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Essays on Economic Modeling of Climate Change

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Cover picture: Gustav's desk.

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Abstract

This thesis consists of three self-contained essays dealing with different aspects of the economics of climate change.

Structural change in a two-sector model of the climate and the economy

This paper introduces issues concerning substitution possibilities among goods into a two-sector macroeconomic growth model where emissions from fossil fuels give rise to a climate externality. Substitution possibilities are modeled using a constant elasticity of substitution (CES) production function where the intermediate inputs differ only in their technologies and the way they are affected by the climate externality. By solving the social planners problem and characterizing the competitive equilibrium I am able to derive a simple formula for optimal taxes and resource allocation over time. The impact of different assumptions regarding the elasticity of substitution on taxes turns out to be a simple function of the size or relative magnitude of the distribution parameter of the CES function, technology and the impact of the climate externality. In particular, it is shown that a higher (lower) elasticity of substitution will result in a higher (lower) optimal unit tax rate *if and only if* the distribution parameter of the most productive sector, multiplied by its total factor productivity and climate damage function, is smaller (larger) than the corresponding term of the other sector. I also present some numerical simulations for a calibrated model based on the U.S. and Indian economy. The results show that the assumptions regarding substitution possibilities plays a much bigger role for optimal fossil fuel consumption in the agriculturally intense Indian economy.

Energy Balance Climate Models and General Equilibrium Optimal Mitigation Policies

In a general equilibrium model of the world economy, we develop a one-dimensional energy balance climate model with heat diffusion and anthropogenic forcing across latitudes driven by global fossil fuel use. This introduces an endogenous latitude dependent temperature function, driving spatial characteristics, in terms of location dependent damages resulting from local temperature anomalies into the standard climate-economy framework. We solve the social planner's problem and characterize the competitive equilibrium for three separate cases differentiated by the degree of market integration and assumptions regarding costs of transfers. We define optimal taxes on fossil fuel use and how they can implement the planning solution. Our results suggest that if the implementation of international transfers across latitudes are not possible or costly, then

optimal taxes are in general spatially non-homogeneous and may be lower at poorer latitudes. The degree of spatial differentiation of optimal taxes depend on heat transportation. By employing the properties of the spatial model, we show by numerical simulations how the impact of thermal transport across latitudes on welfare can be studied.

Energy Balance Climate Models, Damage Reservoirs and the Time Profile of Climate Change Policy

We explore optimal mitigation policies through the lens of a latitude dependent energy balance climate model, featuring an endogenously driven polar ice cap. We associate the movement of the polar ice cap with the idea of a damage reservoir being a finite source of climate related damages affecting the economy only to the extent that there still exists some ice left to melt. We capture this idea by coupling this model with a simple economic growth model. We show that the endogenous ice line characteristic of our energy balance climate model induces a nonlinearity in the model. This nonlinearity when combined with two sources of damages - the conventional damages due to temperature increase and the reservoir damages - generates multiple steady states and Skiba points. When introducing these concepts into the fairly mainstream DICE model of William Nordhaus it is shown that the policy ramp implied by the model calls for high mitigation now. The simulation results further suggests that the policy ramp could be u-shaped instead of a the monotonically increasing gradualistic policy ramp.

Assessing Sustainable Development in a DICE World

This paper investigates a method for assessing sustainable development under climate change in the Dynamic Integrated model of Climate and the Economy (DICE-2007 model). The analysis shows that the results, with respect to sustainability are highly sensitive to the calibration of the social welfare function. When revising the social welfare parameters of the DICE-2007 model to the alternative parametrization approach, used in the DICE-(1994,1999) model, it is only the former that upholds a sustainable productive base. This finding implies that when recalibrating the social welfare function, to match historical rates of return on capital, this can result in inconsistent projections of future social welfare. The robustness of these results are investigated by imposing uncertainty, regarding key parameter estimates. This shows that the social welfare parameters along with total factor productivity growth are much more important as determinants of productive base sustainability than climatic parameters such as the damage or temperature sensitivity coefficients.

Acknowledgments

First and foremost, I would like to express my deepest gratitude to the people at the Beijer Institute of Ecological Economics, in particular, Carl Folke and Anne-Sophie Crépin (co-supervisor). Without their belief in me this thesis would never have been written. Anne-Sophie has provided me with great support and guidance and served as both a professional and private mentor in all types of matters.

The Beijer Institute is a fantastic environment to do research in, with an openness and warmth unheard of. I will never forget my first days at the office. Prior to my arrival I had only had a chance to interact with a Anne-Sophie, Therese and Ingela, mainly through email correspondence concerning the course on Ecological Economics, which I was attending at a distance. On the morning, of my first day Christina and Agneta kindly showed me around the building and where I would be working. After that it didn't take long before my first, what I would like to call, "Beijer moment", occured. This happened when Åsa stumbled into the office. She was wearing a black tank top, a pair of "don't mess with me" type leather boots, hair colored in different shades of blonde/blue and with a piece of saran plastic wrapping around her right shoulder. She was very friendly and talking at about a hundred miles an hour when I finally after 10-20 minutes or so, got around to asking here about the plastic around her shoulder. She responded that she had just been at the tatoo parlour and recieved this awesome tatoo featuring a "morbid angle" that I would get to see in a couple days, when the plastic could be removed. About this time Calle "The Director" came by and presented himself. He was a well dressed guy, wearing nice and tidy clothes, fitting for a director of an international research institute. He was super friendly and insured me several times that he was extremely pleased that I had joined the work force. Åsa, full of energy, was quick to let him know that she had just acquired this amazing piece of ink on her shoulder and insured him that tattoo guy was one of the best in Stockholm and if he ever wanted to get a tattoo she would gladly give him his number, and me too for that matter. The rest of the day continued somewhat in this manner and I can't remember getting much work done. In sum, my first day at Beijer was a heart warming experience with a lot of laughter and I quickly got the feeling that this place did not draw a sharp line between work and private life and that there was an overall acceptance for the strange and unconventional. This is something that I am now glad and proud to be apart of and which I can also still confirm to be true.

Concerning research, my first month at Beijer also became an extraordinary experience. Calle invited me to participate in the annual meeting on the island of Askö which was held in conjunction with the Beijer board meeting. The Askö meeting has become known as an great example of a successful transdiciplinary research, where top scholars from around the world meet once a year to discuss a pressing topic related to global environmental problems. The researchers invited vary, but the year I participated, the guest list included, Kenneth Arrow, a Nobel prize winner in economics, which had made several important contributions to a variety of different fields including almost all of the chapters in the horrible (but also excellent) book by Mas-Colell, Whinston and

Green which strikes fear in the eyes of most graduate students during their first compulsory courses in Micro-economics. The amazing thing about this meeting was that apart from getting to listen in on the fantastic discussions taking place in the meeting rooms, the trip also resulted in me becoming a co-author of the article "Looming global scale failures and missing institutions" which was published in Science Magazine, one of the most prestigious scientific journals on the planet. This was an amazingly inspiring experience and even more the fact that only a couple of years prior to this experience I had been working a low level maintainence job at a stone-crushing factory in Södra Sandby (Skåne) ...

The story of how I got from Stockholm to Skåne and back again, starts with me getting rejected to board the Stockholm to Riga ferry, where I was traveling together with my friends Kalle and Björn in the pursuit of a business venture involving Kayaks and Sunglasses. The staff refused to let me on board the ferry due to the fact that I had forgotten to bring my passport with me. This is a long story, but to make it short it was this slight miscalculation on my part that took me on a journey towards central Skåne, ultimately altering my trajectory in life. In particular, it was my experiences there that several years later caused me to resume my studies in economics (which I had dismissed a year or so before). After several years of trial and error type endeavours, I was thus working the night shift at the stone crushing factory under the supervision of Hillman, an extraordinary man that had spent most his life servicing the factory and thus knew basically everything that one needed to know about it. Meanwhile I was living on a farm, together with Johan, close to Ekeby Möbelvaruhus, with no running drinking water or waste disposal system. We enjoyed a lot of good times at the farm and it was a wonderful way to connect with the beautiful nature and landscape of Skåne. However, regardless of the wonderful friendship and good times I enjoyed with Johan at the farm and Pekka on the factory rooftop, I still felt I wanted to do something else with my life. I decided to turn it all around. In the fall of 2004 it was back to economics but this time at Lund University (my prior studies had been in Stockholm and Uppsala). I finished my Masters in 2005 and joined the ever so big crowd of unemployed economist's. After several months of job rejections I finally got an offer at the department of economics at Lund University. This came as a quite a surprise. My prior years of undergraduate studies, in which I had been much concerned with social gatherings at local university pubs etc., had not surprisingly reflected on my grade levels. However, thanks to the great support of my Master thesis supervisor, Hossein Asgharian, and a much exaggerated self-confidence I managed to convince the entire Professor's Colleague that I wasn't actually as stupid as my grades made me out to be. It felt great I had managed to get the University to pay my salary for the next 4-5 years and as long as I passed at least a few of the courses they couldn't kick me out immediately. I was on a roll!

After a couple of years in Lund my environmental interest forced me to take on, what I believe to be the hardest challenge that the Economic's profession is currently facing: appropriately accounting for nature's role in economic and human development. Carl Folke has a nice way of describing what this interaction is all about by picturing three circles of different sizes. The first "outer" circle consists of the biosphere (life on Earth or sum of all ecosystems); inside this circle we then have an inner circle describing human societies existing as a subset of the biosphere; finally within this social circle we have an innermost circle called the economy. This describes a fundamental relationship between nature and economic development by making it clear that when trying to understand the complex web of life within which we live, these three circles are not merely separate spheres interacting with each other but rather a hierarchy of processes where the inner circles cannot exist without properly functioning (or existing) outer circles. That is without a biosphere there is no economy! Despite its self-evident simplicity this relationship is ever to often forgotten when we are putting together models of the interaction between the economy and the natural environment.

My interest in environmental issues became the main reason for why I decided to move back to Stockholm. The Beijer had granted me the possibility to do research as a part of their institute. Being part of the Beijer Institute is definitely a way to kick-start a career. As I already mentioned the Askö meetings gave me an overwhelming start with the Science paper. However, this is also the place to which I owe half of my thesis. Prior to my second Askö meeting Anastasios Xepapadeas (Tasos) had become aware that I was working with the DICE model by Nordhaus. He and William (Buz) Brock were looking to do some simulations w.r.t. thresholds in this model and asked me if I could help them with this. I managed to successfully complete the simulations before the meeting and after having discussed the results at Askö they decided to make me a co-author of the paper they were writing. The final paper came together a half a year or so later and is now part of this thesis and presented in chapter 4. The collaboration continued and has to date resulted in a total of three papers of which two have become part of this thesis.[1] I am thus deeply indebted to both Buz and Tasos for believing in me and taking on a young and naive scholar like myself. It has been a great privilege to work with them and I have learned so much, from the intense email correspondence we have had over the years.

Another person which deserves his own paragraph here is Dieter Grass. Without Dieter the beautiful graphs of chapter 4 (figure 4.1 and 4.2) would never have seen the light of day. Dieter became without exaggerating my own "private" teacher in Optimal Control Theory. He spent a year or so as a guest researcher at the Beijer Institute where he mainly worked on developing his optimal control toolbox for Matlab. During this time I became his test subject in order to determine whether the toolbox could be run by mere mortals as myself.[2] Dieter has been a great friend and I am happy to see him back here in September.

After a year or so at Beijer I completed my Licentiate Exam, still being officially a

[1]The third paper with Buz and Taso is presented in chapter 3. There is also another paper with the title "Solow meets Lovelock: the economics of Daisyworld" authored by myself and Tasos, which however did not fit within the context of this thesis.

[2]We also have a paper "Poverty traps and economic growth in a two-sector model of subsistence agriculture" from this time that also did not make it into the thesis. For a copy see the Beijer discussion paper series.

graduate student at Lund University. Tommy Andersson, who became my supervisor at Lund, provided me with excellent support in completing my Lic. In the meantime I got accepted at the department of economics at Stockholm University. As my main research interest involved macro economic models of climate change I eventually got involved with the IIES which during recent years have been making great progress in this arena.

After having seen David present at the institute and listening in on the harsh but constructive critic he was receiving from John Hassler and Per Krusell, I felt that this was the kind of atmosphere I needed to face in order to prepare properly for my upcoming dissertation. I thus asked John to supervise me, which he kindly accepted and informed me that this involved taking part in their seminar series for macro grad students. Before joining this group my experience from seminars was fairly painless. Presenting at the macro seminars was completely different. My first presentation resulted in a dire slaughter of the paper I was trying to present. I don't believe I reached much further than slide number five in my presentation. This may seem like a cruel process but it has really helped me in shaping my arguments and I now feel much better prepared for an international research arena. Hence, I want to express my graduate to both John and Per for providing me with several valuable comments over the years and in particular for helping me understand the importance of being absolutely clear with what you are trying to do and what purpose you have with your research.[3]

Finally, trying to finish these acknowledgments is a hard task since there is a lot of people that deserved to be mentioned and I have a tendency to forget.

Therese, I couldn't have received a better roommate. Even though I am perfectly comfortable with everybody at Beijer I would never trade you for someone else ;)

Johan G, we have had some great and fulfilling research discussions and I am thankful that you encouraged me to go discrete. You are also one of the best listeners and with your remarkable sense of working the models in your head makes, this makes it hard to resist the temptation of stepping into your office when mathematical problems gather on the horizon. Thanks for all the helpful discussions!

Li, we have also had some great discussions from which I have learnt a lot. I look forward to working closer together with you during the coming years!

Stephan, thanks for all the great experiences we have gone through together. Most of them have been outside the office just having a good time but research discussions have always been a common denominator. More of this in the future!

Åsa and Johan C, you are my super awesome office neighbors. I love having you both there although I get more work done when its just one at a time :). Also, Åsa thanks for listening I hope I have done the same for you.

Agneta and Christina, you guys are great! Thanks for taking care of me over the years! I hope I am not placing to much of a burden on you...

On the non-work related side there is also a lot of people worth mentioning. I spent six years in Skåne. Björn brought me down there and taught me the black-smith trade

[3]Also, thanks to John for his explanation of what a steady state is... ;)

and introduced me to a bunch of great people including Danne, Jens, Johan x 2, Holma, Max, Muck, Pekka, Paddan and a lot more that I haven't gotten to know as much as I should have. Thanks for all the good times! I will try to visit for at least a week every year despite what happens!

Kalle, we have known each other a long time and had some amazing adventures together. We never thought we would make it past 27 but here we are so I suggest we keep going a while longer...

Ted, we both have vague memories of GC, Kvarnen and more recently sailing. In particular the sailing trips have been important for completing this thesis. Without you, life would have been much more boring especially since I never would have met Gunnar, Juck and Mattis!

CF, you never gave up on our friendship despite that I disappeared for several years at a time without as much as a phone call. It will not happen again! You also introduced me to some great people since I've come back to Stockholm including Magnus, Helena, Romeo... Also I know Nils is still around even though we barely hear from him!

Olle, it just strikes me as strange that we spent nearly two years in Lund without ever picking up the guitar and that we started playing first after moving back to Stockholm. Anyhow, playing music with you is an awesome and inspiring experience! You have also grown to become a close friend and I know Jonas sees us as two brothers harassing each other all the time. Jonas and Olle, I will always try to make time for some musical evenings so you two can get a drink once in a while!

Mom, Dad, Hanna and Jon, Thanks for all the support over the years! I am very greatful despite that I many times don't show my appreciation enough. I am very glad for all you have done for me! I also want to say, "Jag saknar Tuva!!!"...

Sanna, thanks for being my source of joy, comfort and alternative perspectives on life! I'm not sure I could have done this without you...

Finally, I hope that I have covered most of the people that deserve to be mentioned, but if you feel that we have been close friends and you have been left out, then to you I can only say that my close friends know that I have a terrible memory and would never do that on purpose.

To Sanna and my family

Stockholm, August, 2012

Gustav Engström

Contents

Chapter 1

Introduction

The thesis consists of four independent essays on issues dedicated to the economic modeling of climate change. Modeling the climate-economy interaction has become a popular topic during the last decade and this thesis is far from the only one addressing it. Much of the pioneering work on this issue is usually attributed to William Nordhaus who wrote his first article on the matter back in 1977.[1] Since, then an increasing amount of evidence has been gathering on the detrimental impacts from human induced global warming of the planet, and many of the very top economist's have devoted themselves to the subject.[2] The background to economic modeling of climate change rests on several theoretical insights which has been acquired over the past century which are important for understanding the overarching approach. This introduction aims at providing a brief overview of the field and how the articles of my thesis are related to it.

A safe and habitable climate system is a great example of what an economist will usually refer to as a global public good. Public goods are commodities for which the cost of extending the consumption of the good to another individual is zero, while the cost of excluding an individual from consumption is infinite. These conditions are usually referred to as non-rivalry and non-excludability. The climate system of the Earth is characterized by non-rivalry because if we manage to create a stable climate system for at least a few people, then there is no extra cost to letting more people benefit from enjoying the same climate. Likewise, the climate system is also characterized by non-excludability since it will in general not be possible to exclude any individual from reaping the benefits, once a stable climate system has been obtained. The basic problem with public goods and what sets them apart from private goods, such as e.g. food, shoes or clothing, is that markets are generally not capable of producing the good efficiently. By efficient production we mean that for some given amount of production of private goods, the provision of the public good is at its maximum. This means that

[1] See Nordhaus (1977).

[2] Among these several Nobel laureates are to be found e.g. Ellinor Ostrom, Paul Krugman, Joseph Stiglitz and Kenneth Arrow to mention a few. It should also be noted that dealing with climate change is only one of several global environmental challenges that faces humanity. See e.g. Walker *et al.* (2009); Rockström *et al.* (2010) for a discussion on these issues.

in the absence of production efficiency, for a fixed amount of the privately produced goods it is possible to reallocate the factors of production so as to produce more of the public good without reducing the amount of private goods in production.[3]

Another way to think about the climate change problem is as a result of externalities coming from the production of private goods. By externality we mean an unforeseen or unintended consequence of an economic activity experienced by a third party. An externality can be positive, if the third party enjoys benefits from the activity, or negative, if it involves unintended costs. Climate change is in this terminology regarded as a negative externality that arises mainly from production activities involving the use fossil fuels.[4] The climate-economy feedback loop can be described as follows. The burning of fossil fuels, causes a release of carbon dioxide into the atmosphere, accumulating into a total stock of excessive carbon dioxide content, which dissipates evenly throughout the Earth's atmosphere.[5] The total stock of excessive carbon dioxide causes a perturbation in the energy balance of the Earth system thru the reduction in the amount of ougoing long wave radiation leaving the Earth's atmosphere.[6] This perturbation is what is usually proxied by an increase in the global average temperature. The result of this increase in the Eartht́s energy budget is expected to have several severe consequences for human well-being, with e.g. dissruptions in local weather patterns and increases in the amount and variations of extreme weather events such as draughts, floods, storm frequencies, heat waves, etc. spread unevenly across the planet.[7] These unintended impacts completes the climate-economy feedback loop since they will come back and haunt any producer still residing on planet Earth. The negative externality arises since all residents regardless of their contribution to the stock of atmospheric carbon dioxide will bear the costs associated with the changing climate. In economic jargon this is usually referred to as a market failure.

Economic theory has provided much insight into how externalities can be dealt with. The main theoretical insight was first clearly formulated by Pigou (1920) stating that a market failure which is due to negative environmental externalities can be alleviated by an appropriately designed tax scheme where polluters are forced to pay a tax which internalizes the social impacts of their production activities. When all polluters are taxed at an appropriate rate this should cause a reduction in the amount of emissions so that the marginal private costs of production equal the marginal social costs of pollution. In terms of the previous discussion on public goods, this achieves production efficiency in the sense that the reduction in emissions is achieved at a minimum of social

[3]For an excellent exposition of global public good dilemma's see Barett (2007).

[4]Fossil fuel use is not the only reason for climate change. Another major contributor is land-use change e.g. conversion of forest into agricultural land which is estimated to cause a net release of carbon dioxide of approximately $1.6GtC$ a year.

[5]Atmospheric carbon dioxide has a relatively slow decay rate. Archer (2005) estimates that 75% of an excess atmospheric carbon concentration has a mean lifetime of 300 year and the remaining 25% stays there forever.

[6]That is the balance between the amount of incoming short wave radiation and outgoing long wave radiation. See e.g. Trenberth *et al.* (2009) for an introduction.

[7]See e.g. Smith *et al.* (2009) for an update of the current expected impacts.

costs. Since the works of Arthur Pigou much has been accomplished in terms of insights on different types of taxation schemes, and alternatives such as e.g. tradeable emission quota systems.

Based on this short theoretical background I now turn to back to the main topic of the thesis, the economic modeling of climate change. As was previously hinted early models of the climate economy dates back to the early 1970's when the problem at hand started to receive an increasing amount of attention from climate scientists.[8] The early models developed by e.g. William Nordhaus had their basis in economic growth theory featuring what is usually referred to as a neoclassical growth model. These models typically feature the problem of the representative consumer trying to maximize his/her discounted lifetime utility of consumption over time, while renting out endowments of capital and labor to a profit maximizing firm that utilize these as production factors in the process of creating more goods which can be used for either consumption or investment. The extension of these models to include climate change, resulting from the process of economic growth implied that in order to avoid market failure a benevolent social planner had to set taxes so as to internalize the externality caused by the emission producing firms. These models later became known as integrated assessment models (IAMs), since they integrated the scientific knowledge inherent in two academically distinctive fields. The literature on IAMs has since then grown considerably with several well known models which are typically made available to the scientific community for independent simulation/modification.[9] The policy recommendations coming from IAMs have recieved much attention both in the media and academic community. One of the most well known and debated models followed as part of the Stern (2007) review. The review was critized based on its assumption regarding how future consumption streams had been discounted which seemed to be the main driver behind the stringent optimal mitigation policies that followed. Other models that have recieved a lot of attention, at least in the academic community, are the DICE/RICE-models of William Nordhaus. These models have also not passed without criticism. For example, much criticism has been raised with respect to e.g. the market based approach to discounting, the failure to appropriately account for the value and vulnerabilities of ecosystems and the downplaying of scientific uncertainty when it comes to climate damages and the possibilities of catastrophic events (Ackerman *et al.*, 2009).

Coming back to the articles of this thesis and how they relate to the short background provided so far, I now will discuss them in their order of presentation. The first article, deals with issues concerning the substitutability among different goods in the

[8]The climate-economy models introduced in chapter 3 and 4 are both derived based on work from this period by e.g. North (1975a,b). However, at the time global warming was not always seen as the main issue of concern. On the contrary, many writers at the time were concerned with another phenomena that arose from the development of these models. This concerned the existence an possibility of shifting into an alternative stable state featuring a completely ice-covered planet. This climatic state was coined "Snowball Earth". See e.g. Pierrehumbert (2008).

[9]See e.g. the MERGE model (Manne *et al.*, 1995), FUND model (Tol, 1997), DICE model (Nordhaus, 2007) or PAGE model (Hope, 2006).

economy. This is an old topic in economics and relates to how we value different type of goods depending on their relative scarcity. The paper was inspired by the writings of Hoel and Sterner (2007); Sterner and Persson (2008). These papers show that if environmental goods are complements to ordinary consumption goods then if the environment is expected to degrade due to e.g. climate change, while ordinary consumption good production is expected to rise then an optimal policy given the consumers preferences will force a more stringent emission policy than if the two goods had been valued as substitutes. This logic was used in the article by Sterner and Persson (2008) to force a much more stringent optimal emission policy out of the DICE model.

