framing of current energy modeling practice in terms of model uncertainty. As noted in the introduction, this concept entails multiple models of a given system being assigned equal weight, credibility, or validity, whether explicitly or – as in the case of energy modeling – implicitly. However, "validity" and "validation" as such are generally not part of present-day discourse in this modeling domain. This represents a departure from what was the norm in an earlier era.

The 1970s saw the emergence of numerical energy modeling based on optimization or equilibrium principles with an explicitly microeconomic perspective (Murphy and Shaw 1995). Among their other features, such models allowed for the prospective analysis of hypothetical policies much more readily than was the case in a more standard forecasting approach. At the same time, however, in contrast to what was then the standard approach to, for example, macroeconomic modeling, this form of deterministic "system simulation" modeling generally did not employ econometric estimation techniques for assigning values to model parameters (Greenberger et al. 1976). Thus, concepts of validity based upon the empirical grounding of models in data through classical statistical procedures were for the most part not directly applicable.

Nevertheless, during this period numerical model validation was an active area of research and application; indeed, this may have reflected the greater complexity of the problems of validation in the simulation paradigm. A 1978 bibliography on validation of computer models in policy analysis and the social sciences contained over seven hundred entries, with more than one-hundred in the category of "energy and electric power models" (Gruhl and Gruhl 1978). More generally, model "assessment" – both "evaluative" and "non-evaluative" (e.g., model inter-comparisons) – received substantial attention as well as resources (Greenberger and Richels 1979). For example, the U. S. National Bureau of Standards convened multi-disciplinary workshops on energy model validation (NBS 1980).

Subsequently, however, these efforts attenuated. By the mid-1980s, with a change in national policy priorities and the end of the "energy crisis," both regulatory and academic interest in energy-related topics declined. However, energy modeling continued in the U. S. Department of Energy, national laboratories, and a handful of universities and other research centers, and became embedded in the policy process.

In the succeeding decades, a certain tension developed in energy modeling epistemology with respect to criteria for validity and verisimilitude. Experiences in the policy arena in the 1970s, including often difficult problems of communication with decision-makers, had led one prominent modeler to argue that "the purpose of energy modeling is insight, not numbers" (Hogan 1978), which became a widely-accepted precept in the modeling community (e.g., Peace and Weyant 2008). At the same time, however, the path of actual model development and evolution has shown a pronounced emphasis on greater detail and complexity, to an extent indicating that increased model detail *per se* is seen as improving verisimilitude. In many cases, this detail is in the representation of energy technologies, and the resulting change in model output hinges on the particular assumptions made regarding specific technology characteristics - that is, the *numbers* describing technologies and their role in the energy system and its response to policy.

Both of these philosophies are closely related to the use of calibration rather than estimation procedures for parameterizing energy models. "Calibration" here refers to the use of external sources to assign values for elasticities, rates of technical change, technology cost, and other parameters. The calibration philosophy was succinctly described by Dawkins et al. (2001); although specifically directed at numerical general equilibrium models, it applies quite aptly to other types of energy-economic models as well:

> "…modelers typically see their simulations largely as numerical implementations of theoretical structures. To them, the widespread use of a particular structure in the theoretical literature is an indication of its worth, so that they seek less to test or validate models and more to explore the numerical implications of a particular model, conditional on having chosen it…" (Dawkins et al. *op. cit.*, p. 3762).

This epistemology helps to explain the prevalence of energy model uncertainty as we described in the previous section. Economic theory supports a wide range of practical choices regarding numerical model structure, functional forms, and other features, and reliance on calibration in turn allows for a range of equally plausible numerical realizations conditional on such features having been selected. Moreover, different models can yield "insights" that, all else being equal, are equally credible.

While the majority of energy models are deterministic, there have been models that take an explicitly decision-making-under-uncertainty approach (Kann and Weyant 2000). In general, these also reflect the epistemology we have just described, in that the uncertainty is on the part of agents within the model, and is conditional upon prior assumptions regarding model structure and basic parameterization choices. Addressing model uncertainty in the sense defined above, Kahn and Weyant (*op. cit.*) proposed a framework for applying uncertainty analysis to deterministic models to elucidate, e.g., the sources of inter-model differences. This framework has not been adopted among energy modelers, however, and well-known reasons for inter-model output differences, such as contrasting choices pertaining to structure and parameter values, continue to be acknowledged qualitatively without being quantified or analyzed in depth. (See, for example, Clarke et al. 2007) The study of Fischer and Morgenstern (*op. cit.*) is a rare

exception that proves this rule; they performed a regression analysis in an attempt to quantitative gauge the importance of various model features in determining inter-model output variation. More recently, the U. S. Congressional Research Service conducted an insightful and illuminating investigation of multiple models' projections of the costs of long-run CO2 abatement policy (Parker and Yacobucci 2008); this analysis is in effect an effort by the U. S. Congress to address energy model uncertainty in the context of a major policy issue.

3 Modeling energy-related technological change

Early energy modeling studies emphasized the importance of the mechanisms of substitution – i.e., between energy and non-energy economic factors, including capital – in determining the role of energy in the aggregate economy (Hogan and Manne 1977). However, events of the 1970s appeared to demonstrate the importance of overall energy productivity in a manner analogous to that of labor productivity. That is, consider a stylized economy-wide production function $F(\bullet)$ giving gross output Y_t as a function of capital K_t, labor L_t, energy E_t, and factor-augmenting technical change:

$$Y_t = F\left(\alpha_t^K K_t, \alpha_t^L L_t, \alpha_t^E E_t\right). \tag{1}$$

If $d\alpha_t^i / dt > 0$, then over time, all else being equal, a given input level of factor *i* will yield a progressively greater level of output. Having been proposed for macroeconomic labor-output relationships by Solow and Swan in the 1950s, this type of relationship suggested itself to explain the aggregate "de-coupling" between energy and economic growth that occurred in the '70s.

This hypothesis was not completely uncontroversial; Hogan and Jorgenson (1991) argued that, on the contrary, the de-coupling phenomenon could be explained in terms of aggregate substitution between energy and other factors, resulting from energy price shocks. Nonetheless, the need to calibrate numerical energy-economic models to observed trends, combined with the simplicity of factor-augmentation, gave rise to widespread adoption of this type of representation. Colloquially, the term "autonomous energy efficiency index," or "AEEI," came into common use to refer generically to parameters such as α_t^E above. Variations on this mathematical representation were also developed for models without production function-based structure.

That this approach was in effect a reduced form for the complex, not-well-understood dynamics of energy-related technical change was generally if tacitly recognized. However, the advent of the "new growth theory" as developed by Romer (1986, 1990), Lucas (1987) and others in the 1980s and 1990s indirectly weakened its plausibility. The new growth theory insight that technological innovation is to a significant extent a market phenomenon, responsive to economic incentives, and its practitioners' development of mathematical tools to represent it in aggregate models, provided grounds for questioning the standard autonomous technical

change assumptions in energy models as well as indicating how these assumptions might be modified.

While this issue entails a number of challenging theoretical and empirical problems, its policy relevance is quite intuitive. The fundamental policy application of energy models is to project how government actions such as GHG emissions abatement measures will affect the energy system and the economy. Such actions are generically of several types. The first is direct regulation such as technology adoption requirements. In a literal sense, this is not an "autonomous" change, although depending upon the level of model detail, among other considerations, it can be approximately captured by adjusting AEEI-type parameters. In equilibrium models, however, this technique raises questions of interpretation, since it could imply that the government can simply impose productivity increases in the economy. The second type is promotion of energy-related technical change through government or government-sponsored R&D. Here again, the assumption of autonomous technical change poses a *prima facie* hurdle, since R&D is fundamentally a production relationship between inputs of factors such as scientific or engineering expertise, and outputs such as patents or actual technologies.

The third type of government action is market- or price-based interventions – especially, emissions taxes or emissions cap and trade regimes. In a model with autonomous technical change, the only possible response to a perturbation such as an emissions tax is substitution away from emissions-intensive energy sources on the part of consumers and firms - technical change is by assumption unaffected. In particular, the possibility that energy price changes might result in energy-or emissions-reducing innovation is ruled out. Such innovation could arise, for example, from private firms undertaking R&D aimed at creating technologies that would yield profits given a price on emissions. Given that such "induced" or "endogenous" innovation or technical change would yield technologies enabling emissions abatement (for example) at lower cost than existing technologies, the omission of such price-driven innovation would arguably result in systematic over-estimation of the costs of policies.

Over the past fifteen years, research on induced technical change (ITC) in energy models has proliferated; Gillingham et al. (2008) is a thorough and insightful review. A number of new computational models have been created incorporating representations of ITC, such as Buonanno et al. (2003) and Popp (2006). Underlying principles have been analyzed, including the cost-bias problem with AEEI-based models (Pizer and Popp 2008, Popp 2010). This activity has not, however, resulted in either a convergence of energy modeling approaches to ITC nor, perhaps more importantly, in changes in the treatment of technical change in established models. That is, the use of AEEI-type representations remains common, and in particular continues in the most firmly established (and influential) energy as well as integrated assessment models in the U. S.

4 A simple integrated assessment model

In a widely-cited paper, Goulder and Mathai (2000) presented an elegant theoretical framework to capture essential features of the relationship between autonomous and induced technical change, and the consequences of technical change assumptions for optimal policies of CO2

abatement, in a partial equilibrium setting. They analyzed cases based on both a cost-effectiveness criterion applied to meeting a CO2 concentration target, and a cost-benefit criterion when abatement costs are weighed against damages from climate change. In each of these two cases, they further distinguished between technical change induced by R&D investment as well as by learning-by-doing.

To adapt the Goulder-Mathai (hereafter, G-M) framework to study model uncertainty, we will focus specifically on the case of cost-benefit analysis with learning-by-doing. The model is as follows. The setting is deterministic, continuous-time, infinite-horizon optimal control. In the absence of abatement, CO2 emissions are assumed to follow an exogenously given baseline time path E_t^0. Emissions contribute to an atmospheric carbon stock S_t, resulting in damages $D(S_t)$. Emissions abatement A_t can be undertaken at a cost $C(A_t, H_t)$ that depends jointly on A_t and a stock of "abatement knowledge" H_t. This cost function is assumed to be twice continuously differentiable, strictly increasing in abatement, and strictly decreasing in knowledge, and is also assumed to be strictly convex in A_t, $\partial^2 C / \partial A_t^2 > 0$, and to exhibit decreasing marginal costs of abatement with respect to knowledge, $\partial^2 C / \partial A_t \partial H_t < 0$.

The atmospheric carbon stock follows a standard linear decay model, augmented by baseline emissions and abatement:

$$\tfrac{d}{dt} S_t = -\varepsilon S_t + E_t^0 - A_t \tag{2}$$

with initial condition $S_0 = S(0)$. The damage function $D(S_t)$ is assumed to be strictly increasing and strictly convex.

The dynamics of technical change are characterized as follows. A knowledge function $\Psi(H_t, A_t)$, increasing in both H_t and A_t, captures the idea that undertaking abatement results in increased knowledge about abatement. This function is weighted by a parameter κ and combined with a term representing standard autonomous, in this case knowledge-increasing, technical change, giving the equation-of-motion for the knowledge stock

$$\tfrac{d}{dt} H_t = \alpha H_t + \kappa \Psi(H_t, A_t), \tag{3}$$

with $\alpha > 0$ and initial condition $H_0 = H(0)$. The parameter κ controls the influence of induced technical change: $\kappa = 0$ represents the case of autonomous change, and $\kappa > 0$ induced. G-M treat κ as a continuous parameter in their theoretical analysis, while in their numerical examples focus on the two cases $\kappa = 0, \kappa = 1$.

The decision criterion is to minimize the present value (discounted) cost of abatement plus damages. Thus, the complete G-M model in this case is

$$
\begin{aligned}
&\min_{A_t} \int_0^\infty \left[C\left(A_t, H_t\right) + D\left(S_t\right) \right] e^{-\rho t} dt \\
&s.t. \\
&\tfrac{d}{dt} H_t = \alpha H_t + \kappa \Psi\left(H_t, A_t\right) \\
&\tfrac{d}{dt} S_t = -\varepsilon S_t + E_t^0 - A_t \\
&H_0, S_0 \ \ given \\
&A_t, H_t, S_t \geq 0.
\end{aligned}
\tag{4}
$$

G-M's mathematics is informal; they derive first-order optimality conditions, and sensitivity relationships involving the parameter κ, without specifying the precise assumptions under which these derivations are valid. Overall, the model just specified is an infinite-horizon, not-necessarily-convex, parametric optimal control problem with pure control and state constraints. This class of model as such does not appear to have been fully analyzed in the various strands of the optimal control and optimization literature, although there are fairly extensive sub-literatures focused upon specific aspects – such as infinite horizons, parametric control, and different forms of control and state constraints. Because this paper is aimed at describing general concepts and their policy implications rather than addressing technical issues, we adopt an heuristic perspective, assuming little more than the existence of feasible and optimal solutions, and rely on numerical calculation to explore the nature of these solutions with specific functional forms and parameter values. However, some of the technical issues are discussed in the Appendix, along with remarks on previous research that bears on this problem.

G-M analyze the implications for the cost-minimization problem of assuming ITC compared with solely autonomous technical change. First, by simple inspection, it is clear that the presence of ITC – represented by $\kappa > 0$ - decreases the cost of abatement. This is because the cost function is decreasing in knowledge capital H_t - i.e., $\partial C / \partial H_t < 0$ - and it is assumed that there are no costs associated with the learning effect. (With additional assumptions, the sign and magnitude of the cost reduction resulting from ITC can also be inferred by applying a dynamic envelope theorem; see Appendix.) Second, assuming that damages are a convex function of the carbon stock, the time path of the optimal tax falls with the introduction of ITC. Finally, ITC has an ambiguous effect on initial abatement A_0, but increases the level of cumulative abatement over the entire time horizon.