The paper introduced in **chapter 2**, likewise focuses on issues concerning substitutability among goods by extending the models of Hoel and Sterner (2007); Sterner and Persson (2008) to fit within a two-sector macroeconomic growth model with explicit modeling of emissions from fossil fuels which give give rise to the climate externality. The two sectors consist of agricultural and non-agricultural activities. The underlying assumption is that the agricultural sector may provide us with a proxy for the more abstract environmental good of Hoel and Sterner (2007); Sterner and Persson (2008). Further, since agricultural production is more environmentally dependent than e.g. car manufacturing this sector may also suffer harder following developments of climate change.

The motivation for this extension comes from the fact that Sterner and Persson (2008) was not able to deliver an empirical justification for their choice of model parameters based on data. Modern macroeconomics is on the contrary firmly seated in the data and chapter 2 thus seeks to develop a model which can be empirical justified, as a way of making the findings of Hoel and Sterner (2007); Sterner and Persson (2008) more accessible/acceptable to the general macro crowd.

Concerning the technicalities, substitution is modeled using a constant elasticity of substitution (CES) production function where the intermediate inputs differ only in their technologies and the way they are affected by the climate externality. By solving the social planners problem and characterizing the competitive equilibrium I derive a simple formula for optimal taxes and resource allocation over time. The impact of different assumptions regarding the elasticity of substitution on taxes turns out to be a simple function of the size or relative magnitude of the distribution parameter of the CES function, technology and the impact of the climate externality. In particular, it is shown that a higher (lower) elasticity of substitution will result in a higher (lower) optimal unit tax rate *if and only if* the distribution parameter of the most productive sector, multiplied by its total factor productivity and climate damage function, is smaller (larger) than the corresponding term of the other sector. I also present some numerical simulations for a calibrated model based on the U.S. and Indian economy. The results show that the assumptions regarding substitution possibilities plays a much bigger role for optimal fossil fuel consumption in the agriculturally intense Indian economy.

Turn now to the papers presented in **chapter 3** and **chapter 4**. These papers are both co-authored with William A. Brock and Anastasios Xepapadeas. These papers both take a different approach to climate-economy modeling by asking; What can be learnt by taking as a starting point the climate models developed by climate scientists and coupling these to fit with standard models of economic growth? This is different from the statistical models used in e.g. Nordhaus (2007) which consists of a set of difference equations calibrated to match the predictions of larger climate models. On the contrary, the models underlying chapter 3 and 4 are instead based on basic physical laws, governing the balance between the amount of incoming solar radiation and outgoing heat wave radiation, which by human intervention is affected through a change in the atmospheric composition of gases. These models provide us with the possibility to consider alternative endogenous processes such as e.g.the melting of polar ice caps which induces a positive feedback process on the earth system through a decrease in the local albedo (reflectivity) may create further warming. These possibilities are not readily available in statistical models. The physical models I am referring to here are typically known as energy balance climate models (EBCMs) and are usually attributed to the independent works of Budyko (1969) and Sellers (1969). These models can typically be characterized in terms of three over-arching categories, the zero-, one- and two-dimensional model. The number of dimensions of these dynamic models concern the amount of spatial dimensions. The zero-dimensional model is thus a simple model of the global average energy balance since it contains no explicit spatial dimension. The one-dimensional model on the other hand, contains an explicit spatial dimension in terms of a latitude dependent temperature function where the energy content at any given latitude is determined not only by the amount of incoming and outgoing radiation but also how energy diffuses across latitudes. Finally, the two-dimensional model is a spatial model with heat diffusion across both latitudes and longitudes so as to create local average temperatures at each point on the surface of the Earth.[10] In both chapter 3 and 4 we develop climate-economy models based on one-dimensional EBCMs. The introduction of a explicit spatial dimensions featuring a latitude dependent temperature function is to our knowledge new within the climate-economy modeling literature. The typical approach has previously instead been to proxy local temperature related damages to the economy as a direct function of global temperature averages. We believe this approximation might blur some of the interactions between the natural mechanisms related to temperature change and economic mechanisms related to e.g. local production characteristics. Hopefully a more spatially explicit model may be able to shed increased light on these issues.

Finally, it should be mentioned that the purpose of these modeling attempts conducted in both chapter 3 and 4 should be seen as a first pass at uncovering the value added of the integrating economic models with EBCMs. The next step for more a realistic model is of course to move to the more general two-dimensional models EBCMs. Based on our work so far we believe this to be both a feasible and tractable possibility for future research.

[10]EBCMs have also been developed to include seasonal variations in temperatures. See e.g. North *et al.* (1981).

Turn now to the specific contents of the two chapters. Starting with **chapter 3**. In this paper we develop a general equilibrium model of the world economy, featuring a one-dimensional EBCM with heat diffusion and anthropogenic forcing across latitudes driven by global fossil fuel use. As previously described, this introduces an endogenous latitude dependent temperature function, driving spatial characteristics, in terms of location dependent damages resulting from local temperature anomalies into the standard climate-economy framework. We solve the social planner's problem and characterize the competitive equilibrium properties for three separate cases differentiated by the degree of market integration and assumptions regarding costs of transfers. We define optimal taxes on fossil fuel use and how they can implement the planning solution. Our results suggest that if the implementation of international transfers across latitudes are not possible or costly, then optimal taxes are in general spatially non-homogeneous and may be lower at poorer latitudes. The degree of spatial differentiation of optimal taxes depend on heat transportation. By employing the properties of the spatial model, we show by numerical simulations how the impact of thermal transport across latitudes on welfare can be studied.

In **chapter 4** we instead focus more on the processes regarding the movement of an endogenous polar ice cap. We associate this movement with the uncovering of damage reservoirs. Damage reservoirs in the context of climate change can be regarded as sources of damage which eventually will cease to exist when the source of the damage has been depleted. We identify, ice caps and permafrost as typical damage reservoirs, where the state of the reservoir is connected to the latitudinal position of the ice line.[11] We capture this idea by coupling a one-dimensional EBCM with a simple economic growth model. We show that the endogenous ice line characteristic of our energy balance climate model induces a nonlinearity in the model. This nonlinearity when combined with two sources of damages, the conventional damages due to temperature increase and damages ue to the movement of the ice line, may generate multiple steady states and Skiba points which qualitatively changes the behavior of optimal mitigation policies. When introducing these concepts into the fairly mainstream DICE model of William Nordhaus it is shown that the policy ramp implied by the model calls for high mitigation now. The simulation results further suggests that the policy ramp could be u-shaped instead of a the monotonically increasing gradualistic policy ramp which is common feature in many IAMs.

Finally, turning to the **last chapter** of this thesis. The motivation for this paper came out of the literature on national accounting. In particular, many writers on this topic are concerned that GNP is inadequate in the sense that it fails to capture several important aspects related to wealth and economic development. A richer measurement would at the minimum, account for the depreciation of the capital resources in the economy. Dasgupta (2009) notes two particular points of importance here: (1) GNP does not measure wealth. GNP is a flow concept, whereas wealth is a stock.

[11] The ice line is defined as the latitudinal position where one moves between the polar and non-polar region. The polar region is defined as a region featuring average temperatures below approximately -10 degree Celsius (North *et al.*, 1981).

(2) Although it has become a commonplace to regard GNP as a welfare index, it is an aggregate measure of the output of final goods and services, nothing more. There are several alternatives to GNP which have been developed over the years. One of these relates to the idea of measuring the comprehensive wealth of an economy. This involves measuring the change in the value of an economies stock of assets, through the use of shadow prices, which are the projected value of the assets in relationship to other assets of the economy over time. The articles by Hamilton and Clemens (1999), Dasgupta and Mäler (2000) and Arrow *et al.* (2003) cited in chapter 5, provide a thorough analysis of how this measure can be derived. In particular, the later two focus on how the theoretical background, in the face of non-convexities, exogenous trends in e.g. population or technology and under situations when non-optimal decision paths are being pursued for whatever the reason. Further, they show how this measure also is related to the concept of sustainable development. A famous report by the World Commission on Environment and Devolopement (1987) defined sustainable development as "... development that meets the needs of the present without compromising the ability of future generations to meet their own needs". In the light of this statement, sustainable development requires that each generation relative to their populations should leave to their children at least as large an overall productive base (i.e. the base upon which all economic activity depends) as it had itself inherited. This implies that future generations will have at least the same possibilities to generate welfare as the current. The articles by Hamilton and Clemens (1999), Dasgupta and Mäler (2000) and Arrow *et al.* (2003) all show that if the national accounts were redefined to measure changes in comprehensive wealth instead of GNP this would also, given the appropriate definitions and measurements, constitute an index of sustainable development.

The last paper of the thesis takes the theoretical insights from these papers into account in an assessment of sustainability within the DICE-2007 model. The analysis shows that the results, with respect to sustainability are highly sensitive to the calibration of the social welfare function. When revising the social welfare parameters of the DICE-2007 model to the alternative parametrization approach, used in the DICE-(1994,1999) model, it is only the former that upholds a sustainable productive base. This finding implies that when recalibrating the social welfare function, to match historical rates of return on capital, this can result in inconsistent projections of future social welfare. The robustness of these results are investigated by imposing uncertainty, regarding key parameter estimates. This shows that the social welfare parameters along with total factor productivity growth are relatively more important as determinants of productive base sustainability than climatic parameters such as the damage or temperature sensitivity coefficients.

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Chapter 2

Structural and climatic change in a two-sector model of the global economy

2.1 Introduction

In undergraduate courses in economics we learn to classify goods as being either substitutes or complements in order to determine their effect on demand, supply and prices in the market place. When goods are complements an increased scarcity in one good will increase its price relative to other goods and vice versa. In macro economics these differences among goods or factor inputs has shown to be of importance in explaining trends in the movements of capital and labor across different sectors of the economy over time (structural change). This approach to modeling structural change was originally proposed by Baumol (1967) and has recently been developed further by e.g. Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). These later studies show that both productivity and capital intensity differences among sectors can help explain the post-industrial flow of capital and labor from the agricultural sector into the manufacturing and service sectors.[1]

Within environmental economics these properties has also been receiving attention. Recent studies by (Hoel and Sterner, 2007; Sterner and Persson, 2008), shows that when assumptions regarding perfect substitutability between goods are relaxed this can have potentially large effects on optimal mitigation policies in climate economy models. Standard, economic models featuring a climate externality typically ignore these effects. An implicit assumption embedded in these aggregate models is thus that both consumption goods and intermediate inputs to production are perfect substitutes.[2] The paper by Sterner and Persson (2008) experimented with the well-known DICE model developed

[1]Acemoglu and Guerrieri (2008) do not attempt to explain the flow of capital and labor from agriculture into manufacturing, however later unpublished work by Lin and Xu (2011) show that this could have been explained within the context of their model.

[2]Examples of such aggregate models can be found in a recent review by Stanton *et al.* (2009).

by (Nordhaus, 2007), showing that if an alternative environmental good is introduced into this model this can result in a dramatic shift in the optimal emission policy. This was done by replacing the representative consumption good of the DICE model with a composite good consisting of an environmental good and a manufactured good, which are weighted together using a constant elasticity of substitution (CES) utility function. The two intermediate goods were further assumed to be complements in the utility function implying a elasticity of substitution below unity. Since the investment decisions resulting from model simulation implied that the manufactured good was growing over time while the environmental good was becoming increasingly scarce the uneven growth rates together with a CES smaller than one, lead to a rising relative price of the environmental good. The result of this imbalance among growth rates was thus an increase in cost of climate change and hence a more stringent emission policy.[3]

In this paper I continue this line of research exploring how assumptions regarding substitutability among input factors might affect mitigation policies by developing a two-sector general equilibrium model of the climate-economy interaction. The purpose of this exercise is to extend the insights of (Hoel and Sterner, 2007; Sterner and Persson, 2008) into a more tractable general equilibrium model of the macro economy. I differ from them in three important aspects; First, I have replaced the environmental good by an agricultural sector. This is an important first step in making the model more accessible to macro economic researchers since it allows for calibration of the model based on actual macroeconomic data. The alteration of the DICE model by (Sterner and Persson, 2008) differs here since their definition of the environmental good is a much more abstract and difficult concept to find empirical data on. Further, I believe that the agricultural sector can work as a good proxy since this sector is highly dependent upon the surrounding environment such as temperature and precipitation. Second, I allow for endogenous and free mobility of resources between the two sectors. By doing so I follow in the tradition of a vast literature on multi-sector growth models. Within the climate-economy framework considered here this assumption also has a useful interpretation in terms of adaptation costs to climate change. Here, we can think of resources flowing into the most heavily damaged sector as the opportunity cost of mitigation, implying that there exists a trade off between mitigation and adaptation decisions within the model. Finally, I model substitution decisions as a supply side phenomena i.e. I look at substitution among intermediate inputs in final output. This is perhaps more of a technical aspect that increases the analytical tractability of our model. However, as will be shown the equations governing structural change are identical to those of Ngai and Pissarides (2007) when the climate externality is ignored. Further, to my knowledge this has yet not been applied within a climate-economy model.

The model developed here draws upon work by Acemoglu and Guerrieri (2008) which highlights a supply side reason for structural change based on the thesis presented by Baumol (1967). They develop a two-sector model, with a constant elasticity of sub-

[3]Weitzman (2010) shows that under their specific assumptions regarding the elasticity of substitution this specification becomes equivalent to introducing an additive damage function affecting utility directly.

stitution and show that if either capital shares or productivity differs between the two sectors structural change will take place. Further, if the two sectors are complements in production then this implies that resources will be allocated towards the smaller of the two sectors. Our paper also draws upon work by Golosov *et al.* (2011) which develop a stochastic dynamic general-equilibrium of the climate and the economy. They show that given four specific assumptions i) logarithmic utility, ii) climate damages being proportional to output iii) the stock of atmospheric carbon dioxide grows linearly in emissions and iv) a constant saving rate, it is possible to derive a simple formula for the marginal externality cost from the emissions of carbon dioxide. These assumptions also turn out to be particularly useful for deriving analytical results in our two-sector setting.

The numerical section of this paper concludes with a simple calibration and simulation exercise. Here, I calibrate and simulate the model based on data from the U.S. and Indian economy separately. Already in the seminal article by Arrow *et al.* (1961) it was pointed out that systematic inter-sectoral differences in the elasticity of substitution and income elasticities of demand, imply the possibility that the process of economic development itself might shift the over-all elasticity of substitution. It has also for a long time been a well recognized stylized fact that as a country moves out of poverty and economic growth starts to take of, the relative economic importance of the agricultural sector starts to decline (see e.g. Timmer (2009)). Hence, since these two economies differ to a great extent in the size of their agricultural sector calibrating to their respective observed economies shows off some important differences that can prove to be of relevance when considering global optimal emission policies from the perspective of different nations or economic systems. The results show that the optimal global emission policy from the perspective of the Indian economy exhibits a more stringent emission path and is more sensitive to changes in substitution possibilities than the corresponding U.S. economy.

This paper is structured as follows. Section 2.2 introduces the general features of the model, derives the planner and corresponding competitive equilibrium solution. Section 2.3 calibrates and solves the model numerically. Section 2.4 concludes.

2.2 A two-sector model of the climate-economy interaction

In this section the general setting and description of the planning problem and competitive equilibrium of the two-sector model is introduced. The model I develop here is a discrete time version of the model developed in Acemoglu and Guerrieri (2008) extending it to include a climate externality and fossil fuel use. In order to get the analytical results derived in this paper I will make some specific assumptions that although not completely implausible still might be regarded as overly stylized. The reason for this is related to the purpose of this paper which is to clarify the mechanism played by the elasticity of substitution in determining optimal fossil fuel use and taxes within a macroeconomic growth model. Finally, I work out the solution to the planner problem and show how this solution can be implemented in a decentralized setting given an externality correcting taxation policy.

2.2.1 Model description

The objective function of a representative household in the economy is given by

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \tag{2.1}$$

where U is a standard concave the utility function function, C consumption and $\beta \in (0,1)$ is the discount factor.

The economy produces a unique final good which can be thought of as an aggregate/composite good consisting of the two intermediate goods

$$Y_t = \left(w_m Y_{mt}^{(\epsilon-1)/\epsilon} + w_a Y_{at}^{(\epsilon-1)/\epsilon} \right)^{\epsilon/(\epsilon-1)} \tag{2.2}$$

having a elasticity of substitution $\epsilon \in [0,\infty)$ and a distribution parameters (sectoral weights) $w_m \geq 0; w_{at} \geq 0$ and $w_m + w_a = 1$.[4] The economy is thus divided into two sectors. First, the agricultural sector Y_{at} is a proxy for all types of food related production activities. Second, manufacturing production Y_{mt} refers to all other types of production activities that do not fit into agricultural production (i.e. everything else). Both production technologies employ standard production factors such as capital K, labor L and energy E. Production functions are further assumed to be of Cobb-Douglas type with differing technological trends A_{at} and A_{mt}. Finally both sectors are also assumed to be affected differently by climate change in a multiplicative fashion.[5]

$$Y_{at} = \Omega_a(S_t) A_{at} K_{at}^{\alpha_1^a} L_{at}^{\alpha_2^a} E_{at}^{\alpha_3^a} \tag{2.3}$$

$$Y_{mt} = \Omega_m(S_t) A_{mt} K_{mt}^{\alpha_1^m} L_{mt}^{\alpha_2^m} E_{mt}^{\alpha_3^m} \tag{2.4}$$

where $\Omega_i(S_t) \in [0,1]$, A_{it}, K_{it}, L_{it} and E_{it} are the damage function associated with atmospheric carbon dioxide concentration $S_t > 0$, technological growth, capital, labor and energy use in each sector $i = \{a, m\}$ respectively. Note, that the damage function I consider here is a direct function of the atmospheric carbon dioxide stock meaning that I have surpassed several possibly important dynamical relationships such as for example ocean heating etc common in many integrated assessment models. Golosov *et al.* (2011) argue that this is a reasonable assumption given the available intermediate complexity models used in natural sciences. Although, I do not aim to take a stand here this reduced complexity makes it easier to understand the forces of driving the

[4] For the cobb-douglas case where $\epsilon = 1$ these distribution parameters can be interpreted as the income shares of the intermediate goods in final good production.

[5] Hassler *et al.* (2011) point out that, on shorter time horizons, Cobb-Douglas production does not represent a good way of modeling energy demand since it does not capture the joint shorter- to medium- run movements of input prices and input shares. However, on longer time scale we consider here it is more reasonable since input shares do not appear to trend over time.

model we consider here.[6] Further, I will throughout this paper assume that damages are always increasing in the atmospheric carbon stock i.e. $\Omega_i'(S_t) < 0$.

Finally, the economy's budget constraint in final good production is

$$K_{t+1} + C_t = Y_t + (1-\delta)K_t \tag{2.5}$$

where the left hand side denotes next periods resource use (capital and consumption) while the right hand side denotes production and depreciation of capital.

Regarding fossil fuel use dynamics let R_t denote the stock of remaining fossil fuel at the beginning of time period t, where $R_0 > 0$ is given, and $E_t \geq 0$ denotes the total amount of extracted fossil fuel by the two sectors.

$$R_{t+1} - R_t = -E_t,\ R(0) = R_0 > 0 \tag{2.6}$$

the following resource constraint thus applies:

$$R_0 \geq \sum_{t=0}^{\infty} E_t \tag{2.7}$$

Capital, Labor and Energy can be allocated costlessly across both sectors. Market clearing thus requires that

$$K_t = K_{at} + K_{mt} \tag{2.8a}$$
$$L_t = L_{at} + L_{mt} \tag{2.8b}$$
$$E_t = E_{mt} + E_{at} \tag{2.8c}$$

Finally, I let S_t denote the stock of carbon dioxide emitted after the pre-industrial period and assume the following simple structure for the carbon cycle.

$$S_{t+1} = (1-\varphi)S_t + \xi E_t \tag{2.9}$$

This equation is a much simplified expression for the behavior of anthropogenic induced CO_2 emissions following early work on climate economy models (see e.g. Nordhaus (1994)) where φ captures the rate of removal of CO_2 from the atmosphere and ξ the airborne fraction of carbon dioxide emissions. Removal might be due to for example uptake by oceans or the terrestrial biosphere. This is a rather simple an crude way of capturing carbon storage which ignores several possibly important dynamical relationships present in for example Nordhaus and Boyer (2000). However, for the purposes of the present paper these dynamics serve us well as a simplified representation. Further, as a reference Golosov *et al.* (2011) argue that increased complexity of the three box carbon cycle used by Nordhaus is quite well approximated by a simple one-dimensional lag structure.

[6]In the numerical section of this paper I will make a simple logarthmic transformation from carbon dioxide to temperature units found in e.g. IPCC (2001).

2.2.2 The Planning problem

Based on the formulations described above I can now form a social planner problem and characterize a solution. The planner problem becomes

$$\max_{\{K_{t+1},R_{t+1},S_{t+1}E_t,C_t,K_{mt},K_{at},L_{at},L_{mt},E_{mt},E_{at}\}} \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{2.10}$$

$$\text{subject to (2.2), (2.3) (2.4), (2.5), (2.6), (2.8a), (2.8b), (2.8c), (2.9)} \tag{2.11}$$

Inspection of the social planner problem reveals that this maximization problem can be broken down into two parts. First, given the state variables K_t, R_t and S_t the allocation of factors across the two sectors becomes an intratemporal problem of maximizing the aggregate output Y_t in each time period. Second, given this choice of factor allocation in each time period the time path of C_t and E_t can be chosen so as to maximize the value of the objective function. These two parts thus corresponds to the solutions of the static and dynamic maximization problems. I start by characterizing the static equilibrium.

Static equilibrium

As mentioned previously, in order to obtain a tractable model in terms of analytical results I will have to make some rather specific assumptions. The first assumption relates to the factor input shares within the two sectors

Assumption 2.1. $\alpha_j^a = \alpha_j^m \equiv \alpha_j, \ \ for \ \ j = \{1, 2, 3\}$

This assumption is crucial in order to obtain the analytical results derived below. I deviate in this respect from the model derived in Acemoglu and Guerrieri (2008) which relies on differing input shares generating sectoral reallocations. However, as will be seen this assumption serves us well as a baseline case and will help us flesh out the mechanisms that are driving our results. Based on this assumption it is clear that optimal resource allocation will require that the marginal products of capital labor and energy are equalized:

$$\begin{aligned}
w_m \alpha_1 \left(\frac{Y_t}{Y_{mt}}\right)^{\frac{1}{\epsilon}} \frac{Y_{mt}}{K_{mt}} &= w_a \alpha_1 \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\epsilon}} \frac{Y_{at}}{K_{at}} \\
w_m \alpha_2 \left(\frac{Y_t}{Y_{mt}}\right)^{\frac{1}{\epsilon}} \frac{Y_{mt}}{L_{mt}} &= w_a \alpha_2 \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\epsilon}} \frac{Y_{at}}{L_{at}} \\
w_m \alpha_3 \left(\frac{Y_t}{Y_{mt}}\right)^{\frac{1}{\epsilon}} \frac{Y_{mt}}{E_{mt}} &= w_a \alpha_3 \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\epsilon}} \frac{Y_{at}}{E_{at}}
\end{aligned} \tag{2.12}$$

Based on these equations I can solve for the optimal capital, labor and energy shares allocated to each sector. This is allocation is given by the following proposition.