5 Extension to model uncertainty analysis

In a stylized sense, the G-M analysis reflects the modeling epistemology described in the introduction to this paper: Multiple models – in this case, defined by the presence or absence of ITC – yield different policy conclusions, but are in effect assigned equal plausibility by the modeler. The results are intended to be *informative* to a hypothetical decision-maker, but the manner in which they might actually be used for decision-making is left unspecified. That is,

imagine in the present case that an individual or entity sought to decide upon an intertemporal policy of abatement and investment using the G-M model, including the dimension of parametric ITC. How should this be done?

One approach would be to assign a probability distribution to κ, and convert the problem into one of *expected* cost minimization. The problem is how to determine an appropriate distribution, or put differently, how to generate priors for the value of this parameter. As discussed above, years of research on ITC in general in energy-environmental modeling have not resulted in consensus regarding the important issues – how it should be modeled, the values of key parameters, and so forth. With a particular model such as G-M's, it would seem possible to conduct empirical (econometric) analysis to quantify κ. In practice, however, this has proven exceptionally difficult. Even the standard approach of calibration (as opposed to estimation) of numerical models is challenging in the case of ITC, and moreover a calibration approach would not provide stochastic information to support assignment of a probability distribution.

The conceptual and practical difficulties associated with determining model priors are among the issues discussed by Brock et al. (2007a) in motivating a non-Bayesian approach to dealing with model uncertainty (in their analysis, in a macroeconomic context). Non-Bayesian decision rules allow a policy-maker to explicitly incorporate multiple models or model specifications without needing to specify probability distributions, or specific probability magnitudes. While individual models may be of optimization type, the overall decision problem is not – instead the goal is to make decisions that will yield at least acceptable outcomes irrespective of which candidate model may be correct – this is a form of *robustness* to model uncertainty.

Both the technical model details and the economic policy issues studied by Brock et al. differ from those associated with the G-M analysis. Nevertheless, the underlying ideas of fundamental model uncertainty and both ambiguity aversion and non-Bayesian methods to address it are eminently applicable to the technological change problem. We next describe the formulation of this problem – as represented by the G-M optimization model – in a non-Bayesian framework, analogous to the Brock et al. macroeconomic analysis. Intuitively, we interpret the G-M framework as defining a class or family of "candidate" models parameterized by κ, but now the different members of this model set, and their policy implications, are not to be just *considered* by a decision-maker, but rather *used* explicitly in a precisely defined way.

- ***Min-max***

Applied to model uncertainty, the min-max criterion, introduced by Wald (1950), bases the decision upon the "worst-case" model – in the present context, the model associated with the highest cost. A min-max framing of the ITC problem using the G-M model is

$$
\begin{aligned}
&\min_{A_t}\left\{\max_{\kappa}\int_0^{\infty}\left[C\left(A_t,H_t\right)+D\left(S_t\right)\right]e^{-\rho t}dt\right\}\\
&s.t.\\
&\tfrac{d}{dt}H_t=\alpha H_t+\kappa\Psi\left(H_t,A_t\right)\\
&\tfrac{d}{dt}S_t=-\varepsilon S_t+E_t^0-A_t,
\end{aligned}
\tag{5}
$$

subject to initial conditions and non-negativity constraints.

Heuristically, we interpret this problem and characterize its solution as follows. Assume that the support of κ is a closed interval, $\kappa\in\left[0,\bar{\kappa}\right]$. Then note that abatement trajectory $\left\{A_t\right\}$ that is admissible for one value of κ in this interval will be admissible for all values. (The meaning of "admissible" in this instance is discussed in the Appendix.) Now, given admissible $\left\{A_t\right\}$, as κ ranges over its domain the path of H_t also varies, and therefore the time-path of values of $C\left(A_t,H_t\right)$ varies as well. So, the discounted cost is maximized over this set, as a function of κ (continuing to hold the A_t trajectory fixed in the space of trajectories). This maximization yields a function of $\left\{A_t\right\}$, which is then minimized over all admissible such pairs. Regarding the solution, note first that because the discounted cost decreases in κ, the "inner" maximum is attained at $\kappa=0$. Then the "outer" problem is simply the cost minimization with autonomous technical change. Thus, the solution to the min-max problem corresponds to the "worst" case, in which no cost reduction is available from ITC.

- ***Min-max regret***

The idea of the min-max regret (MMR) criterion is to ameliorate the conservatism of the min-max criterion's dependence upon the worst case. The use of this criterion in energy modeling was pioneered by Loulou and Kanudia (1999) in a regional-scale linear programming (LP) model. There has recently been an increase of work on this topic. Li et al. (2011), Dong et al. (2011), and Dong et al. (2014) apply the regret criterion in different variations of LP power system modeling. Iverson (2012) applies MMR in a version of Nordhaus's well-known "DICE" integrated assessment model, while Hall et al. (2012) compare the results of MMR with an "info-gap" analysis as approaches to robustness analysis in DICE. Anthoff and Tol (2013) analyze several decision criteria including MMR using the "FUND" integrated assessment model.

In the present context, for a given policy – in our case, a trajectory $\left\{A_t\right\}$- the "regret" associated with a model – defined by a value of κ - is the difference in discounted cost between that associated with $\left\{A_t\right\}$and the cost of the optimal policy for that model. For notational brevity, define

$$
\Omega\left(A_t,H_t,S_t\right)\equiv C\left(A_t,H_t\right)+D\left(S_t\right). \tag{6}
$$

For a given κ, let A_t^*, H_t^*, S_t^* be the cost-minimizing control and state variables subject to the constraints listed in Equations (4), and define

$$M^*(\kappa) \equiv \int_0^\infty \Omega\left(A_t^*, H_t^*, S_t^*\right) e^{-\rho t} dt = \min_{A_t} \int_0^\infty \Omega\left(A_t, H_t, S_t\right) e^{-\rho t} dt. \tag{7}$$

Then the min-max regret formulation is

$$\begin{aligned} &\min_{A_t} \left\{ \max_{\kappa} \left[\int_0^\infty \Omega\left(A_t, H_t, S_t\right) e^{-\rho t} dt - M^*(\kappa) \right] \right\} \\ &s.t. \\ &\tfrac{d}{dt} H_t = \alpha H_t + \kappa \Psi\left(H_t, A_t\right) \\ &\tfrac{d}{dt} S_t = -\varepsilon S_t + E_t^0 - A_t \end{aligned} \tag{8}$$

For given A_t and κ, we can re-write the "regret" more succinctly as

$$R\left(A_t, \kappa\right) \equiv \int_0^\infty \Omega\left(A_t, H_t, S_t\right) e^{-\rho t} dt - M^*(\kappa), \tag{9}$$

and the overall min-max regret problem as

$$\begin{aligned} &\min_{A_t} \max_{\kappa} R\left(A_t, \kappa\right) \\ &s.t. \\ &\tfrac{d}{dt} H_t = \alpha H_t + \kappa \Psi\left(H_t, S_t\right) \\ &\tfrac{d}{dt} S_t = -\varepsilon S_t + E_t^0 - A_t. \end{aligned} \tag{10}$$

The min-max regret problem appears to be unsolved in the optimal control literature. (See Appendix.) We thus focus on computational solutions of the G-M model with this decision rule, as described in Sections 6 and 7, below.

- ***Expected cost minimization***

While this paper focuses on a non-Bayesian approach to addressing model uncertainty, it will also be instructive to compare min-max regret solutions to the G-M model with the results of standard expected cost minimization applied to this model. In such a formulation, we assume that the uncertainty in κ is captured by a probability distribution, and the model described by equations (4) above becomes

$$
\begin{aligned}
&\min_{A_t} \mathbf{E}_\kappa \int_0^\infty \left[C\left(A_t, H_t\right) + D\left(S_t\right) \right] e^{-\rho t} dt \\
&s.t. \\
&\tfrac{d}{dt} H_t = \alpha H_t + \kappa \Psi\left(H_t, A_t\right) \\
&\tfrac{d}{dt} S_t = -\varepsilon S_t + E_t^0 - A_t \\
&H_0, S_0 \ \ given \\
&A_t, H_t, S_t \geq 0,
\end{aligned}
\tag{11}
$$

where $\mathbf{E}$ is the expectation operator. In the following sections we will implement this solution concept using, in turn, two different distributions for κ, to explore the dependence of the result on this assumption. This exercise is loosely related to the concept of "ambiguity aversion," which refers to a decision maker's model incorporating probabilistic uncertainty regarding one or more parameters, but the decision maker being uncertain as to which of several candidate distributions is the correct one to assign to these inputs. A theory of decision-making under ambiguity has been developed to analyze the positive and normative implications of this type of model uncertainty.

6 Computational model forms

6.1 Functional forms

To implement and solve the G-M model numerically, we switch to discrete time using the following functional forms.

Abatement cost:

$$
C\left(A_t, H_t\right) = M_C \frac{A_t^{\alpha_{C_1}}}{\left(E_t^0 - A_t\right)^{\alpha_{C_2}}} \frac{1}{H_t}, \tag{12}
$$

with $M_C = 83, \alpha_{C_1} = 3, \alpha_{C_2} = 2$. In addition, as described in Section 7 below, we analyze several cases with H_t^4 in the denominator.

Damage function:

$$
D\left(S_t\right) = M_D S^{\alpha_D}, \tag{13}
$$

where $M_D = 0.0012$ and $\alpha_D = 2$.

As above, we denote $\Omega\left(A_t, H_t, S_t\right) \equiv C\left(A_t, H_t\right) + D\left(S_t\right)$.

Technological change – knowledge function:

The knowledge production function is

$$
\Psi\left(I_t, H_t\right) = M_\Psi I_t^\gamma H_t^\phi, \tag{14}
$$

where $\gamma = 0.5, \phi = 0.5,$ and $H_0 = 1$. As described in Section 7 below, we first set $M_\Psi = 0.0022$, but then also analyze cases with $M_\Psi = 0.022$ and $M_\Psi = 0.22$.

Technological change – dynamics:

The rate of autonomous change is assumed to be 0.5% per annum, i.e., $\alpha = 0.005$.

$$H_{t+1} = (1+\alpha) H_t + \kappa M_\psi H_t^\phi A_t^\gamma . \tag{15}$$

Carbon stock dynamics:

$$S_{t+1} = S_t + \beta\left(E_t^0 - A_t\right) - \delta\left(S_t - S_{PRE}\right), \tag{16}$$

where $\beta = 0.64, \delta = 0.008, S_0 = 360,$ and the pre-industrial concentration is $S_{PRE} = 278$ parts-per-million by volume (ppmv).

The baseline emissions abatement scenario is "Representative Concentration Pathway (RCP) 8.5," generated using the intermediate-complexity climate model MAGICC 6 (Meinshausen et al. 2011a-b).

6.2 Min-max regret

We first note that in the discrete-time formulation, the min-max model becomes

$$\begin{aligned}
&\min_{A_t} \max_{\kappa} \sum_{t=0}^{\infty} \Omega\left(A_t, H_t, S_t\right) e^{-\rho t} \\
&s.t. \\
&H_{t+1} = (1+\alpha) H_t + \kappa M_\psi H_t^\phi A_t^\gamma \\
&S_{t+1} = S_t + \beta\left(E_t^0 - A_t\right) - \delta\left(S_t - S_{PRE}\right) \\
&A_t, H_t, S_t \geq 0.
\end{aligned} \tag{17}$$

As in the theoretical, continuous-time version of this problem, the solution is immediately seen to be at $\kappa = 0,$ because the discounted cost declines monotonically as κ increases, for any given abatement path A_t. We therefore turn our attention to the min-max regret problem. To formulate a computational version, we first define

$$f(\kappa) \equiv \min_{A_t} \left(\sum_{t=0}^{\infty} \Omega\left(A_t, H_t(\kappa), S_t\right) e^{-\rho t} \right). \tag{18}$$

Then the regret function is

$$R\left(A, H(\kappa), S, \kappa\right) \equiv \left(\sum_{t=0}^{\infty} \Omega\left(A_t, H_t(\kappa), S_t\right) e^{-\rho t} \right) - f(\kappa), \tag{19}$$

where A, $H(\kappa)$ and S respectively represent the paths of A_t, $H_t(\kappa)$ and S_t over time, and the discrete-time min-max regret model is

$$\begin{aligned}
&\min_{A_t} \max_{\kappa} R\left(A, H(\kappa), S, \kappa\right) \\
&s.t. \\
&H_{t+1} = (1+\alpha) H_t + \kappa M_\psi H_t^\phi A_t^\lambda \\
&S_{t+1} = S_t + \beta\left(E_t^0 - A_t\right) - \delta\left(S_t - S_{PRE}\right) \\
&A_t, H_t, S_t \geq 0.
\end{aligned} \tag{20}$$

For computational implementation, we discretize κ over the unit interval, with values $\kappa_1, \ldots \kappa_n$, so that the objective becomes

$$\min_{A_t} \max_{\kappa \in \{\kappa_1, \ldots, \kappa_n\}} R\left(A, H(\kappa), S, \kappa\right). \tag{21}$$

Next, we transform this to a more standard optimization form as follows:

$$\begin{aligned}
&\min_{A_t} \lambda \\
&s.t. \\
&\lambda \geq R\left(A, H^{(i)}, S, \kappa_i\right), i = 1, \ldots, n \\
&H_{t+1,i} = (1+\alpha) H_{t,i} + \kappa_i M_\psi H_{t,i}^\phi A_t^\gamma, i = 1, \ldots, n \\
&S_{t+1} = S_t + \beta\left(E_t^0 - A_t\right) - \delta\left(S_t - S_{PRE}\right) \\
&A_t, H_{t,i}, S_t \geq 0,
\end{aligned} \tag{22}$$

where $H^{(i)}$ represents the path of knowledge with the given κ_i, i.e., $\left\{H_{t,i} : t = 0, 1, \ldots\right\}$.

Numerically we replace the infinite-horizon problem by a finite but large horizon one. For example, in our numerical implementation in Section 7 below, we use 490 years as the horizon because the present value of the costs after 490 years is almost zero.