Proposition 2.1. *Assuming equal sectoral factor shares and constant returns to scale the intratemporal factor allocation is determined by*

$$\Psi(S_t) = \left(\frac{w_a}{w_m}\right)^{\epsilon} \left(\frac{\Omega_a(S_t)}{\Omega^m(S_t)} \frac{A_{at}}{A_{mt}}\right)^{\epsilon-1}$$

where $\Psi(S_t) \equiv \frac{K_{at}}{K_{mt}} = \frac{L_{at}}{L_{mt}} = \frac{E_{at}}{E_{mt}}$

Proof. see appendix □

The following corollary also follows immediately from the above proposition

Corollary 2.1. *If* $w_a \Omega_a A_a < w_m \Omega_m A_m$ *at some point in time a higher value for the elasticity of substitution* ϵ *would allocate more resources to the manufacturing sector (*$\Psi(S_t)$ *decreases) and vice versa.*

Proof. Applying the envelope theorem I have

$$\frac{d\Psi(S_t)}{d\epsilon} = \left(\frac{w_a}{w_m}\right)^{\epsilon} \left(\frac{\Omega_a A_a}{\Omega_m A_m}\right)^{\epsilon-1} \ln\left(\frac{w_a}{w_m} \frac{\Omega_a A_a}{\Omega_m A_m}\right) \tag{2.13}$$

which is negative *iff* $w_a \Omega_a A_a < w_m \Omega_m A_m$. □

The above proposition and the following corollary gives us an important heads up regarding how resources will be allocated within the two sectors. Depending on the size of ϵ the sectoral ratio of relative damages to total factor productivity multiplied by the distribution parameter will determine the direction of resource flow. Consider first the case when $\epsilon < 1$ so that the two intermediate goods are complements in production. Then the agricultural sector will be relatively larger if and only if $w_a \Omega_a A_a < w_m \Omega_m A_m$ and smaller if and only if $w_a \Omega_a A_a > w_m \Omega_m A_m$. Further, the total productivity of the two sectors will be determined by the terms $\Omega_a A_a$ and $\Omega_m A_m$, which are both endogenous and time dependent, implying that they will determine which of these two sectors is expanding and which is contracting with time.[7] Making use of proposition 2.1 the final goods production can now be substantially simplified. Together with the market clearing conditions I can now write the final goods production function as

$$\tilde{Y}_t = \Gamma_t(S_t) K_t^{\alpha_1} L_t^{\alpha_2} E_t^{\alpha_3} \tag{2.14}$$

where

$$\Gamma_t(S_t) = \left(w_m \Gamma_{mt}^{(\epsilon-1)/\epsilon} + w_a \Gamma_{at}^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)} \tag{2.15}$$

[7] It is interesting to see that the expression of relative income shares proposition 2.1 corresponds, with exception of the damage function, precisely to the the ratio of consumption expenditure on a consumption good to consumption expenditure on the manufacturing (capital building) good given by equation (10) in Ngai and Pissarides (2007). The add on here is the climate externality that heterogeneously effects productivity within each sector.

and

$$\Gamma_{at} = \Omega_a(S_t)A_{at}\left(\frac{\Psi(S_t)}{1+\Psi(S_t)}\right) \tag{2.16}$$

$$\Gamma_{mt} = \Omega^m(S_t)A_{mt}\left(\frac{1}{1+\Psi(S_t)}\right) \tag{2.17}$$

As can be seen from the above equations the solution to the static equilibrium will simplify the dynamic analysis greatly since final goods production is now a function of only carbondioxide S and the aggregate resource inputs $\{K, L, E\}$.

Dynamic equilibrium

In order to proceed with analytical results also in derivation of the dynamic equilibrium I will also assume that utility is logarithmic and a capital depreciation rate of a hundred percent.

Assumption 2.2. $U(C_t) = \ln(C_t),\ \delta = 1$

Logarithmic preferences is rather standard and a common assumption in many models. For example, the earlier climate economy models developed by William Nordhaus all featured logarithmic preferences (see e.g. Nordhaus (1994); Nordhaus and Boyer (2000)). As the main purpose of this paper is more qualitative than quantitative in nature I will not spend time on discussing the robustness of the results to this assumption. However, judging from the results derived in this paper and based on the work of Acemoglu and Guerrieri (2008)[8], modifying this assumption should not affect the qualitative results derived here regarding structural transformation. Golosov *et al.* (2011) also use and discuss these assumptions. They argue that in particular for longer time periods (10 years) suggests a lower curvature of the utility function.

A depreciation rate of a hundred percent is large even for a ten year period. Golosov *et al.* (2011) claim that this does not affect the results of their model remarkably. Together these assumptions are convenient in these types models since it is well known that as long as aggregate capital can be factored out of the production function the saving rate will become a constant.[9]

Given this assumption and the results derived from the static equilibrium we can now write down the lagrangian of the dynamic problem facing the social planner

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t[\ln(\tilde{Y}_t - K_{t+1}) + \lambda_{Rt}(R_t - E_t - R_{t+1}) + \lambda_{St}((1-\varphi)S_t + E_t - S_{t+1})] \tag{2.18}$$

Taking the F.O.C w.r.t. K_{t+1} we have

$$\mathcal{L}_{K_{t+1}} = -\beta_t\frac{1}{C_t} + \beta_{t+1}\frac{1}{C_{t+1}}\alpha_1\frac{\tilde{Y}_t}{K_{t+1}} = 0 \tag{2.19}$$

[8]In particular, see proposition 1 and 2 of this paper.

[9]This assumption was first used by Brock and Mirman (1972) to provide a closed-form solution in a stochastic growth setting.

which gives us

$$\frac{C_{t+1}}{C_t} = \beta\alpha_1 \frac{\tilde{Y}_{t+1}}{K_{t+1}} \tag{2.20}$$

which gives us the following consumption/investment rule:

$$C_t = (1 - \beta\alpha_1)\tilde{Y}_t \tag{2.21}$$

$$K_{t+1} = \beta\alpha_1 \tilde{Y}_t \tag{2.22}$$

From the above calculations we see that optimal investment in capital K remains a fixed fraction $\beta\alpha_1$ of manufacturing production over time. Hence it is unaffected by changes to the parameters in the instantaneous utility function such as the elasticity of substitution ϵ between the two consumption goods.

Concerning optimal fuel use I now proceed with the first order conditions w.r.t. the fossil fuel:

$$\mathcal{L}_{R_{t+1}} = -\beta^t \lambda_{Rt} + \beta^{t+1}\lambda_{Rt+1} = 0 \tag{2.23}$$

$$\mathcal{L}_{E_t} = \alpha_3 \frac{1}{C_t}\frac{\tilde{Y}_t}{E_t} - \lambda_{Rt} + \lambda_{St} = 0 \tag{2.24}$$

$$\mathcal{L}_{S_{t+1}} = \beta^{t+1} \frac{1}{C_{t+1}} \frac{\partial \Gamma_{t+1}}{\partial S_{t+1}} K_{t+1}^{\alpha_1} L_{t+1}^{\alpha_2} E_{t+1}^{\alpha_3} - \beta^t \lambda_{St} + \beta^{t+1}\lambda_{St+1}(1-\varphi) = 0 \tag{2.25}$$

From (2.25) we have:

$$\lambda_{St} = \beta \frac{1}{C_{t+1}} \frac{\partial \Gamma_{t+1}}{\partial S_{t+1}} \frac{\tilde{Y}_{t+1}}{\Gamma_{t+1}} + \beta\lambda_{St+1}(1-\varphi)$$

and thus

$$\lambda_{St} = \sum_{s=1}^{\infty} (1-\varphi)^{s-1}\beta^s \left(\frac{1}{C_{t+s}} \frac{\partial \Gamma_{t+s}}{\partial S_{t+s}} \frac{\tilde{Y}_{t+s}}{\Gamma_{t+s}} \right) + \lim_{s\to\infty} (1-\varphi)^{s-1}\beta^s \left(\frac{1}{C_{t+s}} \frac{\partial \Gamma_{t+s}}{\partial S_{t+s}} \frac{\tilde{Y}_{t+s}}{\Gamma_{t+s}} \right) \tag{2.26}$$

By the transversality condition the limiting term is zero and if we express the marginal damage cost λ_{St} in terms of present day consumption $\Lambda_{St} \equiv \lambda_{St}/U'(C_t)$ we get an expression similar to equation (12) of Golosov *et al.* (2011).

$$\Lambda_{St} = \sum_{s=1}^{\infty} (1-\varphi)^{s-1}\beta^s \left(C_t \frac{\tilde{Y}_{t+s}}{C_{t+s}} \frac{\partial \Gamma_{t+s}}{\partial S_{t+s}} \frac{1}{\Gamma_{t+s}} \right) \tag{2.27}$$

This formula is more complex than the one derived in their paper. In particular the formula does not depend on merly the saving rate but also on fossil fuel use thru the term $\frac{\partial \Gamma_{t+s}}{\partial S_{t+s}} \frac{1}{\Gamma_{t+s}}$ which I will refer to as the "relative damage term". However, as will be shown in the next section this term is not a complete black box and we can learn alot about its properties by analyzing how it is affected by changes in its parameters. Further, if we express the marginal externality costs of emissions as a proportion of GDP i.e. $\hat{\Lambda}_{St} \equiv \Lambda_{St}/\tilde{Y}_t$ and make use of the consumption rule we get a simpler expression which is independent of saving and well suited for examining the role of the elasticity of substitution for climate damages.

$$\hat{\Lambda}_{St} = \sum_{s=1}^{\infty}(1-\varphi)^{s-1}\beta^s \left(\frac{\partial \Gamma_{t+s}}{\partial S_{t+s}}\frac{1}{\Gamma_{t+s}}\right) \tag{2.28}$$

The marginal externality cost of CO_2 and the elasticity of substitution

From (2.2.2) we saw how allocation of factor inputs depended on technical and climatic change within the two sectors and the role played by the elasticity of substitution. In expression (2.28) we see that marginal climate damages also depends on the elastictity of substitution through the relative damage term $\frac{\partial \Gamma_{t+s}}{\partial S_{t+s}}\frac{1}{\Gamma_{t+s}}$. Hence the marginal externality cost of atmospheric carbon dioxide will depend upon both the substitution possibilities amongst the two sectors and how damages are spread between them. The following proposition is usefull in order to understand how this works.

Proposition 2.2. *For $0 \leq \epsilon < \infty$ the marginal externality cost of carbon dioxide per unit of GDP* (2.28) *is always negative and bounded above and below by the marginal damages within each sector.*

$$\hat{\Lambda}_{S_t}(\epsilon) \in \left[\hat{\Lambda}_{S_t}(0), \hat{\Lambda}_{S_t}(\infty)\right] \tag{2.29}$$

where

$$\hat{\Lambda}_{S_t}(0) = \sum_{s=1}^{\infty}(1-\varphi)^{s-1}\beta^s \frac{(\Omega_a A_{at})^{-1}\frac{\Omega_a'}{\Omega_a} + \Omega_m A_{mt})^{-1}\frac{\Omega_m'}{\Omega_m}}{(\Omega^m A_{mt})^{-1} + (\Omega_a A_{at})^{-1}} \tag{2.30}$$

$$\hat{\Lambda}_{S_t}(\infty) = \begin{cases} \sum_{s=1}^{\infty}(1-\varphi)^{s-1}\beta^s \frac{\Omega_a'}{\Omega_\rho} & \text{if } w_a\Omega_a A_a > w_m \Omega_m A_m \\ \sum_{s=1}^{\infty}(1-\varphi)^{s-1}\beta^s \frac{\Omega_m'}{\Omega_m} & \text{if } w_a\Omega_a A_a < w_m \Omega_m A_m \end{cases} \tag{2.31}$$

there are four cases to consider:

i) $w_a\Omega_a A_a > w_m\Omega_m A_m$ *and* $\frac{\Omega_a'}{\Omega_a} < \frac{\Omega_m'}{\Omega_m}$ *then* $\hat{\Lambda}_{S_t}(\infty) < \hat{\Lambda}_{S_t}(0) : \hat{\Lambda}_{S_t}$ *decreasing in* ϵ

ii) $w_a\Omega_a A_a > w_m\Omega_m A_m$ *and* $\frac{\Omega_a'}{\Omega_a} > \frac{\Omega_m'}{\Omega_m}$ *then* $\hat{\Lambda}_{S_t}(\infty) > \hat{\Lambda}_{S_t}(0) : \hat{\Lambda}_{S_t}$ *increasing in* ϵ

iii) $w_a\Omega_a A_a < w_m\Omega_m A_m$ *and* $\frac{\Omega_a'}{\Omega_a} < \frac{\Omega_m'}{\Omega_m}$ *then* $\hat{\Lambda}_{S_t}(\infty) > \hat{\Lambda}_{S_t}(0) : \hat{\Lambda}_{S_t}$ *increasing in* ϵ

iv) $w_a\Omega_a A_a < w_m\Omega_m A_m$ *and* $\frac{\Omega_a'}{\Omega_a} > \frac{\Omega_m'}{\Omega_m}$ *then* $\hat{\Lambda}_{S_t}(\infty) < \hat{\Lambda}_{S_t}(0) : \hat{\Lambda}_{S_t}$ *decreasing in* ϵ

Proof. The proof is derived by examining the limits of $\frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t}$ as $\epsilon \to 0$ and $\epsilon \to \infty$. First, from (2.16), (2.17) and proposition (2.1) we have

$$\Gamma_{at} = \frac{w_a^\epsilon (\Omega_a A_{at})^\epsilon}{w_m^\epsilon \left(\Omega^m A_{mt}\right)^{\epsilon-1} + w_a^\epsilon (\Omega_a A_{at})^{\epsilon-1}}$$
$$\Gamma_{mt} = \frac{w_m^\epsilon (\Omega_m A_{mt})^\epsilon}{w_m^\epsilon \left(\Omega^m A_{mt}\right)^{\epsilon-1} + w_a^\epsilon (\Omega_a A_{at})^{\epsilon-1}}$$

further we can write

$$\frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t} = \gamma_{mt}\frac{\Gamma'_{mt}}{\Gamma_{mt}} + \gamma_{at}\frac{\Gamma'_{at}}{\Gamma_{at}} \tag{2.32}$$

where

$$\gamma_m \equiv \frac{\partial \Gamma_t}{\partial \Gamma_{mt}}\frac{\Gamma_{mt}}{\Gamma_t} = \frac{w_m \Gamma_m^{\frac{\epsilon-1}{\epsilon}}}{\left(w_m \Gamma_m^{\frac{\epsilon-1}{\epsilon}} + w_a \Gamma_a^{\frac{\epsilon-1}{\epsilon}}\right)} = \frac{w_m^\epsilon (\Omega_m A_{mt})^{\epsilon-1}}{w_m^\epsilon \left(\Omega^m A_{mt}\right)^{\epsilon-1} + w_a^\epsilon (\Omega_a A_{at})^{\epsilon-1}} \tag{2.33}$$

$$\gamma_a \equiv \frac{\partial \Gamma_t}{\partial \Gamma_{at}}\frac{\Gamma_{at}}{\Gamma_t} = \frac{w_a \Gamma_a^{\frac{\epsilon-1}{\epsilon}}}{\left(w_m \Gamma_m^{\frac{\epsilon-1}{\epsilon}} + w_a \Gamma_a^{\frac{\epsilon-1}{\epsilon}}\right)} = \frac{w_a^\epsilon (\Omega_a A_{at})^{\epsilon-1}}{w_m^\epsilon \left(\Omega^m A_{mt}\right)^{\epsilon-1} + w_a^\epsilon (\Omega_a A_{at})^{\epsilon-1}} \tag{2.34}$$

further we can also derive

$$\frac{\Gamma'_{at}}{\Gamma_{at}} = \frac{\Gamma_{at}}{\Omega_a A_a}\frac{\Omega'_a}{\Omega_a} + \frac{\Gamma_{mt}}{\Omega_m A_m}\left(\epsilon\frac{\Omega'_a}{\Omega_a} - (\epsilon-1)\frac{\Omega'_m}{\Omega_m}\right) = \gamma_{at}\frac{\Omega'_a}{\Omega_a} + \gamma_{mt}\left(\epsilon\frac{\Omega'_a}{\Omega_a} - (\epsilon-1)\frac{\Omega'_m}{\Omega_m}\right) \tag{2.35}$$

$$\frac{\Gamma'_{mt}}{\Gamma_{mt}} = \frac{\Gamma_{mt}}{\Omega_m A_m}\frac{\Omega'_m}{\Omega_m} + \frac{\Gamma_{at}}{\Omega_a A_a}\left(\epsilon\frac{\Omega'_m}{\Omega_m} - (\epsilon-1)\frac{\Omega'_a}{\Omega_a}\right) = \gamma_{mt}\frac{\Omega'_m}{\Omega_m} + \gamma_{at}\left(\epsilon\frac{\Omega'_m}{\Omega_m} - (\epsilon-1)\frac{\Omega'_a}{\Omega_a}\right) \tag{2.36}$$

substituting (2.35) and (2.36) into (2.32) we have

$$\begin{aligned}\frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t} &= \gamma_{mt}\left(\gamma_{mt}\frac{\Omega'_m}{\Omega_m} + \gamma_{at}\left(\epsilon\frac{\Omega'_m}{\Omega_m} - (\epsilon-1)\frac{\Omega'_a}{\Omega_a}\right)\right) + \gamma_{at}\left(\gamma_{at}\frac{\Omega'_a}{\Omega_a} + \gamma_{mt}\left(\epsilon\frac{\Omega'_a}{\Omega_a} - (\epsilon-1)\frac{\Omega'_m}{\Omega_m}\right)\right)\\ &= \gamma_{mt}^2\frac{\Omega'_m}{\Omega_m} + \gamma_{at}^2\frac{\Omega'_a}{\Omega_a} + \gamma_{mt}\gamma_{at}\left(\frac{\Omega'_a}{\Omega_a} + \frac{\Omega'_m}{\Omega_m}\right) = \gamma_{at}\left(\gamma_{at}+\gamma_{mt}\right)\frac{\Omega'_a}{\Omega_a} + \gamma_{mt}\left(\gamma_{at}+\gamma_{mt}\right)\frac{\Omega'_m}{\Omega_m}\\ &= \gamma_{at}\frac{\Omega'_a}{\Omega_a} + \gamma_{mt}\frac{\Omega'_m}{\Omega_m}\end{aligned}$$

from our assumptions on Ω_a and Ω_m we see that $\frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t}$ is always negative implying that marginal damages (2.28) are also negative.

The limit as $\epsilon \to 0$:

Using standard limit rules for products we evaluate the γ_{at} and γ_{mt} of (2.32) seperatly.

$$\lim_{\epsilon\to 0}\gamma_{at} = \frac{(\Omega_a A_{at})^{-1}}{(\Omega^m A_{mt})^{-1} + (\Omega_a A_{at})^{-1}}$$
$$\lim_{\epsilon\to 0}\gamma_{mt} = \frac{(\Omega_m A_{mt})^{-1}}{(\Omega^m A_{mt})^{-1} + (\Omega_a A_{at})^{-1}}$$

this implies that

$$\lim_{\epsilon\to 0}\frac{\partial\Gamma_t}{\partial S_t}\frac{1}{\Gamma_t} = \frac{(\Omega_a A_{at})^{-1}\frac{\Omega_a'}{\Omega_a} + \Omega_m A_{mt})^{-1}\frac{\Omega_m'}{\Omega_m}}{(\Omega^m A_{mt})^{-1} + (\Omega_a A_{at})^{-1}} \tag{2.37}$$

The limit as $\epsilon \to \infty$:
From (2.33) and (2.34) the limits of γ_a and γ_m follow directly:

$$\lim_{\epsilon\to\infty}\gamma_m = \lim_{\epsilon\to\infty}\frac{1}{1+\frac{\Omega_m A_m}{\Omega_a A_a}\left(\frac{w_a\Omega_a A_a}{w_m\Omega_m A_m}\right)^{\epsilon}} = \begin{cases} 0 & \text{if } w_a\Omega_a A_a > w_m\Omega_m A_m \\ 1 & \text{if } w_a\Omega_a A_a < w_m\Omega_m A_m \end{cases} \tag{2.38}$$

$$\lim_{\epsilon\to\infty}\gamma_a = \lim_{\epsilon\to\infty}\frac{1}{1+\frac{\Omega_a A_a}{\Omega_m A_m}\left(\frac{w_m\Omega_m A_m}{w_a\Omega_a A_a}\right)^{\epsilon}} = \begin{cases} 1 & \text{if } w_a\Omega_a A_a > w_m\Omega_m A_m \\ 0 & \text{if } w_a\Omega_a A_a < w_m\Omega_m A_m \end{cases} \tag{2.39}$$

hence we can conclude that

$$\lim_{\epsilon\to\infty}\frac{\partial\Gamma_t}{\partial S_t}\frac{1}{\Gamma_t} = \begin{cases} \frac{\Omega_a'}{\Omega_a} & \text{if } w_a\Omega_a A_a > w_m\Omega_m A_m \\ \frac{\Omega_m'}{\Omega_m} & \text{if } w_a\Omega_a A_a < w_m\Omega_m A_m \end{cases} \tag{2.40}$$

Finally, to prove that these limits of ϵ also bind the marginal damages (2.28) between their limiting values we make use of the implicit function theorem and take the derivative w.r.t. ϵ.

$$\frac{d\frac{\partial\Gamma_t}{\partial S_t}\frac{1}{\Gamma_t}}{d\epsilon} = \frac{{w_m}^e(\Omega_m A_m)^{e+1}{w_a}^e(\Omega_a A_a)^{e+1}}{({w_m}^e(\Omega_m A_m)^e(\Omega_a A_a) + {w_a}^e(\Omega_a A_a)^e(\Omega_m A_m))^2}\ln\left(\frac{w_a\Omega_a A_a}{w_m\Omega_m A_m}\right)\left(\frac{\Omega_a'}{\Omega_a} - \frac{\Omega_m'}{\Omega_m}\right)$$

From this comparative static it is straightforward to see that $\frac{\partial\Gamma_t}{\partial S_t}\frac{1}{\Gamma_t}$ is monotonically increasing or decreasing in epsilon depending on the sign of both $\left(\frac{\Omega_a'}{\Omega_a} - \frac{\Omega_m'}{\Omega_m}\right)$ and $\ln\left(\frac{w_a\Omega_a A_a}{w_m\Omega_m A_m}\right)$ which gives the four case i-iv. □ □

The main intuition behind Proposition (2.2) can be attained by examining figure (2.1). The figure plots the marginal damage term $\frac{\partial\Gamma_t}{\partial S_t}\frac{1}{\Gamma_t}$ for different values of ϵ and w_m. Each line going from the flat horizontal line in the middle of the figure to the more s-shaped line going from -2×10^{-3} to -1×10^{-3} has a different value for the elasticity of substitution ϵ. Here the horizontal line has $\epsilon = 0$ while the most s-shaped line has an $\epsilon = 4$. The figure was plotted assuming a constant $A_{mt} = A_{at} = 1$ and $S_t = 583$. The damage functions were given an exponential form $\Omega_m = e^{-\theta_m S_t}$ and $\Omega_a = e^{-\theta_a S_t}$ with $\theta_m = 0.001$ and $\theta_a = 0.002$. I am thus assuming that a temperature increase will have a larger impact on the agricultural sector.

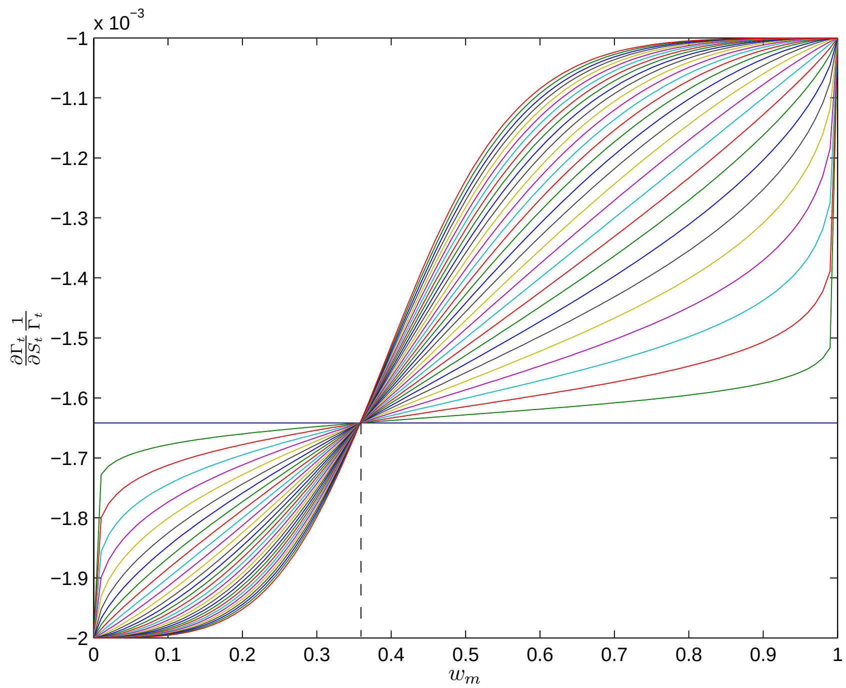

Figure 2.1: Behaviour of the relative damage term $(\frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t})$ for different values of ϵ and w_m. Flatter lines depict small values of ϵ while increasing s-shaped correspond to higher values of ϵ. The figure was plotted assuming a constant $A_{mt} = A_{at} = 1$ and $S_t = 583$. The damage functions were given an exponential form $\Omega_m = e^{-\theta_m S_t}$ and $\Omega_a = e^{-\theta_a S_t}$ with $\theta_m = 0.001$ and $\theta_a = 0.002$.

The figure confirms the results of Proposition (2.2). For an elasticity of substitution equal to zero the marginal damage term is merely a weighted average of the marginal damages within each sector which is independent of the distribution parameter. However, as ϵ increases the marginal damage term moves towards the boundaries discussed in Proposition (2.2) where the direction depends on the conditions $i - iv$. Numerical examination of the behavior when $\epsilon \to \infty$ shows that the steepness of the s-shape increases with an increasing ϵ with a threshold located about the dashed vertical line.

Concerning the interpretation, there are two cases to consider. First, consider the case where w_m lies in the region to the right of the dashed vertical line in figure 2.1. In this case we can see from the figure that for each fixed value of w_m the relative damage term increases with higher values of ϵ. Hence, judging from proposition (2.2) we can conclude that this should correspond to the case where $w_a\Omega_a A_a < w_m\Omega_m A_m$. Further, by proposition (2.1) we also see that in this case resources are moving in the direction of the manufacturing sector and since a large value of ϵ corresponds to a high degree of substitutability this simply means that we are allocating more resources to the sector which suffers the least by climate change.

The second case corresponds to small values of w_m lying to the right of the dashed line. Here, the small value of w_m gives little weight to the manufacturing sector. This implies that when substitutability increases ($\epsilon \uparrow$) the majority of resources are allocated to the agricultural sector since it despite damages makes a greater contribution to overall production. However, because damages are larger in this sector this implies that the marginal damage term $\frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t}$ also becomes larger. Hence, similar to the results obtained by Ngai and Pissarides (2007), it is not only the size of the elasticity of substitution that matters but also the size of the distribution parameter of the CES function that determines the direction of resource allocation and hence as shown here, the externality cost of carbon dioxide.

Finally, it is interesting to see what happens in the cobb-douglas case when $\epsilon \to 1$. In this case it is straightforward to see that $\gamma_m = w_m$ and $\gamma_a = w_a$ implying that the limit follows directly

$$\lim_{\epsilon \to 1} \frac{\partial \Gamma_t}{\partial S_t}\frac{1}{\Gamma_t} = w_a \frac{\Omega_a'}{\Omega_a} + w_m \frac{\Omega_m'}{\Omega_m}$$

Hence we see that the marginal damage term becomes a linear function of the distribution parameters implying that if a sector is valued more in final goods production a higher vulnerability to climate change in this sector also implies an overall higher marginal damage.

Optimal fossil-fuel use

Next we turn to the optimality conditions (2.23) and (2.24) which together become

$$\beta^t \left(\alpha_3 \frac{1}{C_t}\frac{\tilde{Y}_t}{E_t} + \lambda_{St} \right) = \beta^{t+1} \left(\alpha_3 \frac{1}{C_{t+1}}\frac{\tilde{Y}_{t+1}}{E_{t+1}} + \lambda_{St+1} \right) \tag{2.41}$$

Rewriting this expression and making use of (2.20) and (2.27) we have

$$\alpha_1 \frac{\tilde{Y}_{t+1}}{K_{t+1}} = \frac{\alpha_3 \frac{\tilde{Y}_{t+1}}{E_{t+1}} + \Lambda_{St+1}}{\alpha_3 \frac{\tilde{Y}_t}{E_t} + \Lambda_{St}} \tag{2.42}$$

This is a variant of the Hotelling rule stating that the return on capital should be set equal to the return of postponing the extraction. The difference between this expression and the original rule is the appearence Λ_S in the numerator and denominator of the right hand side. Recall that Λ_S is negative hence if Λ_S is falling due to accummulating CO_2 so that $\Lambda_{St+1} < \Lambda_{St}$ then this implies that returns to investment must also be falling. In the market version the price of fossil fuel must equal its marginal product in equilibrium. Hence, in the variant above the terms $\alpha_3 \frac{\tilde{Y}_t}{E_t} + \Lambda_{St}$ can be interpreted as an externality adjusted fossil fuel price at time t.

Further by the investment and consumption rule (2.21) and (2.22) we can write

$$\frac{1}{\beta} = \frac{\alpha_3 \frac{1}{E_{t+1}} + \hat{\Lambda}_{St+1}}{\alpha_3 \frac{1}{E_t} + \hat{\Lambda}_{St}} \tag{2.43}$$

We can now see how a constant saving rate greatly simplifies the analysis. As can be seen from (2.43) the optimal path of fossil fuel use can now be fully determined given the appropriate end constraints and that carbon dioxide accumulation depends on past values of emissions according to (2.9). Hence, the decision regarding capital accumulation has been separated out from that evolving energy use.

2.2.3 Decentralized equilibrium

I will now characterize the how the optimum can be implemented using either a *ad-valorem* (τ_t) or a *per-unit taxes* (θ_t). I assume that the government taxes resources firms in order to implement the optimum. Consider first the problem of the the representive household/individual problem

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t),$$
$$s.t. \;\; C_t + K_{t+1} = r_t K_t + w_t + \Pi_t^e + G_t \tag{2.44}$$

as usual households are assumed to own both production and resource extraction firms and hence profit by renting out capital at the rate r_t, labor at the wage rate w_t, profits from resource extraction $\Pi_t^e \equiv (p_{E_t} - \theta_t)(1-\tau_t)E_t$ and government transfers $G_t \equiv \tau_t p_{E_t} E_t + \theta_t E_t$ where p_{E_t} denotes the before tax price on fossil fuel. The f.o.c. for K_{t+1} is

$$\frac{C_{t+1}}{C_t} = \beta r_{t+1} \tag{2.45}$$

Second, the representative firm within the each sector faces the following problem

$$\max_{K_{it}, L_{it}, E_{it}} p_{y_{it}} Y_{it} - r_t K_{it} - w_t L_{it} - p_{E_t} E_{it}, \quad \forall i \in \{m, a\}$$

with the f.o.c. given by

$$r_t = p_{y_{it}} \frac{\partial Y_t}{\partial K_{it}}, \quad w_t = p_{y_{it}} \frac{\partial Y_t}{\partial L_{it}}, \quad p_{E_t} = p_{y_{it}} \frac{\partial Y_t}{\partial E_{mt}} \tag{2.46}$$

Final good production is also done under profit maximization and perfect competition implying that the marginal product of each good will equal its price. Normalizing the price of the final good to one we thus have that:

$$p_{y_{it}} = w_i \left(\frac{Y_t}{Y_{it}} \right)^{\frac{1}{\epsilon}} \tag{2.47}$$

inserting this into (2.46) we see that

$$r_t = w_m \left(\frac{Y_t}{Y_{mt}}\right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial K_{mt}} = w_a \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial K_{at}}$$
$$w_t = w_m \left(\frac{Y_t}{Y_{mt}}\right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial L_{mt}} = w_a \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial L_{at}}$$
$$p_{E_t} = w_m \left(\frac{Y_t}{Y_{mt}}\right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial E_{mt}} = w_a \left(\frac{Y_t}{Y_{at}}\right)^{\frac{1}{\epsilon}} \frac{\partial Y_t}{\partial E_{at}}$$

These profit maximizing conditions are the same as in the planner solution with the solution derived in appendix and stated explicitly in (2.14). From this solution we thus know that:

$$r_t = \frac{\partial \tilde{Y}_t}{\partial K_t} = \frac{\partial \tilde{Y}_t}{\partial K_{mt}} = \alpha_1 \Gamma_t(S_t) K_t^{\alpha_1 - 1} L_t^{\alpha_2} E_t^{\alpha_3} \tag{2.48}$$

which implies that the same consumption/investment rules will apply as in the social planner solution i.e. (2.21) and (2.22) hold also in the decentralized solution.

Third, the representive resource extraction firm solves the problem

$$\max_{R_{t+1}} \sum_{t=0}^{\infty} (p_{E_t} - \theta_t)(1 - \tau_t) E_t \left(\prod_{s=0}^{t} r_s\right)^{-1}$$
$$s.t.\ R_{t+1} - R_t = -E_t, \quad R_{t+1} \geq 0$$

Once again, this gives us a hotelling type of formula

$$r_{t+1} = \frac{(p_{E_{t+1}} - \theta_{t+1})(1 - \tau_{t+1})}{(p_{E_t} - \theta_t)(1 - \tau_t)} \tag{2.49}$$

I have now characterized the decentralized solution. Next, in order to implement the planning solution we have to set the private return of not using fossil fuels equal to its social return.

Proposition 2.3. *The optimal tax can be implemented by either setting*

$$\theta_t = -\Lambda_{st} \ \textit{ and } \tau_t = \tau \ \ \forall t$$

or by setting

$$\tau_t = -\frac{\Lambda_{st}}{\partial \tilde{Y}_t / \partial E_t} \ \ \textit{ and } \theta_t = 0$$

Proof. Setting the rental price of capital from (2.49) equal to that of the marginal product of capital in from the planner problem (2.42) we can write

$$\frac{(p_{E_{t+1}} - \theta_{t+1})(1 - \tau_{t+1})}{(p_{E_t} - \theta_t)(1 - \tau_t)} = \frac{\alpha_3 \frac{\tilde{Y}_{t+1}}{E_{t+1}} + \Lambda_{St+1}}{\alpha_3 \frac{\tilde{Y}_t}{E_t} + \Lambda_{St}}$$

from this expression its immediate that if $\tau_t = \tau$ then $\theta_t = \Lambda_{st}$ implements the planner optimum. Likewise, if $\theta_t = 0$ then $\tau_t = \frac{\Lambda_{st}}{\partial \tilde{Y}_t / \partial E_t}$ implements the optimum. □

2.3 Numerical analysis

In this section I undertake a simple calibration and simulation exercise in order to illustrate how the optimal fossil fuel consumption problem is affected by underlying model assumptions. I have chosen to calibrate two sets of parameter estimates based on data from India and the U.S. economy respectively. These countries differ vastly in the size of their agricultural sector compared to other sectors of the economy and thus serves well as illustrative examples of economies having different sectoral compositions.[10] The result of this calibration excercise can thus be seen as the policy suggestion resulting from two types of planners trying to adress the economic realities adherent in two economies under different levels of economic development.[11]

2.3.1 Model calibration

In the analytical section above I neglected that economic damages from climate change are typically measured as a function of temperature. This was done in order to avoid complexity that does not impact on qualitative modeling results. A simple way to correct for this simplification is thru the Arrhenius equation, after the Swedish physicist Svante Arrhenius (1859-1927), which states that when CO_2 increases in a geometric progression, the augmentation of the temperature will increase in a nearly arithmetic progression (see e.g. IPCC (2001)). This fairly simple way of capturing the relationship between CO_2 is still in use today in simplified climate system representations approximating global temperature rise from a doubling of atmospheric CO_2 (see e.g. Nordhaus (2007))

$$T_t \equiv T(S_t) = \lambda \ln \left(1 + \frac{S_t}{\bar{S}}\right) / \ln 2$$

where $\bar{S}$ is the pre-industrial atmospheric CO_2 level and λ is a climate sensitivity parameter. Following Nordhaus (2007) (see page 13) I assume a preindustrial concentration of carbon dioxide of $\bar{S} \approx 596Gtc$ (gigagtons of carbon).[12] Taking the fairly standard value of 3 for λ we have that for a doubling of CO_2 in the atmosphere temperature will rise by approximately 3 degrees Celsius.[13] Tans (2011) reports a current CO_2 concentration as of December 2011 of $831Gtc$. Hence, using the above formula this implies that global

[10]For example, the world bank estimates that agricultures share of value added amounts to approximately 1% of the U.S. economy while in India the share is approximatly 20% (`http://data.worldbank.org/indicator/NV.AGR.TOTL.ZS?page=1`).

[11]It should be noted that the assumption of a constant elasticity of substitution is not uncontroversial. Already in the seminal article Arrow *et al.* (1961) it was pointed out that given the systematic inter-sectoral differences in the elasticity of substitution and in income elasticities of demand, the possibility arises that the process of economic development itself might shift the over-all elasticity of substitution. Hence, estimates for the elasticity of substitution may vary greatly depending on the level of development.

[12]I report atmospheric carbon dioxide in gigatons of carbon (GtC) with a conversion factor 1 ppm by volume of atmospheric $CO_2 = 2.13$ GtC.

[13]There is considerable uncertainty surrounding the appropriate estimate for climate sensitivity see for example Roe and Baker (2007).

temperature has increased by approximately 1.44 degrees since pre-industrial levels lies well in the ballpark of more accurate climate model runs found in e.g. the IPCC (2007). Concerning the carbon dioxide accumulation equation (2.9) two parameters need to be calibrated. First, the so called airborne fraction ξ which is the fraction of anthropogenic carbon dioxide emissions that remain in the atmosphere. I model this fraction as constant hence assuming that there is no trend in the biosphere's and oceans ability to absorb human induced emissions. Based on a recent study by Knorr (2009) I set the airborne fraction $\xi = 0.43$.[14] Second, the parameter φ captures the rate at which carbon is absorbed by the deep oceans. Archer (2005) claims that "...75% of an excess atmospheric carbon concentration has a mean lifetime of 300 year and the remaining 25% stays there forever". Although my simple carbon cycle is unable to account for the 25% always remaining in the atmosphere by setting $\varphi = 0.05$ this implies that after 300 years (30 periods) approximately 75% of the carbon dioxide has been removed.

Next, I need to specify the damage functions for the two sectors. Although there exists a large and growing literature concerned with sector specific damages at the local scale it is difficult to find similar assessments at the global scale. Reviewing and aggregating such studies is a large undertaking and lies beyond the scope of the current paper.[15] I follow Nordhaus (2007) here and specify a simple quadratic damage function for each of the two sectors and rely upon the aggregation results found in his accompanying notes for my benchmark estimation.[16] On page 24 of his accompanying notes he presents damage estimates in percent of GDP disaggregated into 12 geographical regions and seven damage sectors including an estimate of catastrophic damages. These damage estimates are then aggregated, using output weights based on predicted 2105 year's GDP levels, into a single estimate of GDP loss due to a 2.5 degree warming of approximately 1.77%. The exact size of these weights are however not a supported part of the DICE model.[17] Following, Nordhaus (2007) I proceed by calibrating two different quadratic damage functions for both the U.S. and Indian economy based on the estimates found in his notes. For the U.S. economy he estimates an economic impact of 0.03% of GDP from the agricultural sector and 0.88% of GDP from the rest of the economy from a 2.5 degree warming. Similarly for the Indian economy he estimates an economic impact of 0.32% of GDP from the agricultural sector and 2.75% of GDP from the rest of the economy. Based on these estimates the damage functions

$$\Omega_a(T_t) = \frac{1}{1+\theta_a T_t^2}, \quad \Omega_m(T_t) = \frac{1}{1+\theta_m T_t^2} \tag{2.50}$$

are calibrated for the U.S. and Indian economy where the parameters are calculated as $\theta_a = 0.0003/2.5^2 \approx 4.8 \times 10^{-5}$ and $\theta_m = 0.0088/2.5^2 \approx 0.0014$ for the U.S. economy and set to $\theta_a = 0.0032/2.5^2 \approx 0.000512$ and $\theta_m = 0.0275/2.5^2 \approx 0.0044$ for the Indian economy.

[14]Although there exists several studies have reported an apparent increasing trend in the airborne fraction the study by Knorr (2009) claim that this trend is statistically insignificant.

[15]See e.g. Tol (2009) for a review of such estimates.

[16]See `http://nordhaus.econ.yale.edu/Accom_Notes_100507.pdf`

[17]Personal communication with William Nordhaus.

For the parameters of the agricultural and manufacturing production functions I follow Golosov *et al.* (2011) and set $\alpha_1 = 0.3$, $\alpha_2 = 0.67$ and $\alpha_3 = 0.03$ for both economies. These estimates are fairly standard for manufacturing or as average estimates for the entire economy. However, if land is considered a capital good, empirical studies point to a much larger estimate for the capital income share in the agricultural sector. Valentinyi and Herrendorf (2008) estimate income shares of capital and labor at the sectoral level for the US economy and find that the capital share of the agricultural sector is approximately 50% larger than the capital share of the aggregate economy. For developing countries the share is somewhat smaller (Irz and Roe, 2005). Concerning the TFP growth rates, Martin and Mitra (2001) estimate that the overall growth rate of TFP in manufacturing varies between 1.13% and 1.86% between 2.34% and 2.91% for agriculture for a sample of 50 countries between the years 1967-92. This is one of the few global studies I could find with the same sectoral disaggregation as I consider here. I thus use the mean of these intervals, i.e. $g_m = 0.015$ and $g_a = 0.026$, as my initial benchmark estimates in both economies. It should however be noted that there is a significant amount of disagreement in the literature whether this relatively high rate of TFP growth in agriculture can continue also in the future. The spread of knowledge on the productive use of pesticides and fertilizers in farming increased output considerably in this sector during the second half of the 20th century but it is unclear whether this level of growth can continue at the same pace in the future due to e.g. biophysical constraints in plant life (Ruttan, 2002).

Concerning the elasticity of substitution and distribution parameters I calibrate these following Acemoglu and Guerrieri (2008). Since, Y_{it} corresponds to the quantity produced in sector $i \in \{m, a\}$ the value of output (nominal value added) produced in sector i follows from $Y_{it}^n \equiv p_{it} Y_{it}$ where p_{it} is given by equation (2.46). This implies the following way of estimating these parameters using a simple linear regression by taking the log of the ratio of sectoral nominal value added and real value added

$$\ln\left(\frac{Y_{mt}^n}{Y_{at}^n}\right) = \ln\left(\frac{w_m}{w_a}\right) + \frac{\epsilon - 1}{\epsilon} \ln\left(\frac{Y_{mt}}{Y_{at}}\right) \tag{2.51}$$

I estimate the above equation using data from the Groningen Growth and Development Centre (GGDC) 10-sector database for the years 1950-2005 for the U.S. and Indian economy. The data set contains annual series of value added, real value added, and persons employed for 10 broad sectors of the economy (Timmer and de Vries, 2009). I let the *agriculture, forestry, and fishing* sector of the 10 sector database represent the agricultural sector of my model while the aggregate of the remaining sectors excluding government services serves as a proxy for the manufacturing sector. Using these aggregate estimates of real value added and value added I obtain by linear regression from (2.51) an estimate of $\epsilon \approx 1.62$ with a standard error of 0.147 for the U.S. economy. For the Indian economy the corresponding estimate is $\epsilon \approx 2.13$ with a standard error of 0.15.[18] I thus choose these values as my benchmark estimates and then calibrate w_m

[18] Both estimates are significant on the 1% level.

to ensure that (2.51) holds for the year 2005 which gives me an estimate of $w_m \approx 0.95$ for the U.S. economy and $w_m \approx 0.64$ for the Indian economy. The above estimation process has thus given us a benchmark estimate of the elasticity of substitution and distribution parameters. In the result section we will thus also consider alternative calibrations based with an elasticity of substitution above and below these estimates.

Next, we also need a to calibrate A_{m0} and A_{a0}. As above I consider 2005 as the initial time 0 and proceed as follows. First, the input factor share is determined by making use of (2.8) and equation (2.53) in appendix i.e. we have that $K_{a0} = \frac{\Psi_0}{1+\Psi_0}K_0$, $L_{a0} = \frac{\Psi_0}{1+\Psi_0}L_0$ and $E_{a0} = \frac{\Psi_0}{1+\Psi_0}E_0$ where $\Psi_0 \equiv \frac{w_a}{w_m}\left(\frac{Y_{a0}}{Y_{m0}}\right)^{\frac{\epsilon-1}{\epsilon}}$. Substituting these input factors in the agricultural production function and the corresponding manufacturing inputs into the manufacturing production function we have that $Y_{a0} = \frac{\Psi_0}{1+\Psi_0}\Omega_a(T_0)A_{a0}K_0^{\alpha_1}L_0^{\alpha_2}E_0^{\alpha_3}$ and $Y_{m0} = \frac{1}{1+\Psi_0}\Omega_m(T_0)A_{m0}K_0^{\alpha_1}L_0^{\alpha_2}E_0^{\alpha_3}$. Dividing the left and right hand sides of these production functions we can solve for the ratio

$$\frac{A_{a0}}{A_{m0}} = \frac{w_m}{w_a}\frac{\Omega_m(T_t)}{\Omega_a(T_t)}\left(\frac{Y_{a0}}{Y_{m0}}\right)^{1/\epsilon} \tag{2.52}$$

Using our estimated values for the elasticity of substitution and distribution parameters together with the 2005 real value added estimates for each of the two sectors as initial values we arrive at an estimate of $\frac{A_{a0}}{A_{m0}} \approx 1.46$ for the U.S. economy and $\frac{A_{a0}}{A_{m0}} \approx 0.94$ for the Indian economy. We can thus without loss of generality normalize and set $A_{m0} = 1$ when calculating optimal emission paths based on equation (2.43). Finally, fossil fuel use in our model also requires an estimate of the current stock R_0. We use the global reserve estimate of $5000GtC$ from Rogner (1997) which also accounts for technical progress in extraction. This estimate is assumed to consist of both coal and oil resources. It should be noted that the way I model fossil fuel production in this paper is highly simplistic as it ignores important issues such as e.g. extraction and refinement costs which are generally much higher for coal than oil. Finally, the last parameter that needs mentioning is the discount factor β which I set equal to 0.985^{10}, where the tenth power is due to the fact that I am considering each time step to be of ten years length. This follows previous work by Nordhaus (2007) and Golosov *et al.* (2011).