6.3 Expected cost minimization

Adopting our notation to reflect the dependence of H_t on κ, a discrete-time formulation of this version of the model is

$$
\begin{aligned}
&\min_{A_t} \mathbf{E}_\kappa \sum_{t=0}^{\infty} \left(\Omega\left(A_t, H_t\left(\kappa\right), S_t \right) \right) e^{-\rho t} \\
&s.t. \\
&H_{t+1}\left(\kappa\right) = \left(1+\alpha\right) H_t\left(\kappa\right) + \kappa M_\psi H_t\left(\kappa\right)^\phi A_t^\gamma \\
&S_{t+1} = S_t + \beta\left(E_t^0 - A_t\right) - \delta\left(S_t - S_{PRE}\right) \\
&A_t, H_t, S_t \geq 0.
\end{aligned} \tag{23}
$$

Assuming that the distribution of κ is continuous, this model can be solved using an integration rule such as Simpson's:

$$
\begin{aligned}
&\min_{A_t} \sum_{i=1}^{n} w_i \left(\sum_{t=0}^{\infty} \Omega\left(A_t, H_t\left(\kappa\right), S_t \right) e^{-\rho t} \right) \\
&s.t. \\
&H_{t+1,i} = \left(1+\alpha\right) H_{t,i} + \kappa_i M_\psi H_{t,i}^\phi A_t^\gamma \\
&S_{t+1} = S_t + \beta\left(E_t^0 - A_t\right) - \delta\left(S_t - S_{PRE}\right) \\
&A_t, H_{t,i}, S_t \geq 0,
\end{aligned} \tag{24}
$$

where the κ_i are the quadrature nodes and the w_i are the quadrature weights, which depend upon the prior distribution.

As presented in Section 7 below, we implemented the expected cost minimization model in two ways:

- Uniform distribution: We assumed that κ is uniform and continuous on $[0,1]$, so that the weights in equation (24) are $w_1 = w_n = h/3, w_2 = w_4 = \ldots = w_{n-1} = 4h/3$, and $w_3 = w_5 = \ldots = w_{n-2} = 2h/3$.
- Beta distribution: We assumed that the prior distribution of κ is Beta, with density

$$
f\left(x;\alpha,\beta\right) = \frac{1}{B\left(\alpha,\beta\right)} x^{\alpha-1}\left(1-x\right)^{\beta-1}, \; x \in [0,1], \tag{25}
$$

where $\alpha > 0$ and $\beta > 0$ are shape parameters and $B\left(\alpha,\beta\right)$ is the Beta function,

$$
B\left(\alpha,\beta\right) = \frac{\left(\alpha-1\right)!\left(\beta-1\right)!}{\left(\alpha+\beta-1\right)!}. \tag{26}
$$

In this case, the mean of κ is $1/\left(1+\beta/\alpha\right)$.

Because it may be time consuming to solve the model (24) for large-size problems, the following deterministic model, in which κ is assigned its mean value $\bar{\kappa}$, is an example of an often-used alternative way to estimate the solution. We note, however, that our numerical results

show that the resulting approximation to the solution of the expected cost minimization problem may be very coarse in this case:

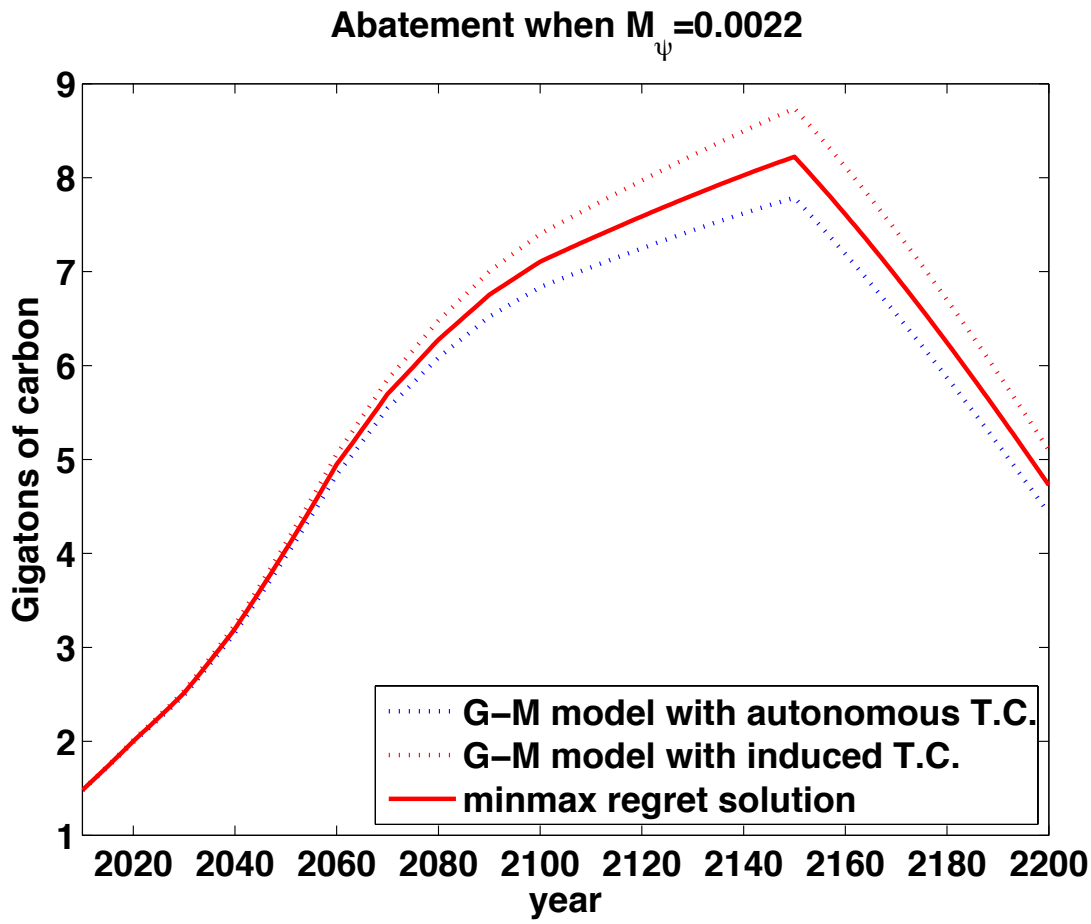
$$
\begin{aligned}
&\min_{A_t} \sum_{t=0}^{\infty} \left(\Omega\left(A_t, H_t, S_t \right) \right) e^{-\rho t} \\
&s.t. \\
&H_{t+1} = \left(1+\alpha\right) H_t + \bar{\kappa} M_{\psi} H_t^{\phi} A_t^{\gamma} \\
&S_{t+1} = S_t + \beta\left(E_t^0 - A_t \right) - \delta\left(S_t - S_{PRE} \right) \\
&A_t, H_t, S_t \geq 0.
\end{aligned}
\tag{27}
$$

7 Numerical results

For all of our computational modeling, we use annual time steps from year 2010 to year 2500. We recall that the baseline emissions abatement scenario is "Representative Concentration Pathway (RCP) 8.5" (Meinshausen et al. 2011a-b). We use CONOPT to solve the models. For the model (24), we use Simpson's rule with n = 101 equally-spaced quadrature nodes over $[0,1]$, i.e., $\kappa_i = \left(i-1\right)h$ for i = 1,…101, with $h = 1/\left(n-1\right)$.