2.3.2 Results

In section 2.2, I showed, that finding the optimal fossil fuel path involves solving a fairly simple difference equation given by equation (2.43) which was shown to be a decision independent of the saving policy.[19] In this section I will present some results for two

[19]The numerical simulations are done in Matlab and the code is readily available from the author upon request. The basic solution strategy for equation (2.43) is as follows: a) Use the decentralized path for E as an initial guess of a solution until some time T (large enough to approximately exhaust the resource) b) Use this path to derive the S_t from (2.9) and $\hat{\Lambda}_s$ from (2.28) c) Using $\hat{\Lambda}_s$ use (2.43) to generate a new path for $E_t + 1, E_t + 2$, etc... which also satisfies the resource constraint (2.7). d) Use

alternative calibrations based on the U.S. and Indian economy. The results for the first calibration based on the estimates for the U.S. economy are given in figure 2.2.

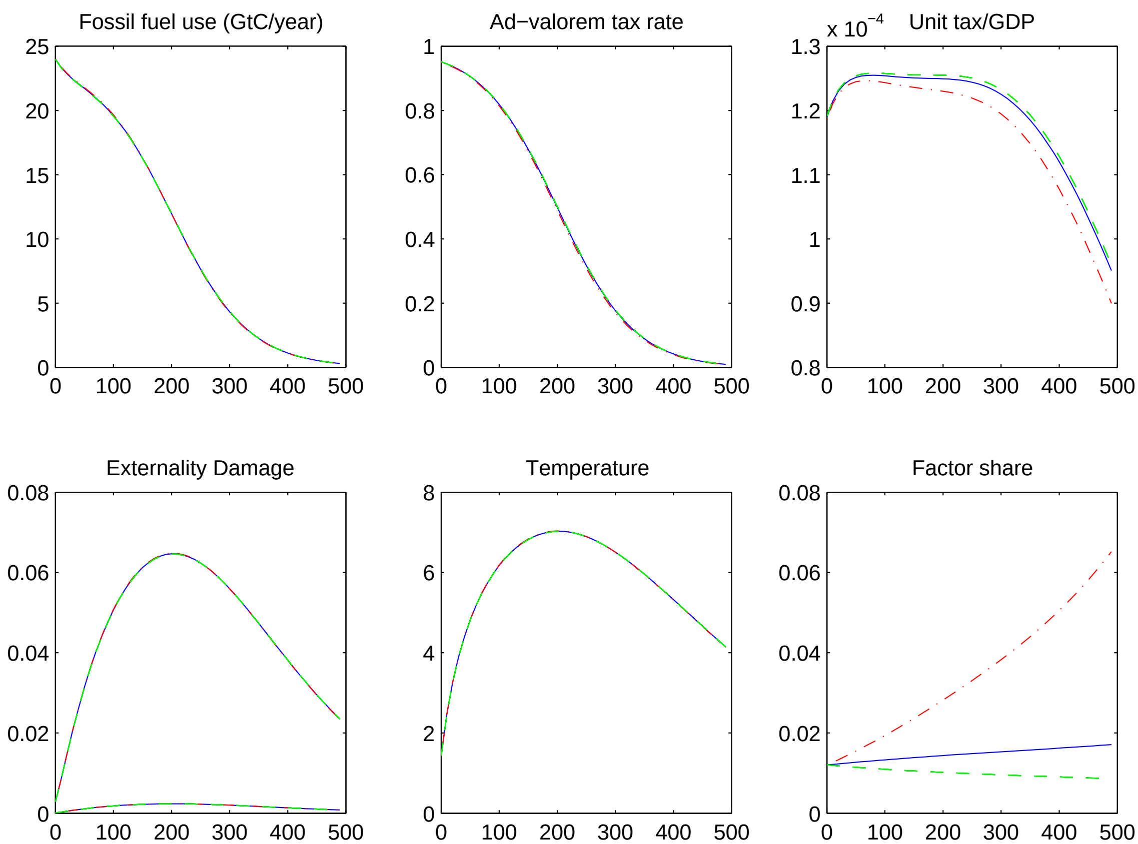

Figure 2.2: Optimal transition paths for the U.S. economy calibration.

The figure depicts optimal transition paths for several of the key variables of the economy for a 500 year time horizon. Each figure contains simulation results for three estimates of $\epsilon = 4$ red dash-dotted line and $\epsilon = 0.4$ green dashed line together with a solid blue lines which depicts the paths for the benchmark calibration.[20] As can be seen the simulation results for are almost inseparable for the paths based on the different values of ϵ. This will be differ when we consider the calibration based on India below. Starting from the upper left graph this depicts the optimal fossil fuel use of the economy. As can be seen the path declines over time and approaches zero as $t \to \infty$. The middle upper graph depicts the ad-valorem tax rate. Alternatively we could have referred to this as a unit tax per market price of fossil fuel. Using this terminology taxes thus start at approximately 95% of the market price and remain high for quite some time.

this path to update S_t from (2.9) and $\hat{\Lambda}_s$ from (2.28) e) Now repeat from step c) and continue until convergence.

[20]For $\epsilon = 0.4$ and 4, I also recalibrate equation (2.51) and equation (2.52) based on the GGDC output data for 2005.

The upper left graph depicts the unit tax per unit of GDP. This graph is displayed in a scale that shows off the results for the different values of ϵ. As can be seen the upper line displaying the results for a $\epsilon = 0.4$ implies a slightly higher tax for the entire time period. Likewise, lower line displaying the results for a $\epsilon = 4$ gives the lowest tax. The middle graph (blue line) depicts the benchmark case. Hence, we see that with increasing values of ϵ the optimal tax rate declines. A carefull inspection of the optimal fossil fuel path graph also reveals that larger values of ϵ decreases the optimal fossil fuel use early on. Next the lower left graph displays the externality damage which is the percent of damage to GDP as a function of tempertature increase. The lower line displays the amount of damage coming from the agricultural sector. Damages are lower in this sector due to the fact that it constitutes a smaller proportion of GDP than the manufacturing sector. As can be seen for the manufacturing sector damages peaks at slightly above 6% of GDP after 200 years while the agricultural damages never goes above 1% of GDP. The lower middle graph depicts the global temperature increases that would result following the emissions associated with the optimal path.[21] Finally, the lower rightmost graph depicts the income factor shares corresponding to the expression in proposition 2.1. As can be seen the differing values for ϵ creates clear trends when it comes to the allocation of factor inputs. Here, the dash-dotted (red line) having an $\epsilon = 4$ creates a clear upward trend in factor input allocation implying that more resources are allocated to agriculture over time. This is due to the larger TFP estimate for the agricultural sector implying that the agricultural sector becomes more productive over time. With an elasticity of substitution larger than one (i.e. substitutes) this implies that factor inputs are flowing in to the more productive sector which can also be seen from the positive slope of the solid blue line. On the contrary the green dashed line is downward sloping implying that resources are flowing into the least productive sector as it continues to grow relatively less productive.

Turn now to the Indian economy. Figure 2.3 displays the corresponding paths of for the variables when the model has been calibrated to match the data from the Indian economy. As can be seen this figure displays a great deal more of variation in all of the depicted variables. One major difference between the Indian and U.S. calibrations was that the damage estimates for a 2.5 degree warming were substantially higher for the Indian economy. A comparison can be made in terms of ad-valorem taxes which for the Indian economy constitutes approximately 100% of the market price at the outset as opposed to 95% for the U.S. economy. This also results in a flatter path of fossil fuel use since the higher damages makes it optimal to postpone fossil fuel consumption to the future. As can be seen from the upper right graph the optimal paths vary a great deal with the size of the elasticity of substitution. The red dash-dotted line with $\epsilon = 4$ the path exhibits a slight u-shape. Here damages early on are significant enough to warrant a lower level of fossil fuel use at the outset than after 200 years. From the lower right graph we see that this is likely to be connected to the increasing factor share

[21] These temperature levels are by many climate scienticts regarded as highly dangerous when it comes to human survival on the planet (see e.g. Hansen (2005); Rockström *et al.* (2010)).

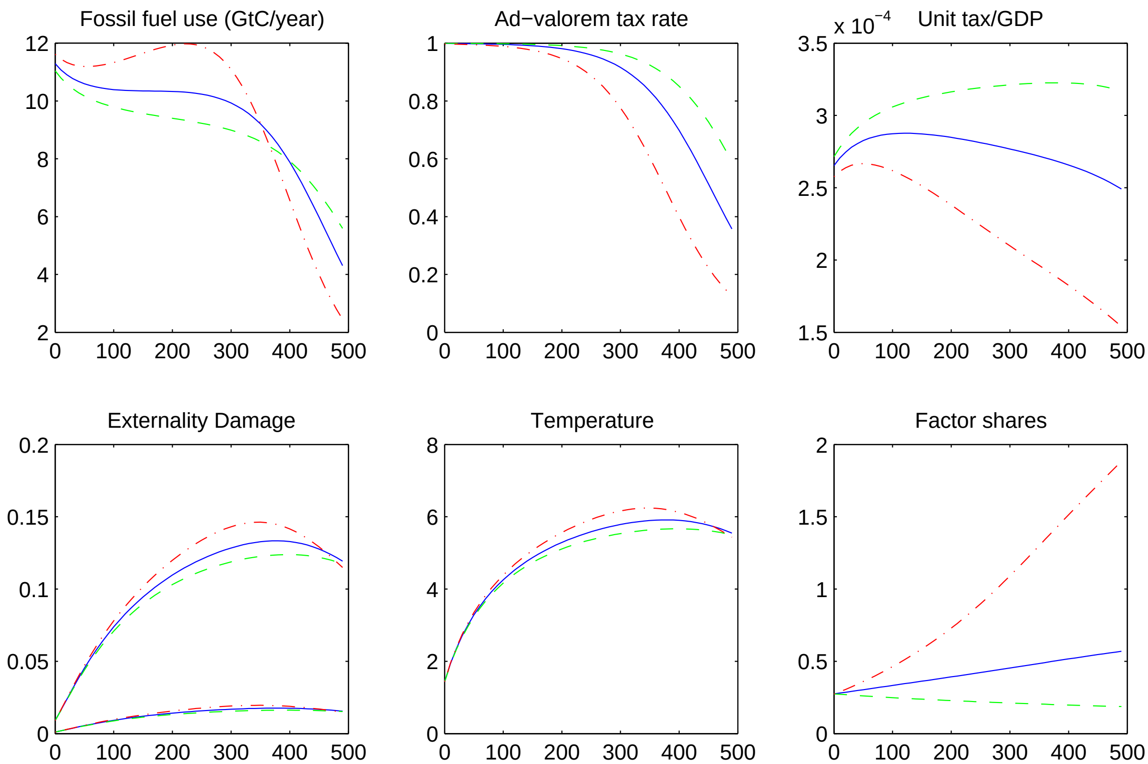

Figure 2.3: Optimal transition paths for the Indian economy calibration.

implying that resources are flowing into the agricultural sector over time. Since this sector is less plagued with damages this explains why fossil fuel use starts to rise again after approximately 100 years. The final downturn after 250 years is of course due to resource scarcity leading to a rising relative price for fossil fuel consumption. From the lower lefthand graph depicting the externality damage as a percent of GDP we see that damages to the Indian economy are substantially higher than those observed for the U.S. economy peaking at roughly 15% of GDP after 300 years. From the temperature graph we see that this extra damage has however resulted in a more restrictive optimal fossil fuel use policy which keeps the global temperature from rising much higher than 6 degrees as opposed to the near 7 degree peak in the U.S. economy.

2.4 Concluding remarks

In this paper I have developed a two-sector general equilibrium model featuring a global climate externality arising from the use of fossil fuels in production. I derive analytical and numerical results for the optimal fossil fuel use in the social planner setting and the corresponding unit and ad-valorem tax rates that implement the planner solution in a decentralized market equilibrium. I have shown that when the elasticity of substitution between the two sectors is constant the same economic forces giving rise to structural change also impact the externality costs of climate change. The analytical results reveal the behavior of optimal tax rates subject to sectoral differences in damages, total factor productivity and sectoral weights between the two sectors. It is shown that a higher (lower) elasticity of substitution will result in a higher (lower) optimal unit tax rate if and only if the sectoral weight of the most productive sector, where productive refers to total factor productivity net climate damages, is small (large) enough. The model and results derived here draws upon the results and findings of papers by Ngai and Pissarides (2007), Sterner and Persson (2008), Acemoglu and Guerrieri (2008) and Golosov *et al.* (2011).

Given a set of simplifying assumptions following Golosov *et al.* (2011) I am able to derive i) a simple formula for the marginal externality cost of emissions and ii) a structural change mechanism which is almost identical to demand side mechanism driving the results in Ngai and Pissarides (2007).[22] However, in contrast to Ngai and Pissarides (2007), the mechanism driving structural change in this paper is a combination of damages and productivity assumptions which enter the model as a supply side phenomena.

The numerical section of the paper gives a simulated example based on the U.S. and Indian economy. Calibrating the model to two economies at different stages of development is a crude way of accommodating that systematic inter-sectoral differences in the elasticity of substitution, imply the possibility that the process of economic development itself might shift the over-all elasticity of substitution (Arrow *et al.*, 1961). The results suggest that the elasticity of substitution plays a large role for optimal fossil fuel consumption in the Indian economy where the agricultural sector constitutes approximately 20% of GDP. On the contrary the U.S. economy where the agricultural sector constitutes only 1% of GDP this parameter plays only a peripheral role.

This paper has been a first attempt at developing a climate-economy model where it is possible to explore how substitution possibilities among goods might impact growth and marginal externality costs. The model was developed in the tradition of modern macroeconomic growth models with the intention of making it more accessible to empirical studies exploring the role of substitution possibilities for calculating the costs of climate change. A step for future research could be a more rigid calibration of the model where for example the assumption of equal capital shares are also relaxed.

[22] If the climate externality is removed from the model developed here these mechanisms would have been identical.

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2.A Appendix

2.A.1 Static problem

Proof. of Proposition (2.1)
Rewrite equation (2.12) as:

$$\begin{aligned}
\frac{K_{at}}{K_{mt}} &= \frac{w_a}{w_m}\left(\frac{Y_{mt}}{Y_{at}}\right)^{\frac{1}{\epsilon}}\frac{Y_{at}}{Y_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{Y_{at}}{Y_{mt}}\right)^{\frac{\epsilon-1}{\epsilon}} \\
\frac{L_{at}}{L_{mt}} &= \frac{w_a}{w_m}\left(\frac{Y_{mt}}{Y_{at}}\right)^{\frac{1}{\epsilon}}\frac{Y_{at}}{Y_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{Y_{at}}{Y_{mt}}\right)^{\frac{\epsilon-1}{\epsilon}} \\
\frac{E_{at}}{E_{mt}} &= \frac{w_a}{w_m}\left(\frac{Y_{mt}}{Y_{at}}\right)^{\frac{1}{\epsilon}}\frac{Y_{at}}{Y_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{Y_{at}}{Y_{mt}}\right)^{\frac{\epsilon-1}{\epsilon}}
\end{aligned} \tag{2.53}$$

where $w_a \equiv \tilde{w}_a^{\frac{\epsilon-1}{\epsilon}}$ and $w_m \equiv \tilde{w}_m^{\frac{\epsilon-1}{\epsilon}}$. Further solving for the respective ratios we have

$$\begin{aligned}
\frac{K_{at}}{K_{mt}} &= \left(\left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\frac{L_{at}^{\alpha_2}}{L_{mt}^{\alpha_2}}\frac{E_{at}^{\alpha_3}}{E_{mt}^{\alpha_3}}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-\alpha_1(\epsilon-1)}} \\
\frac{L_{at}}{L_{mt}} &= \left(\left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\frac{K_{at}^{\alpha_1}}{K_{mt}^{\alpha_1}}\frac{E_{at}^{\alpha_3}}{E_{mt}^{\alpha_3}}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-\alpha_2(\epsilon-1)}} \\
\frac{E_{at}}{E_{mt}} &= \left(\left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\frac{K_{at}^{\alpha_1}}{K_{mt}^{\alpha_1}}\frac{L_{at}^{\alpha_2}}{L_{mt}^{\alpha_2}}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-\alpha_3(\epsilon-1)}}
\end{aligned}$$

denote $\nu_1 \equiv \frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)}$, $\nu_2 \equiv \frac{\epsilon-1}{\epsilon-\alpha_2(\epsilon-1)}$ and $\nu_3 \equiv \frac{\epsilon-1}{\epsilon-\alpha_3(\epsilon-1)}$.

$$\frac{K_{at}}{K_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)}}\frac{L_{at}}{L_{mt}}^{\frac{\alpha_2(\epsilon-1)}{\epsilon-\alpha_1(\epsilon-1)}}\frac{E_{at}}{E_{mt}}^{\frac{\alpha_3(\epsilon-1)}{\epsilon-\alpha_1(\epsilon-1)}} \tag{2.54}$$

$$\frac{L_{at}}{L_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\epsilon-1}{\epsilon-\alpha_2(\epsilon-1)}}\frac{K_{at}}{K_{mt}}^{\frac{\alpha_1(\epsilon-1)}{\epsilon-\alpha_2(\epsilon-1)}}\frac{E_{at}}{E_{mt}}^{\frac{\alpha_3(\epsilon-1)}{\epsilon-\alpha_2(\epsilon-1)}} \tag{2.55}$$

$$\frac{E_{at}}{E_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\epsilon-1}{\epsilon-\alpha_3(\epsilon-1)}}\frac{K_{at}}{K_{mt}}^{\frac{\alpha_1(\epsilon-1)}{\epsilon-\alpha_3(\epsilon-1)}}\frac{L_{at}}{L_{mt}}^{\frac{\alpha_2(\epsilon-1)}{\epsilon-\alpha_3(\epsilon-1)}} \tag{2.56}$$

Substituting the labor share equation (2.55) into the capital share equation (2.54) and solving for the capital share gives us

$$\begin{aligned}
\frac{K_{at}}{K_{mt}} &= \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)}\left(1+\frac{\alpha_2(\epsilon-1)}{\epsilon-\alpha_2(\epsilon-1)}\right)}{1-\alpha_1\alpha_2\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)}\frac{\epsilon-1}{\epsilon-\alpha_2(\epsilon-1)}}}\frac{E_{at}}{E_{mt}}^{\frac{\frac{\alpha_3(\epsilon-1)}{\epsilon-\alpha_1(\epsilon-1)}\left(1+\frac{\alpha_2(\epsilon-1)}{\epsilon-\alpha_2(\epsilon-1)}\right)}{1-\alpha_1\alpha_2\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)}\frac{\epsilon-1}{\epsilon-\alpha_2(\epsilon-1)}}} \\
&= \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)-\alpha_2(\epsilon-1)}}\frac{E_{at}}{E_{mt}}^{\frac{\alpha_3(\epsilon-1)}{\epsilon-\alpha_1(\epsilon-1)-\alpha_2(\epsilon-1)}}
\end{aligned}$$

Now, substitute this into the labor share equation (2.55) to obtain

$$\begin{aligned}\frac{L_{at}}{L_{mt}} &= \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\frac{E_{at}^{\alpha_3}}{E_{mt}^{\alpha_3}}\right)^{\frac{\epsilon-1}{\epsilon-\alpha_2(\epsilon-1)}+\frac{\alpha_1\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)}\frac{\epsilon-1}{\epsilon-\alpha_2(\epsilon-1)}\left(1+\frac{\alpha_2(\epsilon-1)}{\epsilon-\alpha_2(\epsilon-1)}\right)}{1-\alpha_1\alpha_2\nu_1\nu_2}} \\ &= \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\epsilon-1}{\epsilon-\alpha_1(\epsilon-1)-\alpha_2(\epsilon-1)}} \frac{E_{at}}{E_{mt}}^{\frac{\alpha_3(\epsilon-1)}{\epsilon-\alpha_1(\epsilon-1)-\alpha_2(\epsilon-1)}}\end{aligned}$$

Substituting both these expressions into the emission share equation (2.56) we obtain after som algebra

$$\frac{E_{at}}{E_{mt}} = \left(\frac{\tilde{w}_a}{\tilde{w}_m}\frac{\Omega_a(S_t)}{\Omega_m(S_t)}\frac{A_{at}}{A_{mt}}\right)^{\frac{\epsilon-1}{\epsilon-(\epsilon-1)(\alpha_1+\alpha_2+\alpha_3)}}$$

Further, from (2.53) we have that $\frac{K_{at}}{K_{mt}} = \frac{L_{at}}{L_{mt}} = \frac{E_{at}}{E_{mt}}$ which implies that the solution will be equivalent for all factor inputs. Further it is clear that the above exponents simplifies to $\epsilon - 1$ if we assume constant returns to scale. □ □

Chapter 3

Energy Balance Climate Models and General Equilibrium Optimal Mitigation Policies*

3.1 Introduction

The impact of climate change is expected to have a profound regional structure in terms of temperature and damage differentials across geographical regions.[1] The spatial dimension of damages can be associated with two main factors: (i) Natural mechanisms which produce a spatially non-uniform distribution of the surface temperature across the globe. These mechanisms relate mainly to the heat flux that balances incoming and outgoing radiation and in the differences among the local heat absorbing capacity - the local coalbedo - which is relatively lower in ice covered regions; (ii) economic related forces which determine the damages that a regional (local) economy is expected to suffer from a given increase of the local temperature. These damages depend on population size and production characteristics (e.g. agriculture vs services) or local natural characteristics (e.g. proximity and elevation from the sea level). The interactions between the spatially non-uniform temperature distribution and the spatially non uniform economic characteristics will finally shape the spatial distribution of damages.

Existing literature and in particular the DICE/RICE models (e.g. Nordhaus and Boyer (2000), Nordhaus (2007, 2010, 2011)) provide a spatial distribution of damages where the relatively higher damages from climate change are concentrated to the zones around the equator.[2] These models as well as the big majority of Integrated Assessment Models (IAMs) do not account for the first factor, the natural mechanism generating

*This paper has been co-authored with William A. Brock and Anastasios Xepapadeas

[1]Detailed reports of climate change effects on different parts of the world can be found at http://www.metoffice.gov.uk/climate-change/policy-relevant/obs-projections-impacts

[2]For example, Nordhaus's RICE 2010 divides the world into US, EU, Japan, Russia, Eurasia, China, India, Middle East, Africa, Latin America, Other high income, Other developing Asia. In the DICE model spatial damages are implicit in the aggregate representation since regional impacts are aggregated to a single measure using a bottom-up approach.

temperature distribution across the globe. The DICE/RICE models do not include the spatial transportation of heat, nor the albedo differentials across locations, and perform their analysis in terms of the global mean surface temperature which does not vary across regions during their planning horizons.

In climate science terminology these IAMs can be referred to as zero-dimensional models since they do not include spatial aspects such as heat diffusion. This contrasts to the one- or two-dimensional energy balance climate models (EBCMs) developed by climate scientists which model heat diffusion across latitudes (one-dimensional) or across latitudes and longitudes (two-dimensional) (see e.g. Budyko (1969), Sellers (1969, 1976), North (1975a,b), North *et al.* (1981), Kim and North (1992), Wu and North (2007)). One-dimensional EBCMs predict a concave temperature distribution across latitudes with the maximum temperature at the equator. This non uniform temperature distribution is important for understanding the so called "temperature anomaly" which is the difference between the temperature distribution at a given benchmark period and the current period. Data indicate (Hansen *et al.*, 2010) that since 1880 the anomaly has been relatively higher in high latitude zones, relative to zones around the equator, which suggest spatial non-uniformity in the distribution of temperature over time.

The temperature anomaly is however the basis for estimating regional damages. Regional damages are obtained by mapping a given change in the temperature of a region relative to a benchmark period (the temperature anomaly) to the damages that this change is expected to bring given the characteristics of the region's economy. In the context of a zero-dimensional model this temperature anomaly will be spatially homogeneous, or flat across regions, since climate change acts on the global average temperature which is spatially homogeneous. In the context of a one- or two-dimensional model, climate change acts on the spatially non uniform temperature distribution. This is expected to result in a spatially non homogeneous distribution of the temperature anomaly which in turn will differentiate the distribution of damages from those implied by a zero dimensional model.

In this paper we study the economics of climate change by coupling a one-dimensional EBCM with heat diffusion and albedo differentiation across latitudes, with an economic growth model. We believe that this approach that integrates solution methods for one-dimensional spatial climate models, with methods of solving economic models, can provide new insights regarding issues such as the profile of optimal mitigation policies and the spatial distribution of damages, relative to the more conventional integrated assessment models with carbon cycle but without heat diffusion. We focus on a popular class of EBCMs, with analytical solutions derived in North (1975a,b); North *et al.* (1981)). These models feature (i) the explicit spatial dimension in the form of heat diffusion or transportion across latitudes using a heat diffusion operator which allows for the use of approximation methods based on Legendre polynomial expansions, and (ii) the spatial dependency of earths albedo due to the presence of an endogenous ice line where latitudes north (south) of the ice line are solid ice and latitudes south (north) of the ice line are ice free.[3]

The economic part of the model is an infinite horizon Ramsey-type model allowing for basic heterogeneity among consumers and firms at each respective latitude denoted by "x". Considering each latitude as an individual economic zone is not the most realistic of assumptions but we believe this to be an important first step before moving to more complex areas involving two-dimensional climate-economy models. Our basic analytical approach is however very general and we need not confine x to meaning "latitude". At an analytical level of abstraction we just have to use a different basis set than Legendre polynomials.[4] Hence, the results derived analytical in terms of e.g. optimal tax rates still hold with more complex diffusion processes although numerical computations would have been more costly. The approach taken here and the results derived should thus be seen as an attempt to lay down a basic groundwork for future studies involving more complex interaction among economic zones and temperatures located on the sphere where x instead is interpreted as a "location" defined by both a latitude and a longitude.

Thus, in the context described above the main contribution of our paper is the coupling of a spatial climate model, implied by climate science, with a standard economic growth model. This allows us to explore what insights can be gained from the use of economic-EBCMs when analyzing the temporal and spatial allocation decisions associated with the climate change mitigation problem. Since EBCMs most likely are new to most economist we have chosen to focus on the basic general equilibrium properties of the derived model under different assumptions regarding international capital markets and their integration. This approach shows of the basic welfare properties of the model and how the fundamental theorems of welfare economics apply within the context of our energy balance climate economy model. The approach also makes clear how the optimal carbon tax rates should be chosen in order to implement a social planning problem and should thus constitute an important first step for economists working with these types of models. We have chosen to look at three different cases here. The first two cases concern economies that are either completely open or completely closed at each respective latitude. In the first case when all economies are open capital returns and interest rates will be equal across latitudes. In this case we show that the optimal carbon tax will be uniform or equal across latitudes. In the second case when the economy is closed, interest rates and tax rates will generally differ across latitudes. In this case if some countries are rich and some poor and international transfers are assumed to be restricted across latitudes, this will in general imply that optimal carbon taxes will be

[3] We could of course had considered other approaches alternative to EBCMs for approximating temperature fields which are based on more complex and computationally costly models, such as pattern scaling (Lopez *et al.*, 2012) or emulation theory (Challenor *et al.*, 2006). Because the purpose of this paper is however to construct the simplest coupled climate economy model with a climate feedback response mechanism in space that responds to changes in the spatiotemporal structure of taxes on fossil fuels, which is still analytically tractable, we considered the EBMCs framework as more appropriate.

[4] For example, Wu and North (2007) show how similar approximation methods can be used in a two-dimensional model featuring a "Fickian" diffusion process across both latitudes and longitudes.

spatially differentiated. This result that in the absence of international transfers a spatially uniform optimal tax rate is in general not possible was first noted by Chichilnisky and Heal (1994). Our result provides new insights into this issue by characterizing the spatial distribution of fossil fuel taxes and linking the degree of spatial differentiation of optimal fossil fuel taxes to the diffusion of heat across latitudes. The third and intermediate case we consider, extends our results beyond what was considered in the static model of Chichilnisky and Heal (1994). This concerns the case of costly transfers i.e. an open economy where transfers are available but come at a cost. In this case we show that under the assumption that the marginal costs of international transfers does not change over time we can have both a spatially differentiated tax rate and equality among interest rates across latitudes. Using these three separate scenarios we are able to show how heat transport across latitudes matters regarding the prediction of the spatial distribution and the corresponding temporal evolution of temperature, damages and optimal mitigation efforts. Our one-dimensional model further allows us to show how heat diffusion across geographical zones impacts on the size of the spatial differentiation of fossil fuel taxes between poor and wealthy regions. In the numerical section of the paper we finally attempt a tentative calibration exercise and show how the open economy model can be solved numerically in order to obtain graphical representations of e.g. damage and temperature distributions.

To sum up, the main objective of this paper has been to introduce the economics profession to spatial EBCMs with heat transport as a potentially useful tool for studying the economics of climate change relative to alternative zero dimensional models. By deriving the spatiotemporal profile for optimal taxes from the one-dimensional coupled climate economic model, we show how the spatial EBCMs can contribute to the current debate regarding how much to mitigate now, whether mitigation policies should be spatially homogeneous or not, and how to derive geographically specific information regarding damages and policy measures.[5]

[5]Another issue that can be addressed by latitude dependent climate models is damage reservoirs. Damage reservoirs in the contest of climate change can be regarded as sources of climate damages which will eventually cease to exit when the source of the damages is depleted. Damage reservoirs are latitude dependent and ice lines and permafrost can be regarded as such reservoirs.

As the ice lines move closer to the poles, due to climate change, we might expect that marginal damages from this moving will be large at first and then diminish as the ice line approaches the Poles. When there will be no ice left on the Poles this damage reservoir would have been exhausted. The presence of an endogenous ice line in the EBCM allows us to model these type of damages explicitly given the relevant information

Permafrost is soil at or below the freezing point of water for two or more years. The permafrost feedback suggests that permafrost carbon emissions could affect long-term projections of future temperature change. Studies indicate that up to 22 % of permafrost could be thawed already by 2100. Once unlocked under strong warming, thawing and decomposition of permafrost can release amounts of carbon until 2300 comparable to the historical anthropogenic emissions up to 2000 (approximately 440 GTC) (Schneider *et al.*, 2011).

The modeling of damage reservoirs is beyond the scope of the present paper but we have addressed this with in the context of a one-dimensional in a separate paper of ours (see Brock *et al.* (2012a)). Our findings are that the introduction of damage reservoirs into the climate-economy framework may

Since these models are new in economics we proceed in steps that we believe make this methodology accessible to economists.[6] In section 3.2 we present a basic energy balance climate model which incorporates human impacts on climate which result from carbon dioxide accumulation due to the use of fossil fuels, that blocks outgoing radiation. In developing the model we follow North (1975a,b) and use his notation. We use the model to expose solution methods and especially the two mode approach which transforms systems of partial differential equations (PDEs) in infinite dimensional spaces resulting from spatial modeling, into systems of ordinary differential equations (ODEs) in finite dimensional spaces. The two mode approach will be used to solve, and numerically approximate latitude dependent temperature and damage functions.

Section 3.3 couples the spatial EBCM with an economic growth model, where a finite stock of fossil fuel is an essential input along with capital and labor. Fossil fuels are extracted by fossil fuel firms which pay taxes on profits and/or taxes per unit of fossil fuel extracted. We solve the model for the social planner and for the competitive equilibrium with taxes. We derive the optimal taxes and their temporal profiles. We show that under a mild assumption about a slow decay of the CO_2 in the atmosphere the profit tax on fossil fuel firms decline over time and the unit taxes on extracted fossil fuels grow at rate less than the rate of return on capital. Furthermore we derive the latitude dependent temperature function and the impact of heat transport on damages across latitudes.

In section 3.4 we use approximate solutions, we simulate the climate *open economy* model and we derive explicit numerical solutions for the latitude dependent temporal and damage functions. The last section concludes.

3.2 An Energy Balance Climate Model with Human Inputs

In this section we develop a one-dimensional Energy Balance Climate Models with a "human input". The addition of a human input will form the connection to the economic model developed in the following sections. The term "one-dimensional" means that there is an explicit spatial dimension in the model so that our unified model of the climate and the economy evolves both in time and space.[7] We follow North (1975a,b) and North *et al.* (1981) in this development. North developed his model in continuous time. Here we work out the corresponding model in discrete time. The discrete time framework offers a great deal of tractability given a specific set of assumptions not equally applicable in a continuous time framework. This will become clear when we

give rise to multiple steady states and Skiba points which can generate u-shaped optimal mitigation policies. Hence, we believe this represents an important area for further research.

[6]For more on EBCMs see for example Pierrehumbert (2008) (chapters 3 and 9, especially sections 9.2.5 and 9.2.6 and surrounding material). North *et al.* (1981) is a very informative review of EBCMs while Wu and North (2007) is a recent paper on EBCMs.

[7]In contrast, a "zero-dimensional "model does not explicitly account for the spatial dimension. On the other hand more complicated spatial structures could include two-dimensional spherical models. Our methods can be readily extended to a two dimensional spherical model as in Wu and North (2007).

couple the model to the economic growth model in the following section.[8]

Let x to denote the *sine* of the latitude. We shall abuse language and just refer to x as "latitude". Following North (1975a,b) let $I_{x,t}$ denote outgoing infrared radiation flux measured in W/m^2 at latitude x at time t, $T_{x,t}$ denote surface (sea level) temperature measured in °C at latitude x at time t. The outgoing radiation and surface temperature can be related through the empirical formula.[9]

$$I_{x,t} = A + BT_{x,t} \tag{3.1}$$

The basic energy balance equation developed in (North (1975a), equation (29)) can be written, with human input added $h_{x,t}$, as:

$$c(I_{x,t+1} - I_{x,t}) = QS(x)\alpha(x, x_{s,t}) - [I_{x,t} - h_{x,t}] + D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial I_{x,t}}{\partial x}\right] \tag{3.2}$$

where units of x are chosen so that $x = 0$ denotes the Equator, $x = 1$ denotes the North Pole, and $x = -1$ denotes the South Pole; Q is the solar constant divided by 4;[10] $S(x)$ is the mean annual meridional distribution of solar radiation which is normalized so that its integral from -1 to 1 is unity; $\alpha(x, x_{st})$ is the absorption coefficient or co-albedo function which is one minus the albedo of the earth-atmosphere system, with x_{st} being the latitude of the ice line at time t; and D is a thermal diffusion coefficient. Following later work by North *et al.* (1983) c can be interpreted as an absorption factor. More about this below.

Equation (3.2) states that the rate of change of outgoing radiation is determined by the difference between the incoming absorbed radiant heat $QS(x)\alpha(x, x_{st})$ and the outgoing radiation $[I_{x,t} - h_{x,t}]$. Note that the outgoing radiation is reduced by the human input $h_{x,t}$. We associate human input with injection of carbon dioxide into the atmosphere at latitude x. We will throughout this paper assume that emissions of carbon dioxide from human activities at latitude x disperses rapidly across latitudes so that $h_{x,t} = h_{x',t}, \ \forall x, t$. This implies that we can replace $h_{x,t}$ in (3.2) with h_t, which will represent the aggregate human forcing on the climate system coming from the accumulation of carbon dioxide in the atmosphere defined as $h_t \equiv \int_{-1}^{1} h_{x,t} dx$.

We approximate human related forcing based on table 6.2 of the IPCC (2001) report. Define human related forcing as $h_t = \xi \ln\left(1 + \frac{M_t}{\bar{M}}\right)$ where $\bar{M}$ denotes the pre-industrial atmospheric CO_2 concentration and M_t stock of carbon dioxide in the atmosphere at time t above pre-industrial levels, where ξ denotes the approximate radiative forcing (W/m^2).

[8] For a continuous time analog of our derivations in this section see Brock *et al.* (2012b).

[9] It is important to note that the original Budyko (1969) formulation cited by North parameterizes A, B as functions of fraction cloud cover and other parameters of the climate system. North (1975b) points out that due to non-homogeneous cloudiness A and B should be functions of x. There is apparently a lot of uncertainty involving the impact of cloud dynamics (e.g. Trenberth *et al.* (2010) versus Lindzen and Choi (2009)). Hence robust control in which A, B are treated as uncertain may be called for but this is left for further research.

[10] The solar constant includes all types of solar radiation, not just the visible light. It has been measured by satellite to be roughly 1.376 kilowatts per square meter (kW/m^2) North *et al.* (1981).

The stock of the atmospheric carbon dioxide evolves according to

$$M_{t+1} - M_t = \sigma \int_{x=-1}^{x=1} q_{x,t} dx - m M_t,\ M_0 = M_{00} \tag{3.3}$$

where $\sigma q_{x,t}$ are emissions generated at latitude x, with emissions being proportional to the amount of fossil fuels used by latitude x at time t. The coefficient σ reflects emission intensity of the fossil fuels and m is the carbon dioxide removal rate from the atmosphere.

We assume that the total stock of fossil fuel available is fixed or,

$$\int_{x=-1}^{x=1} q_{x,t} dx = q_t \ ,\ \ \sum_{t=0}^{\infty} q_t \leq R_0 \tag{3.4}$$

where q_t is total fossil fuels used across all latitudes at time t, and R_0 is the total available amount of fossil fuels on the planet. Thus in this model the use of fossil fuels generates emissions, emissions increase the stock of atmospheric carbon dioxide, which in turn increases the temperature by blocking the amount of outgoing "long-wave" radiation.

As pointed out by North (1975b), in equilibrium at a given latitude the incoming absorbed radiant heat is not matched by the net outgoing radiation and the difference is made by the meridional divergence of heat flux which is modeled by the term $D\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial I_{x,t}}{\partial x}\right]$. This term explicitly introduces the spatial dimension, stemming from the heat transport, into the climate model.

Returning to the description of (3.2), the ice line is determined dynamically by the condition (Budyko, 1969; North, 1975a,b):

$$\begin{array}{ll} T > -10°\text{C} & \text{no ice line present at latitude } x \\ T < -10°\text{C} & \text{ice present at latitude } x \end{array} \tag{3.5}$$

and 'below' the ice line absorption drops discontinuously because the albedo jumps discontinuously. For example North (1975a) specifies, discontinuous co-albedo function as:

$$\alpha(x, x_s) = \begin{cases} \alpha_0 = 0.38\, |x| > x_s \\ \alpha_1 = 0.68\, |x| < x_s \end{cases} \tag{3.6}$$

3.2.1 Approximating Solutions for the Basic Energy Balance Equation

We turn now to a more detailed analysis of the solution process. Equation (3.2) is a PDE. One might think that we are going to have to deal with the complicated mathematical issues of the solution or the optimal control of PDEs when we need to discuss the economic optimization problems over space and time. But, as we shall see, the climate problem reduces to the optimal control of a small number of "modes" where each "mode" follows a simple ODE. We believe this decomposition is another important and new contribution of our paper to the study to coupled economic and climate models. Let us continue with the development of the solution procedure for equation (3.2) before turning to optimization.

North (1975b) approached the solution of (3.2) by using approximation methods.[11] In this case the solution is approximated as $\hat{I}_{x,t} = \sum_{n\text{even}} I_{n,t} P_n(x)$, where $I_{n,t}$ are solutions to appropriately defined ODEs and $P_n(x)$ are even numbered Legendre polynomials. A satisfactory approximation of the solution for (3.2) can be obtained by the so called two mode solution where $n = \{0, 2\}$. We develop here a two mode solution given the human forcing function h_t. Since we are going to use the temperature as the basic state variable we redefine (3.2) using (3.1), in terms of temperature $T_{x,t}$ and we have

$$cB(T_{x,t+1} - T_{x,t}) = QS(x)\alpha(x, x_s) - [(A + BT_{x,t}) - h_t] + DB\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial T_{x,t}}{\partial x}\right] \tag{3.7}$$

Using the approximation $\hat{T}_{x,t} = \sum_{n\text{ even}} T_{n,t} P_n(x)$, where now $T_{n,t}$ are solutions to appropriately defined ODEs the two mode solution is defined as:

$$\hat{T}_{x,t}(D) = T_{0,t} + T_{2,t}(D)P_2(x) \tag{3.8a}$$

$$cB(T_{0,t+1} - T_{0,t}) = -A - BT_{0,t} + \int_{-1}^{1} QS(x)\alpha(x, x_s)dx + \xi \ln\left(1 + \frac{M_t}{\bar{M}}\right) \tag{3.8b}$$

$$cB(T_{2,t+1} - T_{2,t}) = -B(1 + 6D)T_{2,t} + 5\int_{-1}^{1} QS(x)\alpha(x, x_s)P_2(x)dx \tag{3.8c}$$

$$T_{0,0} = T_{00}, T_{2,0} = T_{20}, P_2(x) = \frac{(3x^2 - 1)}{2} \tag{3.8d}$$

$$S(x) = \frac{1}{2}[1 + S_2P_2(x)]\ ,\ S_2 = -0.482 \tag{3.8e}$$

The derivation of the solution is presented in Appendix 1.[12] As can be seen the human input $\xi \ln\left(1 + \frac{M_t}{\bar{M}}\right)$ does not appear in the temperature dynamics of the second mode. This is since when human input is independent of x, or as we model it here given by an aggregate stock. The fact that $\int_{-1}^{1} P_2(x)dx = 0$ implies that human input disappears from at all modes except mode zero. Note, that we assume c is such that $|1 - 1/c| < 1$ for stability when there is no ice line. If $c = 1$, $T_{0,t}$ cancels from both sides of (3.8b). Given the definitions of the functional forms the two mode solution is

[11] From here on we denote approximating solutions such as $\hat{I}_{x,t}$ with a hat. For a general approach to approximation methods see for example Judd (1998).

[12] We can develop a series of approximations of increasing accuracy by solving this problem for expansions using a "two mode" solution, a "three mode" solution and so on. North's results suggest that the two mode solution is an adequate approximation. We use the two-mode approximation in our optimal control setting. A topic of further research could be an investigation of how many modes are needed for a good quality approximation in an optimal control setting.

tractable and can be calculated given initial conditions T_{00}, T_{02} which are determined by the initial climate state.

In the two-mode solution, the ice line function $x_s(t)$ which determines the co-albedo solves the equation $I_s = I(x_s(t), t)$. In terms of temperature and the using the two-mode solution, the ice line function solves

$$\hat{T}_{x,t}(D) = T_{0,t} + T_{2,t}(D)P_2(x_{st}) = T_s,\ T_s = -10°\text{C} \tag{3.9}$$

and the ice line function is given by a solution of (3.9), i.e.

$$x_{st} = P_+^{-1}\left(\frac{T_s - T_{0,t}}{T_{2,t}(D)}\right) \tag{3.10}$$

Where the subscript "+" denotes the largest inverse function of the quadratic function $P_2(x) := (1/2)(3x^2 - 1)$. Notice that the inverse function is unique and is the largest one on the set of latitudes $[-1, 1]$. Thus there exist a nonlinear feedback from changes in temperature to the co-albedo thought the endogeneity of the ice line. This feedback can be simplified by making the co-albedo function $\alpha(x, x_s)$ a smooth function of the temperature, $\alpha(x, \hat{T}_{x,t}(D))$ which can be highly nonlinear around -10°C.[13] A more simplified and tractable specification of the co-albedo is the one introduced by North *et al.* (1981) (p.95 equation (18)), where the co-albedo depends only on geographical location or

$$a(x) = 0.681 - 0.202P_2(x) \tag{3.11}$$

In this case the co-albedo function retains its latitude dependence and provides a significant simplification that helps tractability.

Use of global mean temperature and potential bias

The two-mode solution defines the climate module by (3.8a)-(3.8e), and (3.3),(3.4). Although the climate module does not contain the PDE (3.7) that incorporates temperature diffusion, spatial interactions are incorporated through the mode-2 part of the solution the ODE (3.8c). Thus the contribution of the second mode into the full solution can be regarded as the "importance of space" through heat transport, in the analysis of climate change. This can be seen by the following argument.

The size of diffusion coefficient D determines the speed of spatial diffusion in (3.7). If $D = 0$ then are no spatial interactions, if $D \to \infty$ then we have instantaneous mixing and spatial homogeneity and thus the heat transport across latitudes is not relevant for our problem. In this case, the mode two solution vanishes. To show this note that since the total amount of fossil fuel is finite and the contributions to the stock of atmospheric carbon dioxide is due to the use of fossil fuels, the stock of the atmospheric carbon dioxide $M(t)$ must be bounded above. Thus the second term of the right hand side of (3.8c) is bounded above. Then the following proposition can be stated

[13] For example the co-albedo function $\alpha(x, T(x,t)) = c_0 + c_1 \tanh(T(x,t) + 10)$ for $(c_0, c_1) = (.525, .195)$ provides a good approximation of the discontinuous function defined by (3.5)-(3.6).