Figure 1 presents the basic comparison of abatement paths in the solutions of the G-M model with autonomous and induced technical change, respectively, and with the min-max regret criterion. It shows that the latter decision rule in a sense "balances" between the two polar technical change assumptions.

Figure 1. Abatement paths in computational G-M model with induced and autonomous technical change (minimum cost), and with min-max regret criterion



This initial comparison raises several questions. Is the approximate symmetry of the outcome – i.e., with the min-max regret solution roughly "midway" between the two standard solutions – a consequence of the decision rule, the particular characteristics of the model, or some combination? How is the "spread" of abatement paths – that is, the difference between the induced and the autonomous cases, respectively – related to our assumptions? Finally, per our discussion above, how does the non-Bayesian solution compare with a conventional expected cost-minimization solution?

In our next set of results, we expand our set of comparisons in two ways to address these issues. First, we solve the model for additional specifications of the knowledge production function – specifically, recalling that

$$H_{t+1} = (1+\alpha)H_t + \kappa M_\psi H_t^\phi A_t^\gamma, \tag{28}$$

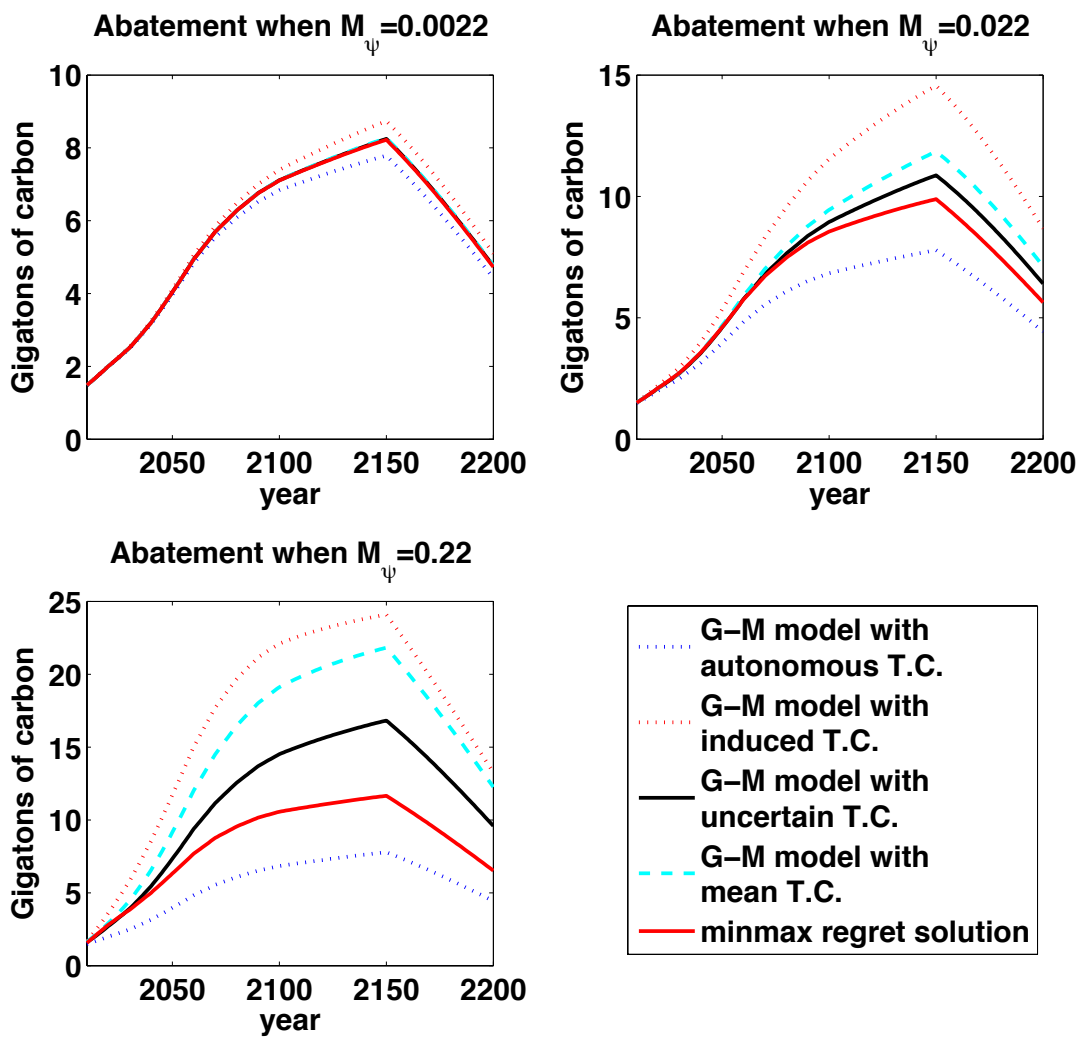
and that $M_\Psi = 0.0022$ in the results displayed in Figure 1, we also solve for $M_\Psi = 0.022$ and $M_\Psi = 0.22$. For any magnitude of $\kappa > 0$, these higher values imply a great relative contribution of induced technical change relative to autonomous in determining the level of the knowledge

stock H_t in each period. The induced, autonomous, and min-max regret solutions are computed for each of these three values.

Second, we also now solve the expected cost minimization version of the model assuming a uniform distribution for κ as described in Section 6.2. For this case also, we solve for the three values of M_Ψ, as well as for the model with κ fixed at its mean value.

The results are shown in Figure 2. In the first panel, we see that the min-max regret, expected cost, and minimum cost with κ set to its mean value essentially coincide exactly. The second and third panels show that increasing the value of M_Ψ not only results in substantial divergence of abatement paths overall, but also an overall increase in their average magnitude. Moreover, it is interesting to note that the min-max regret solution with higher values of M_Ψ becomes relatively "conservative" – i.e., lower in magnitude than all of the other cases except autonomous technical change.

Figure 2. Abatement paths in computational G-M model – varying M_ψ with induced and autonomous technical change (minimum cost), min-max regret criterion, and expected minimum cost with uniform prior on κ



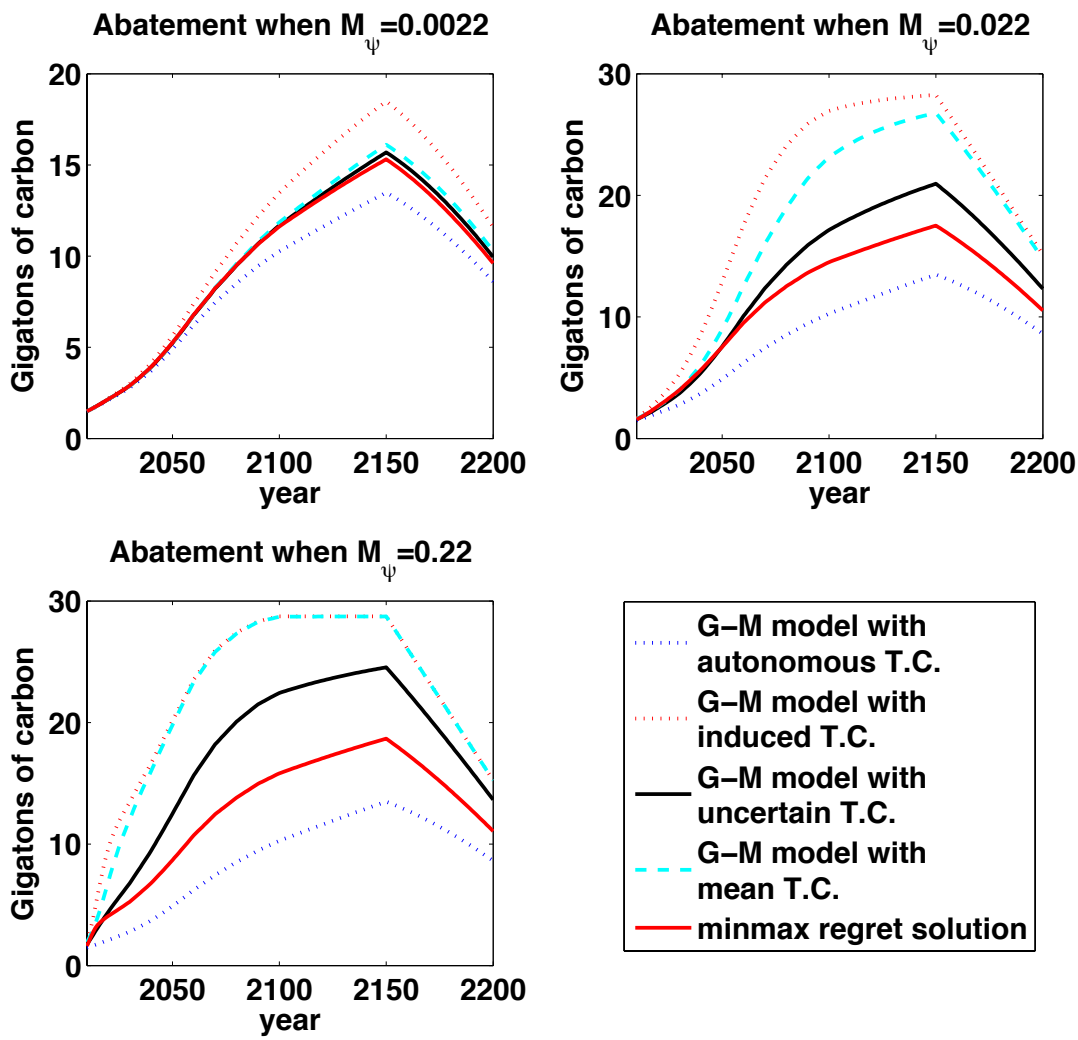
To further explore the interactions between the structure of the model and the solution concept, we next conduct the same set of computations but with increased curvature in the cost function – specifically, with the knowledge stock H_t raised to the 4th (rather than the 1st) power:

$$C\left(A_t, H_t\right) = M_C \frac{A_t^{\alpha_{C_1}}}{\left(E_t^0 - A_t\right)^{\alpha_{C_2}}} \frac{1}{H_t^4}. \tag{29}$$

The results are shown in Figure 3. Again we see a divergence among paths as M_ψ increases, as well as an even greater increase in the overall magnitudes of abatement – that is, increasing the curvature of the cost function raises not only min-max regret abatement, but also optimal

abatement in every case. It is also interesting to note that the abatement paths for induced and mean technical change, respectively, are essentially the same for the highest value of M_Ψ.

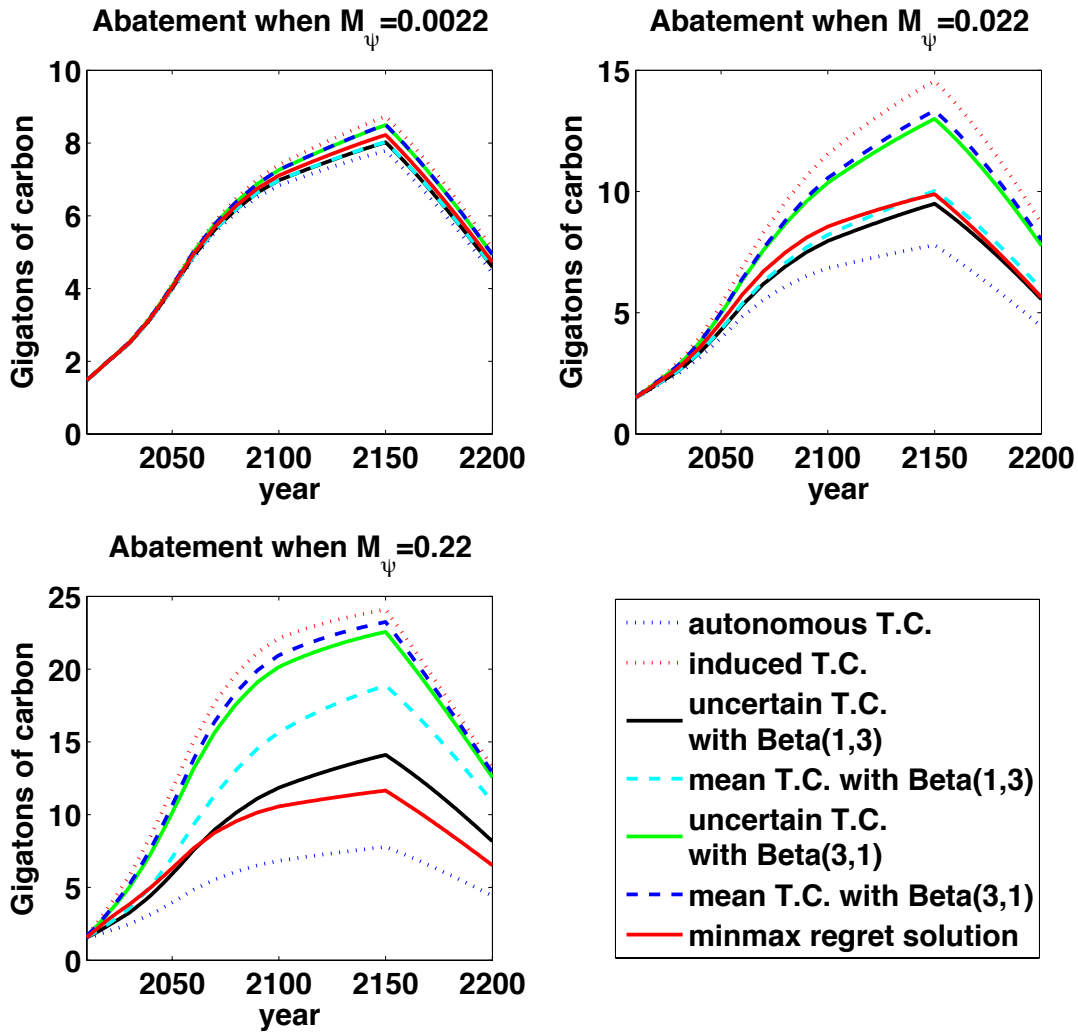
Figure 3. Abatement paths in computational G-M model – "quartic" H_t in cost function, and varying M_Ψ with induced and autonomous technical change (minimum cost), min-max regret criterion, and expected minimum cost with uniform prior on κ