Proposition 3.1. *Assume that* $\int_{-1}^{1} QS(x)\alpha(x,x_s)P_2(x)dx \leq UB < \infty$, *and that* $D \to \infty$. *Then the solution* $T_{2,t}$ *of (3.8c) vanishes.*

Proof. equation (3.8c) can be written as
$c(T_{2,t+1} - T_{2,t}) = -(1+6D)T_{2,t} + (5/B)\int_{-1}^{1} QS(x)\alpha(x,x_s)P_2(x)dx$. As $D \to \infty$ any steady state of (3.8c) defined as $T^{+}_{2,t} = \frac{5}{B(1+6D)}\int_{-1}^{1} QS(x)\alpha(x,x_s)P_2(x)dx \to 0$. Furthermore, consider the ODE

$$c(\bar{T}_{2,t+1} - \bar{T}_{2,t}) = -(1+6D)\bar{T}_{2,t} + (5/B)UB. \tag{3.12}$$

Since $c(T_{2,t+1} - T_{2,t}) \leq -(1+6D)T_2 + (5/B)\,UB$, then by Gronwall's inequality the solution of (3.8c) will be bounded above by the solution $\bar{T}_{2,t}$ of (3.12). This solution however goes to zero as $D \to \infty$. Therefore $T_{2,t} \to 0$ as $D \to \infty$. □

Thus for a given diffusion $D < \infty$ the relative contribution of $T_{2,t}$ to the solution $\hat{T}_{x,t}$ can be regraded as an a measure of whether the heat transport is important in the solution of the problem.

This result can be used to suggest that the use of the global mean temperature alone in IAMs may introduce a bias. From the two mode approximation of the temperature, we obtain the global mean temperature as $m_T = T_{0,t}$.[14] This result, along with proposition 1, indicates that the zero - dimensional IAMs can be regarded as a special case of a one-dimensional model when $D \to \infty$. Thus the second mode that provides that spatial distribution of temperature is omitted in the zero - dimensional IAMs. Since scientific evidence indicate that D is small (less than one according to North *et al.* (1981)) our result suggest that the dropping of the second mode by the IAMs introduces a kind of bias. In our paper we correct for this underlying bias by keeping that second mode, and we also provide a basis for a quantitative representation of this bias. The variance of the global mean temperature is:

$$V_T = \int_{-1}^{1} \left[\hat{T}_{x,t}(D) - T_{0,t}\right]^2 dx = \int_{-1}^{1} (T_{2,t}(D)P_2(x))^2 dx = \frac{2}{5}(T_{2,t}(D))^2 \tag{3.13}$$

In an IAM this variance will be zero since the second mode is dropped.

Local temperature means at latitudes $(x, x+dx)$ and the mean of temperature over the set of latitudes $Z = [a,b]$ are defined by

$$[T_{0,t} + T_{2,t}(D)P_2(x)]\,dx, \quad m\,[a,b] = \int_a^b [T_{0,t} + T_{2,t}(D)P_2(x)]\,dx \tag{3.14}$$

while the variance of temperature over the set of latitudes $Z = [a,b]$ is

$$V\,[a,b] = \int_a^b [T_{0,t} + T_{2,t}(D)P_2(x) - m\,[a,b;t]]^2\,dx \tag{3.15}$$

[14] This is because $m_T = \int_{-1}^{1} \hat{T}_{x,t}dx = \int_{-1}^{1} [T_{0,t} + T_{2,t}P_2(x)]\,dx$ and $\int_{-1}^{1} P_2(x)dx = 0$.

It might be plausible to assume that utility in each area $[a,b]$ depends upon both the mean temperature and the variance of temperature in that area. For example we may expect increases in mean temperature and variance to have negative impacts on output in any area Z, if it is located in tropical latitudes. Whereas mean temperature increases in some areas Z (e.g. Siberia) may increase utility rather than decrease utility.[15]

3.3 An Economic EBC Model

In this section we characterize the solution to the planning problem. Based on the results of section 3.2 this implies that a well defined planning problem should consider resource allocation across both time and space. For the remainder of the paper we drop the hat ($\hat{T}_{x,t}$) notation for local temperature to ease up on notation. If nothing else is stated $T_{x,t}$ henceforth refers to the approximate solution given by (3.8). We first describe the general features of the economy and then proceed with the problem of the planner. In the proceeding section we then characterize the market equilibrium.

3.3.1 General features of the economy

The economy is assumed to be inhabited by a representative household at latitude x having preferences defined by the following utility function.[16]

$$U(C_{x,t}/L_{x,t}) = \frac{(C_{x,t}/L_{x,t})^{1-\theta} - 1}{(1-\theta)} \tag{3.16}$$

where $C_{x,t}$ denotes aggregate consumption and $L_{x,t}$ the size of the representative household (equal to population) at time t and for latitude x. U is thus a standard concave utility function having a constant inter-temporal elasticity of substitution $1/\theta$. Labor is supplied inelastically and is equal to population, which grows at a constant factor n so that $L_{x,t} = L_{x,0}n^{t-1}$. Production takes place separately at each latitude

$$Y_{x,t} = \mathbb{A}_{x,t}\Omega(T_{x,t})F(K_{x,t}, L_{x,t}, q_{x,t}) \tag{3.17}$$

where $K_{x,t}, L_{x,t}, q_{x,t}$ denote capital, labor and fossil fuels respectively used at latitude x, and time t, a is the TFP growth so that $\mathbb{A}_{x,t} = \mathbb{A}_{x,0}a^{t-1}$, and $\Omega(T_{x,t})$ are the damages to output due to climate change occurring at latitude x and time t as a function of the

[15] In a stochastic generalization of our model, we could introduce a stochastic process to represent "weather," i.e. very high frequency fluctuations relative to the time scales we are modeling here. Here the "local variance" of high frequency phenomena like "weather" may change with changes in lower frequency phenomena such as mean area Z temperature and area Z temperature variance. We leave this task to future research. Existing dynamic integrated models of climate and economy, (e.g. Nordhaus's well known work (2007, 2010)) can not deal with these kinds of spatial elements, such as impacts of changes in temperature variance, generated by climate dynamics over an area Z.

[16] We will throughout the rest of the paper think of x as being a country when discussing economic activity along a specific latitude. This is of course a ludicrous abstraction from reality. However, as this paper constitutes a first stab at bringing in latitude dependent temperatures into climate-economy models we have preferred to stick to a general setting, prioritizing mathematical neatness and thus leaving more realistic descriptions for future research.

local temperature at the that latitude, with $\frac{\partial \Omega(T_{x,t})}{\partial T_{x,t}} < 0$. The total amount of fossil fuels available is finite and given by equation (3.4) of the previous section. The spatial aspect of our model also induces the possibility to consider the possibility of intertemporal trade. We denote the net stock of foreign bond or asset holdings at latitude x by $B_{x,t}$ which generate a return R_t. Negative values thus implies that latitude x has an outstanding debt to other latitudes. The budget constraint of each country is thus given by

$$C_{x,t} + K_{x,t+1} + B_{x,t+1} \leq Y_{x,t} + (1-\delta)K_{x,t} + R_t B_{x,t} \tag{3.18}$$

from which we can write the global budget constraint by integrating over x

$$C_t + K_{t+1} \leq Y_t + (1-\delta)K_t \tag{3.19}$$

where[17]

$$0 = \int_x B_{x,t}dx, \quad C_t = \int_x C_{x,t}dx, \quad K_t = \int_x K_{x,t}dx, \quad Y_t = \int_x Y_{x,t}dx \tag{3.20}$$

3.3.2 Global welfare maximization

Given the description of the economy and climate dynamics provided so far, it is clear that the welfare maximization problem of a planner, should involve both spatial and intertemporal decision making. We formalize this by defining a global social welfare function where each individual country is pre-assigned a specific welfare weight, so that by varying these weights we can trace out different distributional outcomes for the global economy. The welfare function of the planner can be written as

$$\sum_{t=0}^{\infty} \beta^t \int_x \upsilon(x) L_{x,t} U(C_{x,t}/L_{x,t}) dx dt \tag{3.21}$$

where β represents the discount factor and $\upsilon(x)$ denotes the latitude specific welfare weight assigned to the planner.[18] An early result noted by Chichilnisky and Heal (1994) shows that in a multi-country optimal planning problem featuring both a private and a public good, whether the solution to the problem will equalize marginal abatement costs across countries will largely depend upon whether international transfers are available or not.[19] In the absence of such transfers a spatially uniform carbon tax is thus in general not possible.[20] Within the one-dimensional energy balance model we consider

[17] To ease notation we introduce the $\int_x$ which denotes the integral over all x's and is equivalent to $\int_{-1}^{1}$ used previously. We will use these interchangeably from here on.

[18] The welfare weights are assigned so that $\int_x \upsilon(x)dx = 1$. An example of a weight independent of x would be $\upsilon(x) = 1/2$.

[19] Similar results have also been obtained by (Chichilnisky *et al.*, 2000; Shiell, 2003b; Sandmo, 2006; Anthoff, 2011; Keen and Kotsogiannis, 2011).

[20] Transfers of this kind are significant and observed in the real world not only in terms of foreign aid and development assistance but also in terms of payments for the supply of public goods examples include the Joint implementation and Clean development mechanisms incorporated into the Kyoto protocol. However, it is also true that countries to a great extent are limited by their own resources and incomes.

here, the heat diffusion across latitudes creates a spatially differentiated damage function which makes the results of Chichilnisky and Heal (1994) increasingly relevant. In particular, since international transfers may many times prove to be hard to implement in practice.[21] In order to highlight the role of international transfers we will consider three specific cases.

In the first case, which we will refer to as the *open economy* problem, the economy is completely open with free flows of capital, fossil fuel and consumption goods across latitudes. This implies that returns to capitals and bonds will be equal across latitudes.[22] The planner has the power to administer lump-sum transfers of goods across latitudes which implies that all Pareto optimal allocations corresponding to a particular set of welfare weights can be traced out. Due to his ability to freely transfer goods across latitudes the investment decisions of this planner is thus not limited by the capital budget constraint (3.18) involving each individual latitude but rather the sum, or as we have chosen to model it here, the integral over all latitude specific capital budget constraints.

Turning to the second case. We refer to this case as the *closed economy* problem since each latitude is limited by its own budget constraint. As will be shown, in this case, depending on whether the planner has access to international transfers or not, optimal tax rates may differ across latitudes. The particular assumptions connected to this scenario are restrictive and perhaps not so realistic but they help bring out the forces that can generate spatially differentiated taxe rates. In particular we show that if wealth transfers are restricted and welfare weights are assumed to be equal across latitudes this implies that optimal tax rates will be spatially differentiated.

Finally the last case we consider concerns when transfers are *costly*. In this case international transfers are possible but no longer free. This case constitutes a middle way between the *closed economy* and *open economy* case. Here we show that as with the case of the *closed economy*, the tax or social price connected to the use of fossil fuels may differ across latitudes.

We start by solving the *open economy* problem involving free international transfers. We then characterize the competitive equilibrium and derive the optimal tax rates which implement the planning solution. In the following sections we will explore the alternative representations. For all these cases we will employ the two mode approximation defined in section 3.2 when introducing the economic model to the climate dynamics of EBCM's.

[21]Shiell (2003a) points to e.g. to corruption as one possibility that can make large international transfers unfeasible. In this case, the opportunity to help the poor through untied transfers may be seriously compromised by misappropriation or theft by officials who are charged with administering the transfers in recipient countries.

[22]We neglect migration and assume labor is completely immobile across latitudes. Labor immobility at a global scale could be regarded as a reasonable approximation given restrictions on labor mobility relative to capital and fossil fuel mobility.

3.3.3 Optimal planning problem with open markets and free international transfers

Given the economy and climate dynamics described above we start by considering the welfare maximization problem of an *open economy* where the planner has free access to international transfers across latitudes. In this case the planner maximizes the welfare function defined in equation (3.21) subject to (3.3),(3.4), (3.8), (3.19) and (3.20). As can be seen from inspection of this maximization problem, it can be broken down into a static and dynamic problem which can be considered separately. We start by characterizing the static problem.

Static problem

The spatial allocation decision facing the planner involves the allocation of consumption, capital and fossil fuels to each latitude so as to maximize global output and social welfare in each time period subject to the global capital budget constraint (3.19). As will be seen the solution to this problem implies that the marginal products of $K_{x,t}$ and $q_{x,t}$ must be equal across latitudes and that the weighted marginal utility of $C_{x,t}$ at each latitude must also be equal.

First, in order to determine how capital and fossil fuels are allocated across latitudes given the global intertemporal investment and fossil fuel use decisions governed by the capital budget constraint, we proceed by solving the following problem; assuming a Cobb-Douglas production function:[23]

$$\begin{aligned} F_{total}(K_t, q_t, \{T_{x,t}\}_{x=-1}^{x=1}; t) &\equiv \max\{\int_x (an^{\alpha_L})^{t-1} \mathbb{A}_{x,0} L_{x,0}^{\alpha_L} \Omega(T_{x,t}) K_{x,t}^{\alpha_K} q_{x,t}^{\alpha_q} dx\} \\ &= (an^{\alpha_L})^{t-1} K_t^{\alpha_K} q_t^{\alpha_q} \max\{\int_x \mathbb{A}_{x,0} L_{x,0}^{\alpha_L} \Omega(T_{x,t}) (K_{x,t}/K_t)^{\alpha_K} (q_{x,t}/q_t)^{\alpha_q} dx\} \\ \text{s.t.} \int_x K_{x,t}/K_t dx &\leq 1, \ \int_x q_{x,t}/q_t dx \leq 1 \end{aligned} \tag{3.22}$$

We define $F_{total}(K_t, q_t, \{T_{x,t}\}_{x=-1}^{x=1}; t)$ as the "*potential world GDP at date t*". This concept represents the maximum output that the whole world can produce given total world capital K_t available and total world fossil fuel q_t used, for a given distribution of temperature $T_{x,t}$ across the globe, with labor growing with the constant factor n, and treated as immobile. Thus F_{total} can be regarded as a natural base line under ideal world conditions where there's no barriers to capital and fossil fuel flows to their most productive uses across latitudes.[24] We abuse notation and write $F_{total}(K_t, q_t, \{T_{x,t}\}_{x=-1}^{x=1}; x, t) = F_{total}(K_t, q_t, T; t)$. Next, define the shares of K and q at

[23] Hassler *et al.* (2011) argue that, on shorter time horizons, Cobb-Douglas production does not represent a good way of modeling energy demand since it does not capture the joint shorter- to medium-run movements of input prices and input shares. However, On longer time horizons considered here it is more reasonable since input shares do not appear to trend.

[24] This notion can be regarded as similar to the notions of "potential GDP" "potential output" etc used by macro economists.

date t by $S^K_{x,t} \equiv K_{x,t}/K_t$ and $S^q_{x,t} \equiv q_{x,t}/q_t$. It is now easy to check that the optimal shares solving problem (3.22) will be given by[25]

$$S^K_{x,t} = S^q_{x,t} = (\mathbb{A}_{x,0} L^{\alpha_L}_{x,0} \Omega(T_{x,t}))^{\frac{1}{\alpha_L}} / \int_x (\mathbb{A}_{x,0} L^{\alpha_L}_{x,0} \Omega(T_{x,t}))^{\frac{1}{\alpha_L}} dx \tag{3.23}$$

Substituting the optimal shares back into the objective function we obtain the following expression for potential world GDP

$$F_{total}(K_t, q_t, \{T_{x,t}\}^{x=1}_{x=-1}; t) = (an^{\alpha_L})^{t-1} K_t^{\alpha_K} q_t^{\alpha_q} \left(\int_x (\mathbb{A}_{x,0} L^{\alpha_L}_{x,0} \Omega(T_{x,t}))^{\frac{1}{\alpha_L}} dx \right)^{\alpha_L} \tag{3.24}$$

As it can be seen from (3.24) the Cobb-Douglas specification allows the "separation" of the climate damage effects on production across latitudes. Define

$$J\left(\{T_{x,t}\}^{x=1}_{x=-1}\right) \equiv \left(\int_x (\mathbb{A}_{x,0} L^{\alpha_L}_{x,0} \Omega(T_{x,t}))^{\frac{1}{\alpha_L}} dx \right)^{\alpha_L} \tag{3.25}$$

This expression depends on the thermal diffusion coefficient D which multiplies a production function that is independent of x. Thus population growth and technical change affect the "macrogrowth component" $(an^{\alpha_L})^{t-1} K_t^{\alpha_K} q_t^{\alpha_q}$, while changes in the size of D have a direct effect on the "climate component". The combination of the macrogrowth and the climate component determine the potential world input.

Second, the allocation consumption across latitudes is solved in an equivalent manner by maximizing (3.21) subject to $\int_{x'} C_{x',t} \leq C_t$. Hence, $C_{x,t}$ is restricted by the total amount of available consumption C_t which in turn is restricted by the global capital budget constraint. The first order conditions with respect $C_{x,t}$ thus implies

$$v(x)\beta^t U'(C_{x,t}/L_{x,t}) = v(x')\beta^t U'(C_{x',t}/L_{x',t}) \tag{3.26}$$

which further implies that

$$\begin{aligned} C_t = \int_{x'} C_{x',t} dx' &= \int_{x'} \left(\frac{v(x)}{v(x')} \frac{L^\theta_{x,t}}{L^\theta_{x',t}} \right)^{\frac{-1}{\theta}} C_{x,t} dx' \\ &= (v(x) L^\theta_{x,0})^{\frac{-1}{\theta}} \int_{x'} \left(v(x') L^\theta_{x',0} \right)^{\frac{1}{\theta}} dx' C_{x,t} \end{aligned} \tag{3.27}$$

as with the capital and fossil fuel shares we define consumption shares as $S^C_{x,t} \equiv C_{x,t}/C_t$.

The dynamic problem

Together the static solutions (3.24) and (3.27) allows us to write the ideal planning problem completly in aggregate terms. Integrating (3.18) over x after having substituted in the static solutions we can thus express the ideal planning problem as

$$\max \sum_{t=0}^{\infty} (\beta n^\theta)^t U(C_t), \tag{3.28}$$

$$\text{s.t. } C_t + K_{t+1} \leq \tilde{Y}_t + (1-\delta) K_t \tag{3.29}$$

[25]See appendix for a complete derivation.

and (3.3),(3.4),(3.8) where $\tilde{Y}_t \equiv F_{total}(K_t, q_t, T; t)$. The first order necessary conditions w.r.t. K_{t+1} gives us the standard Euler equation

$$U'(C_t) = \beta n^\theta U'(C_{t+1}) \left(\frac{\partial \tilde{Y}_{t+1}}{\partial K_{t+1}} + (1-\delta) \right) \tag{3.30}$$

Since the problem is concave in K this is sufficient for optimum over K. Turn now to optimization w.r.t. q. At each date t a small increase in q_t gets a marginal gain in output at t but sets of a sequence of marginal changes in damages for periods $t+1, t+2, .$ The first order necessary conditions for q_t at date t is given by

$$(\beta n^\theta)^t U'(C_t) \frac{\partial \tilde{Y}_t}{\partial q_t} = -\sum_{k=2}^{\infty} (\beta n^\theta)^{t+k} U'(C_{t+k}) \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t} + \mu_0 \tag{3.31}$$

This expression is fundamental to our paper as it characterizes how the spatial aspects affect optimal fossil fuel use.[26] It is also straightforward to interpret. The first order condition simply states that the marginal social utility of using an extra unit of fossil fuel (left hand side) must equal the marginal social cost (right hand side). The marginal social cost consists of both the capitalized sum of marginal damages to output at $t+1, t+2, ...$ plus the marginal cost of not having that unit of fossil fuel available in the future (when R_0 is ultimately all used up).[27]

Alternatively, the capitalized sum of marginal damages could have been expressed as in Golosov *et al.* (2011) in terms of a shadow price (λ^s)

$$\lambda_t^s \equiv -\sum_{k=2}^{\infty} (\beta n^\theta)^{t+k} U'(C_{t+k}) \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t} \tag{3.32}$$

or alternatively, in consumption units $\Lambda_t^s \equiv \lambda_t^s / U'(C_t)$

$$\Lambda_t^s \equiv -\sum_{k=2}^{\infty} (\beta n^\theta)^{t+k} \frac{U'(C_{t+k})}{U'(C_t)} \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t} \tag{3.33}$$

Equation (3.33) is thus our analog of equation 11 in Golosov *et al.* (2011). We can now from equation (3.31) write down a spatial version of the climate externality adjusted

[26] The formula was derived using $c = 1$ in (3.8). This simplifies the f.o.c. which otherwise would have been given by

$$(\beta n^\theta)^t U'(C_t) \frac{\partial \tilde{Y}_t}{\partial q_t} = -\sum_{k=2}^{\infty} (\beta n^\theta)^{t+k} U'(C_{t+k}) \frac{\partial \tilde{Y}_{t+k}}{\partial T_{t+k}} \left(\sum_{s=1}^{k-1} \frac{\partial T_{t+k}}{\partial M_{t+s}} \frac{\partial M_{t+s}}{\partial q_t} \right) + \mu_0$$

We will keep this assumption throughout the rest of the paper.

[27] When there are damages to consumption, there will be an extra term on the right hand side representing the capitalized sum of marginal damages to future social utility of climate during the following dates. This term can be potentially important for optimal mitigation decisions as shown in Sterner and Persson (2008). For simplicity we abstract from this here.

Hotelling formula

$$\frac{U'(C_t)}{\beta n^{\theta} U'(C_{t+1})} = \frac{\frac{\partial \tilde{Y}_{t+1}}{\partial q_{t+1}} - \Lambda^s_{t+1}}{\frac{\partial \tilde{Y}_t}{\partial q_t} - \Lambda^s_t} \tag{3.34}$$

which by the Euler equation (3.30) above we know equals the return on investments or equivalently the real interest rate. Hence, once again we see how the return to postponing extraction equals the return on capital investments.

In order to provide a more clear picture of the impact of thermal diffusion on optimal fossil fuel use we simplify the climate dynamics following section 3.2.1, assuming that the co-albedo function is independent of the iceline i.e. $\alpha(x, x_s) = \alpha(x)$. Then from (3.8a) the zero mode dynamics can be written as

$$cB(T_{0,t+1} - T_{0,t}) = -A - BT_{0,t} + \int_{-1}^{1} QS(x)\alpha(x)dx + \xi \ln\left(1 + \frac{M_t}{\bar{M}}\right) \tag{3.35}$$

with the approximating temperature at each latitude determined by $T_{x,t} = T_{0,t} + T_{2,t}P_2(x)$ where $T_{2,t}P_2(x)$ is thus independent of M, which thus also implies that $\frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} = \frac{\partial \tilde{Y}_{t+k}}{\partial T_{0,t+k}} \frac{\partial T_{0,t+k}}{\partial M_{t+k-1}}$ which greatly simplifies the analysis while still maintaining implicitly the temperature at each latitude.