In our final set of computations, we revert to the original cost function (with H_t entering linearly), but explore the effect of changing the prior distribution on κ, from uniform to Beta (as described in Section 6), with two different combinations of values for the shape parameters. The results are shown in Figure 4. Overall, the magnitudes of the abatement paths are comparable to those shown in Figure 2 with a uniform prior. It is interesting to note that in the $M_\Psi = 0.022$ case, the min-max regret solution exceeds both the optimal (minimum expected cost) and mean

solutions with Beta(1,3) until the maximum point of each, then crosses below the mean, but not the optimal, solution.

Figure 4. Abatement paths in computational G-M model – linear H_t in cost function, and varying M_ψ with induced and autonomous technical change (minimum cost), min-max regret criterion, and expected minimum cost with two instances of Beta prior on κ



8 Conclusion

The results we have presented demonstrate the insights available from using a non-Bayesian, robust decision-making approach to the technological change problem; in the context of the remarks at the beginning of Section 5, above, it shows that interpreting the technical change problem in terms of model uncertainty in principle enables a policy-maker to formally

incorporate the assumptions of both autonomous and induced technical change in an integrated manner, without having to overcome the difficult problem of assigning numerical prior probabilities to these alternative representations.

As in many other fields of application, computational modeling has become an indispensable analytical methodology for energy and environmental policy. The development and widespread adoption of energy modeling over the past several decades accompanied by the attenuation of serious, sustained work on model validation, however, has arguably given rise to fundamental epistemological issues, with significant implications for model-based policy analysis, that have yet to be systematically addressed. The emergence of multiple models of implicitly equal validity, without formal guidance to policy-makers regarding their joint application, is an important example. If this state-of-affairs reflects the persistence of irreducible uncertainties in our understanding of the energy system and its relationship to the economy, then developing methods to enable policy makers to rationally deal with it is a high priority. The pioneering work of macroeconomists on model uncertainty provides a compelling starting point for such an effort. The work described in this paper is only a modest first step, but we hope to have demonstrated both the importance and the feasibility of bringing the macroeconomists' insights and techniques to bear on energy modeling.

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Appendix

We pointed out in the text that the basic G-M model, while seemingly straightforward, is of a particular type that does not appear to have been previously analyzed as such: An infinite-horizon, not-necessarily-convex, parametric optimal control problem with pure control and state constraints. In this appendix we briefly note some of the technical points that would be entailed in a more rigorous treatment of this problem, and mention some background relevant to the min-max and min-max regret problems specifically.

To re-state: The basic model is

$$\begin{aligned}
&\min_{A_t} \int_0^\infty \left[C\left(A_t, H_t\right) + D\left(S_t\right) \right] e^{-\rho t} dt \\
&s.t. \\
&\tfrac{d}{dt} H_t = \alpha H_t + \kappa \Psi\left(H_t, A_t\right) \\
&\tfrac{d}{dt} S_t = -\varepsilon S_t + E_t^0 - A_t \\
&H_0, S_0 \ \ given \\
&A_t, H_t, S_t \geq 0.
\end{aligned}$$

To ensure that this problem is convex, one would first assume that the integrand $C\left(A_t, H_t\right) + D\left(S_t\right)$ is strictly convex, which in addition to the previously stated assumptions requires that $\partial^2 C \big/ \partial H_t^2 > 0$ and $\partial^2 C \big/ \partial H_t^2 \, \partial^2 C \big/ \partial A_t^2 > \left(\partial^2 C \big/ \partial A_t \partial H_t \right)^2$. One would also assume that the function defining the dynamics of H_t is strictly concave, which requires that $\Psi\left(H_t, A_t\right)$ be strictly concave. We also note that the assumptions on α, $\Psi\left(\bullet\right)$ and H_0 imply that the non-negativity constraint on H_t will never be binding, so that this constraint can be omitted from the specification of the model. It is also possible to ensure that the pure state constraint on the carbon stock S_t is never binding, as follows. Integrating the equation-of-motion yields

$$S\left(t\right) = \exp\left(-\varepsilon t\right) \int_0^t \exp\left(\varepsilon s\right) \left(E_s^0 - A_s\right) ds + S_0,$$

so that $S\left(t\right) > 0$ for all t if and only if

$$\int_0^t \exp\left(\varepsilon s\right) A_s ds < \int_0^t \exp\left(\varepsilon s\right) E_s^0 ds + \exp\left(\varepsilon t\right) S_0$$

for all t. Thus, it is convenient to restrict the space of admissible abatement trajectories to lie within the space of positive time paths $\left\{A_t\right\}$ satisfying this inequality.

We noted in the text that because $\partial C / \partial H_t < 0$ and it is assumed that there are no costs associated with the learning effect, the presence of ITC - represented by $\kappa > 0$ - decreases the cost of abatement. The magnitude as well as the sign of the cost reduction resulting from ITC (relative to the autonomous technical change case of $\kappa = 0$) can be inferred by applying the dynamic envelope theorem (LaFrance and Barney 1991): If the minimum cost (with initial time 0) as a function of the initial conditions and κ is denoted as $V\left(H_0, S_0, \kappa, 0\right)$, then that result implies that

$$\frac{\partial V\left(H_0, S_0, \kappa, 0\right)}{\partial \kappa} = -\int_0^\infty \lambda_t \Psi\left(H_t, A_t\right) dt,$$

where λ_t is the co-state variable for the equation-of-motion for the state variable H_t.

The analysis of min-max solutions to optimal control problems has a long history; for example, Salmon (1968) and Papageorgiou and Yannakakis (2002) are early, and more recent, examples, respectively. The min-max regret problem in an optimal control setting appears to not have been studied *per se*; however, the literatures on sensitivity analysis and sub-optimality in optimal control dating back to the 1960s contain insights and techniques that may be relevant to this problem. For example, Rekasius (1964) and Durbeck (1965) proposed Lyapunov-type techniques for estimating bounds on sub-optimal solutions using the optimal-value function, anticipating more recent work in the area of "dissipation inequalities." Witsenhausen (1970) is an interesting attempt to develop a general theory relating uncertainty and sub-optimality. These two topics were also jointly analyzed by Wierzbicki (1984), in another general theoretical treatment, as well as by Ronge (1985).

About RDCEP

The Center brings together experts in economics, physical sciences, energy technologies, law, computational mathematics, statistics, and computer science to undertake a series of tightly connected research programs aimed at improving the computational models needed to evaluate climate and energy policies, and to make robust decisions based on outcomes.

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