To proceed further with analytical results we make some more assumptions in line with Golosov *et al.* (2011), i.e. we assume log utility, Cobb-Douglas production, exponential damages and full depreciation of capital, which implies that the Euler equation (3.30) can be solved for the optimal investment and consumption rate. It follows immediately that the following rules solves the optimal saving decisions.

$$K_{t+1} = \alpha_K \beta n^{\theta} \tilde{Y}_t, \quad C_t = (1 - \alpha_K \beta n^{\theta})\tilde{Y}_t \tag{3.36}$$

Next, given an exponential damage function as this implies that damages can be written in the following multiplicative form $\Omega(T_{x,t}) = \Omega(T_{0,t} + T_{2,t}P_2(x)) = \Omega(T_{0,t})\Omega(T_{2,t}P_2(x))$. Under these assumptions we may now write (3.25) as

$$\begin{aligned} J(\{T_{x,t}\}_x) &= \left(\int_x \Omega(T_{0,t} + T_{2,t}P_2(x))^{1/a_L} A_{x,0}^{1/a_L} L_{x,0} dx\right)^{a_L} \\ &= \Omega(T_{0,t}) \left(\int_x \Omega(T_{2,t}P_2(x))^{1/a_L} A_{x,0}^{1/a_L} L_{x,0} dx\right)^{a_L} \end{aligned} \tag{3.37}$$

Hence, given the damage function $\Omega(T_{x,t}) = e^{-\gamma T_{0,t+k}}$ we have

$$\frac{\partial \tilde{Y}_{t+k}}{\partial T_{0,t+k}} = \frac{\partial \tilde{Y}_{t+k}}{\partial T_{0,t+k}} \frac{\tilde{Y}_{t+k}}{\tilde{Y}_{t+k}} = \frac{\Omega'(T_{0,t})}{\Omega(T_{0,t})} \tilde{Y}_{t+k} = -\gamma \tilde{Y}_{t+k} \tag{3.38}$$

substituting this expression into (3.33) and making use of (3.36) we have

$$\Lambda^s_t = \gamma \tilde{Y}_t \sum_{k=2}^{\infty} (\beta n^{\theta})^{t+k} \frac{\partial T_{0,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t} \tag{3.39}$$

Thus we see that damages as a fraction of GDP, $\Lambda_t^s/\tilde{Y}_t$, is given by a very simple formula because $\frac{\partial M_{t+k-1}}{\partial q_t}$ is a constant, and if the dynamics of $T_{0,t}$ are forced by a constant times M_t then $\frac{\partial T_{0,t+k}}{\partial M_{t+k}}$ is a constant and if the dynamics are forced by log function as defined in section 2 then $\frac{\partial T_{0,t+k}}{\partial M_{t+k}}$ is a constant divided by $(1 + M_{t+k}/M_0)$. Finally the Hotelling equation becomes completly independent of capital investment and takes the simple form.

$$\frac{1}{\beta n^\theta} = \frac{\alpha_q \frac{1}{q_{t+1}} - \gamma \sum_{k=2}^{\infty} (\beta n^\theta)^k \frac{\partial T_{0,t+k+1}}{\partial M_{t+k}} \frac{\partial M_{t+k}}{\partial q_{t+1}}}{\alpha_q \frac{1}{q_t} - \gamma \sum_{k=2}^{\infty} (\beta n^\theta)^k \frac{\partial T_{0,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t}} \tag{3.40}$$

Hence, we see that by adopting the same assumptions used by Golosov *et al.* (2011) we can arrive at a similar expression for optimal fossil fuel use, independent of decisions on saving, also with our more complex one-dimensional climate model.

3.3.4 The Market Equilibrium and Fossil Fuel Taxes

In this section we characterize the decentralized market equilibrium which given an appropriate set of transfers can implement the planning solution described above. As will be shown this is possible given that taxes rates are set equal to the emission externality. We start by characterizing the market equilibrium with taxes and then discuss optimal taxation within this setup. In the following, we implicitly assume the existence of a government that not only imposes externality taxes on latitude x-firms but also transfers resources. The government must thus satisfy its own budget constraint with taxes collected on the income side and lump sum redistributions on the outgoing side.

Consumers

Consumers at latitude x can borrow and lend on world bond markets at the gross interest rate R_t to solve the following problem

$$\max \sum_{t=0}^{\infty} \beta^t L_{x,t} U(C_{x,t}/L_{x,t}) \tag{3.41}$$

subject to

$$\sum_{t=0}^{\infty} p_t^k (C_{x,t} + K_{x,t+1} + B_{x,t+1}) = \sum_{t=0}^{\infty} p_t^k (R_{x,t} K_{x,t} + R_t B_{x,t} + I_{x,t}) \tag{3.42}$$

$$B_{x,0} = B_{x0}, K_{x,0} = K_{x0} \tag{3.43}$$

$$I_{x,t} \equiv w_{x,t} L_{x,t} + p_t q_{x,t} + \tau_{x,t} q_{x,t} \tag{3.44}$$

where $B_{x,t}$ denotes the amount of foreign bonds held at location x and time t.[28] The competitive price of consumption goods at time t is denoted by p_t^k. Labor $L_{x,t}$ is

[28] B_{x0} and K_{x0} denote the initial amounts of bonds and capital holdings by latitude x at the outset. We further assume that initial bond holdings are zero for all latitudes x.

perfectly inelastically supplied at the wage rate $p_t^k w_{x,t}$. Profits from fossil fuel firms (Hotelling rents) and taxes on fossil fuel use in production expressed in consumption prices are redistributed lump sum to latitude x consumers in the fraction $p_t q_{x,t}$ and $\tau_{x,t} q_{x,t}$ respectively.[29] $R_{x,t}$ and R_t denote the rental rate of capital and the return on government bonds respectively defined in terms of consumption prices. Further, the transversality conditions of capital and bonds are given by

$$\lim_{t\to\infty} \beta^t B_{x,t} \prod_{i=1}^{t} R_i^{-1} = 0, \quad \lim_{t\to\infty} \beta^t K_{x,t} \prod_{i=1}^{t} R_i^{-1} = 0 \tag{3.45}$$

From the first order necessary condition for consumption, capital and bonds we have

$$U'(C_{x,t}/L_{x,t}) = \Lambda_x^k p_t^k \tag{3.46}$$

$$R_{x,t+1} = \frac{p_t^k}{\beta p_{t+1}^k} = R_{t+1} \tag{3.47}$$

this gives us the usual Euler equation as

$$\frac{U'(C_{x,t})}{\beta n^\theta U'(C_{x,t+1})} = R_{x,t+1} = R_{t+1} \tag{3.48}$$

Consumption Goods Producing Firms

Consumption goods producing firms at latitude x solve the following problem

$$\max \left\{ p_t^k \left[\mathbb{A}_{x,t}\Omega(T_{x,t}) F(K_{x,t}, L_{x,t}, q_{x,t}) - (R_{x,t} - (1-\delta))K_{x,t} - w_{x,t}L_{x,t} - (p_t + \tau_{x,t})q_{x,t} \right] \right\} \tag{3.49}$$

where p_t denotes the world price fossil fuels plus the unit tax $\tau_{x,t}$ levied at x at date t, $w_{x,t}$ is the wage at location x and time t.[30] $F(K, L, q)$ is constant returns to scale, hence profits will be zero at each x for firms that produce consumption goods. The optimality conditions for (3.49) the optimal choices for K and q imply:

$$\mathbb{A}_{x,t}\Omega(T_{x,t}) F_K'(K_{x,t}, L_{x,t}, q_{x,t}) = R_{x,t} - (1-\delta) \tag{3.50}$$

$$\mathbb{A}_{x,t}\Omega(T_{x,t}) F_L'(K_{x,t}, L_{x,t}, q_{x,t}) = w_{x,t} \tag{3.51}$$

$$\mathbb{A}_{x,t}\Omega(T_{x,t}) F_q'(K_{x,t}, L_{x,t}, q_{x,t}) = p_t + \tau_{x,t} \tag{3.52}$$

Thus in any decentralized problem latitude x firms will choose demands $K_{x,t}$, $L_{x,t}$ and $q_{x,t}$ according to (3.50), (3.51) and (3.52). Note that the marginal product of $q_{x,t}$ is equated across x's for every date t only if taxes on fossil fuels are equal across locations so that $\tau_{x,t} = \tau_t$.

[29] In baseline analysis using Arrow Debreu private ownership economies, it is standard to assume perfect markets (borrowing and lending with no frictions, defaults, etc.) with profits and taxes redistributed lump sum to consumers.

[30] Wages are not equated across locations due to labor immobility.

Fossil fuel firm

A representative competitive fossil fuel firm solves the following problem

$$\max_{q_{x,t}} \sum_{t=0}^{\infty} \prod_{i=1}^{t} R_i^{-1} p_t q_{x,t}, \quad \text{s.t.} \sum_{t=0}^{\infty} \int_x q_{x,t} dx \leq R_0 \tag{3.53}$$

This implies that the fossil fuel firm maximizes the discounted sum of profits under zero extraction costs implying the following rule

$$p_{t+1} = R_{t+1} p_t, \quad t = 0, 1, 2, ... \tag{3.54}$$

3.3.5 Optimal carbon taxes in the *open economy*

Given the competitive market equilibrium described above we now proceed by showing that given an appropriately defined tax the competitive equilibrium described above can implement the solution to the planning problem. We start by defining a tax τ_t^* given by

$$\tau_t^* \equiv \Lambda_t^s, \quad \forall x \tag{3.55}$$

where Λ_t^s is given by equation (3.33) of the planning problem. By setting $\tau_{x,t} = \tau_t^*$, we see that as in the planning problem the first order conditions of the representative firm (3.50) and (3.52) become equal for all latitudes. Hence, from the static solution to the planning problem it thus follows that the shares of capital and fossil fuels in the competitive equilibrium will also be allocated according to $K_{x,t} = S_{x,t}^K K_t$ and $q_{x,t} = S_{x,t}^q q_t$ where $S_{x,t}^K$ and $S_{x,t}^q$ are defined in expression (3.23) of the planning problem. Comparing the competitive first order condition with the planning solution it also follows that $\frac{\partial Y_{x,t}}{\partial q_{x,t}} = \frac{\partial \tilde{Y}_t}{\partial q_t}$ and $\frac{\partial Y_{x,t}}{\partial K_{x,t}} = \frac{\partial \tilde{Y}_t}{\partial K_t}$. Further, since $S_{x,t}^C$ cancels out when inserting (3.27) into the Euler condition (3.30) of the planning problem we also have that $\frac{U'(C_{x,t})}{\beta n^\theta U'(C_{x,t+1})} = \frac{U'(C_t)}{\beta n^\theta U'(C_{t+1})}$. This implies that after combining condition (3.50) and (3.48) the resulting expression matches the planners Euler equation (3.30) of the planning problem.

Turning to the optimality condition of fossil fuel firms we can after inserting (3.48) and (3.52) into (3.54) write out a market type Hotelling rule

$$\frac{U'(C_{x,t})}{\beta n^\theta U'(C_{x,t+1})} = \frac{\frac{\partial Y_{x,t+1}}{\partial q_{x,t+1}} - \tau_{x,t+1}}{\frac{\partial Y_{x,t}}{\partial q_{x,t}} - \tau_{x,t}} \tag{3.56}$$

Using the definition of optimal taxes (3.55) and our previous results regarding the allocation of $K_{x,t}$ and $q_{x,t}$ in competitive equilibrium it is thus clear by inspection of (3.34) that if we set $\tau_{x,t} = \tau_t^*$ in (3.56), this expression satisfies the externality adjusted Hotelling equation of the planner. From the results derived so far we see that the competitive equilibrium produces the same necessary optimality conditions as in the planning problem. We are now ready to state the following proposition

Proposition 3.2. *The set of welfare weights $\{\upsilon(x)\}_{x\in[-1,1]}$ that produce the Pareto efficient outcome corresponding to the competitive equilibrium with zero transfers must satisfy $\upsilon(x) = 1/\Lambda_x^k$.*

This is simply an application of the first theorem of welfare economics. To see it clearer, denote λ_t^k as the Lagrangian multiplier of the planners budget constraint (3.29). From the results derived so far we see that the price of the consumption good in competitive equilibrium must rise at the same rate as the multiplier of the planners budget constraint i.e. $p_t^k/p_{t+1}^k = \lambda_t^k/\lambda_{t+1}^k$. Hence, by normalizing prices so that $p_0^c = \lambda_0^k$ implies that equilibrium prices will correspond exactly to the Lagrangian multipliers of the planning problem and it follows directly that by setting $\upsilon(x) = 1/\Lambda_x^k$ the first order condition with respect to consumption in the planning problem matches exactly the first order condition of the representative consumer.[31] Finally, from the first order conditions of the goods producers it also follows that after inserting these into the consumers budget constraint the resulting expression satisfies the planners budget constraint.

From the discussion above we have thus asserted that the competitive equilibrium described in section 3.3.4 is indeed Pareto optimal given that welfare weights are defined appropriately. However this is not the end of the story, following the second theorem of welfare economics all other Pareto efficient allocations associated with different welfare weights can also be attained as a competitive equilibrium with transfers. All that is required is a set of transfers that adjust the multiplier Λ_x^k so that its reciprocal equals the weight $\upsilon(x)$ assigned to latitude x. We summarize this in the following proposition which is a consequence of the second theorem of welfare economics

Proposition 3.3. *For any arbitrary set of welfare weights $\{\upsilon(x)\}_{x\in[-1,1]}$ the planning problem can be decentralized as a competitive equilibrium with transfers.*

This result thus confirms within the context of our dynamic one-dimensional energy balance model, the static result of e.g. Chichilnisky and Heal (1994); Chichilnisky *et al.* (2000); Sandmo (2006), that if lump sum transfers between regions are possible, optimal Pigouvian taxes on the externality producing activity should be harmonized across countries. It also follows that if taxes are set according to (3.55), by the Second Welfare Theorem any pareto optimal allocation defined for some set of welfare weights $\upsilon(x)$ can be attained, as a market equilibrium after the appropriate shuffling of endowments across latitudes have been made.[32]

3.3.6 Optimal planning problem in a closed market economy

Turn now to the *closed economy*. In this case each country's capital budget constraint must be independently satisfied so that $B_{x,t} = 0, \forall x, t$. The price of the consumption good is also assumed to be latitude dependent an thus denoted by $p_{x,t}^k$ which replaces p_t^k

[31] The f.o.c. of the planning problem is $\upsilon(x)U'(C_{x,t}/L_{x,t}) = \lambda_t^k$, hence, it is clear that setting $\upsilon(x) = 1/\Lambda_x^k$ this implies that the competitive equilibrium condition (3.46) is satisfied.

[32] See e.g. Kehoe *et al.* (1990) for an application witin a standard macro economic context without the extra complication of externality adjusting taxes.

in the consumers budget constraint of section 3.3.4. In this case each specific country is essentially a closed economy where the planners role is only to determine the optimal tax rate at each latitude. As will be shown below, this scenario can lead to the emergence of a spatially differentiated optimal carbon tax. The appearance of this result is most easily shown by assuming that each location has its own deposit of fossil fuels $R_0(x)$ where the extraction from this resource leads to latitude specific prices $p_{x,t}$ and extraction flows $q_{x,t}$ which must satisfy $\sum_{t=0}^{\infty} q_{x,t} = R_0(x)$.[33] This is not a specifically realistic case but it helps bring out the forces that can generate spatially differentiated carbon taxes. The welfare maximization problem of the planner can be written as:

$$\max \sum_{t=0}^{\infty} \beta^t \int_x \upsilon(x) L_{x,t} U(C_{x,t}/L_{x,t}) dx dt \tag{3.57}$$

subject to (3.3),(3.8),(3.18) and $\sum_{t=0}^{\infty} q_{x,t} = R_0(x)$ with $B_{x,t} = 0, \forall x, t$. Each country is thus restrained by its own permanent income constraint. The first order necessary conditions w.r.t. $K_{x,t+1}$ gives us the standard Euler equation

$$U'(C_{x,t}) = \beta n^{\theta} U'(C_{x,t+1}) \left(\frac{\partial Y_{x,t+1}}{\partial K_{x,t+1}} + (1-\delta) \right) \tag{3.58}$$

Note that this condition is equivalent to the one obtained by combining (3.48) and (3.50) from the market equilibrium which implies that given that taxes and transfers are administered by the market constrained planner this implies that each country is an independent market with its own interest rates and saving/investment decisions.

The first order necessary conditions w.r.t $q_{x,t}$ is given by

$$\begin{aligned}(\beta n^{\theta})^t \upsilon(x) L_{x,0}^{\theta} U'(C_{x,t}) \frac{\partial Y_{x,t}}{\partial q_{x,t}} & \\ = - \sum_{k=2}^{\infty} (\beta n^{\theta})^{t+k} \int_x \upsilon(x) L_{x,0}^{\theta} U'(C_{x,t+k}) & \frac{\partial Y_{x,t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_{x,t}} dx + \mu_0(x)\end{aligned} \tag{3.59}$$

where $\mu_0(x)$ denotes the lagrangian multiplier for the resource constraint at latitude x. The interpretation of this expression is as in the previous section. The first order condition states that marginal social utility of using an extra unit of fossil fuel (left hand side) must equal the marginal social cost (right hand side) at each respective latitude. As in the case of the planning problem in section 3.3.2 we can write this in terms of a latitude specific externality adjusted Hotelling rule

$$\frac{U'(C_{x,t})}{\beta n^{\theta} U'(C_{x,t+1})} = \frac{\frac{\partial Y_{x,t+1}}{\partial q_{x,t+1}} - \Lambda_{x,t+1}^s}{\frac{\partial Y_{x,t}}{\partial q_{x,t}} - \Lambda_{x,t}^s} \tag{3.60}$$

[33] The competitive equilibrium is thus modified by replacing R_0 with $R_0(x)$ in (3.53) and p_t by $p_{x,t}$.

where the marginal externality costs of fossil fuel use per latitude $\Lambda_{x,t}$ is given by

$$\Lambda^s_{x,t} \equiv -\frac{1}{\upsilon(x)L^\theta_{x,0}U'(C_{x,t})}\sum_{k=2}^{\infty}(\beta n^\theta)^k\int_x \upsilon(x)L^\theta_{x,0}U'(C_{x,t+k})\frac{\partial Y_{x,t+k}}{\partial T_{x,t+k}}\frac{\partial T_{x,t+k}}{\partial M_{t+k-1}}\frac{\partial M_{t+k-1}}{\partial q_{x,t}}dx \tag{3.61}$$

To see that the outcome of the planning problem can also be achieved as a competitive market equilibrium we start by observing that the Euler equation (3.58) of the planning problem can also be achieved thru the first order conditions (3.48) and (3.50) of the market equilibrium where $B_{x,t} = 0,\ \forall x, t$ by assumption. Next, by comparing the competitive market Hotelling rule, stated in equation (3.56), to the corresponding planning version (3.60) it follows directly that for the competitive market equilibrium to be Pareto optimal taxes must be set according to

$$\tau_{x,t} = \Lambda^s_{x,t}, \quad \forall x, t \tag{3.62}$$

As with the *open economy* planning problem, the first welfare theorem requires setting welfare weights so that induced transfers are zero which implies that welfare weights must satisfy $\upsilon(x) = 1/\Lambda^k_x$.[34] For a different arbitrary set of welfare weights $\tilde{\upsilon}(x)$ the corresponding Pareto optimal allocation can however also be attained given the appropriate amount of transfers which adjusts the multiplier Λ^k_x of the consumer until $\tilde{\upsilon}(x) = 1/\Lambda^k_x$ holds. Once again, this is the second theorem of welfare economics at work.

As we have now shown that this market constrained planning problem is attainable as a competitive equilibrium with transfer, we now turn to studying how different welfare weights impact on optimal fossil fuel use and welfare across space. From equation (3.59) we see how the solution to this problem may generate spatially differentiated taxes. To see this we start by defining

$$\lambda^s_t \equiv -\sum_{k=2}^{\infty}(\beta n^\theta)^{t+k}\int_x \upsilon(x)L^\theta_{x,0}U'(C_{x,t+k})\frac{\partial Y_{x,t+k}}{\partial T_{x,t+k}}\frac{\partial T_{x,t+k}}{\partial M_{t+k-1}}\frac{\partial M_{t+k-1}}{\partial q_{x,t}}dx \tag{3.63}$$

Next, substituting the f.o.c. of the firm w.r.t. fossil fuel use, which for the *closed economy* problem is given by $\frac{\partial Y_{x,t}}{\partial q_{x,t}} = p_{x,t} + \tau_{x,t}$ into equation (3.59) we have.

$$\frac{\mu_0(x) + \lambda^s_t}{(\beta n^\theta)^t\upsilon(x)L^\theta_{x,0}U'(C_{x,t})} = p_{x,t} + \tau_{x,t} \tag{3.64}$$

where the right hand side can be interpreted as the *social price* of fossil fuel use payed by latitude x, i.e. the final price after the externality has been internalized. By this

[34] As in section 3.3.5, by inspection of the f.o.c. the price of the consumption good in competitive equilibrium must rise at the same rate as the multiplier of the planners budget constraint i.e. $p^k_{x,t}/p^k_{x,t+1} = \lambda^k_{x,t}/\lambda^k_{x,t+1}$. Normalizing prices so that $p^c_{x,0} = \lambda^k_{x,0}$ thus implies that $p_{x,t} = \lambda^k_{x,t}$ and that equilibrium prices exactly equal the Lagrangian multiplier of the plannerŠs problem.

expression we see that it is only in special cases that the right hand side of the above expression will be equal across latitudes. This requires that $\mu_0(x) = \mu_0(x')$ and that the welfare weights are so-called Negishi weights defined as the inverse of marginal utility of consumption. Further, we can see that if $\mu_0(x) = \mu_0(x')$ and welfare weights instead are set equally and independent of latitudes, the latitude having the highest marginal utility of consumption should pay a lower social price of fossil fuel use. This will be the case when for example, the resource of x and the resource of x' are very large, i.e. the "Hotelling rent" is small in x, x'.[35] We formalize this in the following proposition

Proposition 3.4. *Assume that $\mu_0(x) = \mu_0(x')$ and $C_{0,t}/L_{0,t} < C_{x,t}/L_{x,t}$ and that welfare weights $\upsilon(x)$ are set equal and independent of x at all latitudes. Then if the planner is unable to administer transfers, so that the weighted marginal utilities of consumption are equated across latitudes, then the latitude located at the equator ($x = 0$) will pay a lower social price for fossil fuel use relative to a latitude $x \neq 0$.*

The result of proposition (3.4) is most easily seen by taking the ratio between latitude x and the equator ($x = 0$) from (3.64). Under the assumptions of proposition (3.4) we thus have

$$\frac{U'(C_{0,t}/L_{0,t})}{U'(C_{x,t}/L_{x,t})} = \frac{p_{x,t} + \tau_{x,t}}{p_{0,t} + \tau_{0,t}} \tag{3.65}$$

Hence, we see that by proposition (3.4) if the equator is expected to be poorer in the sense that $C_{0,t}/L_{0,t} < C_{x,t}/L_{x,t}$ this implies by the concavity of the utility function that the equator should also pay a lower social price relative to other latitudes.

3.3.7 Optimal planning problem with costly international transfers

We now consider the planning problem with costly international transfers. In this version of the planning problem the planner has the ability to administer transfers across locations but as opposed to the *open economy* problem he now has to take into account the cost of such transfers. This section thus extends the results of e.g. Chichilnisky and Heal (1994) to include the case when transfers are available but costly. We show that within the context of our one-dimensional climate model one can argue that under fairly plausible assumptions the social price of fossil fuels around the equator should be set relatively lower compared to northern or southern latitudes.

[35] Another example of when $\mu_0(x) = \mu_0(x')$ holds is when we as in section 3.3.3 assume, a logarithmic utility function, Cobb-Douglas production and full capital depreciation then it is clear that we can attain a fixed rule for consumption $C_{x,t} = (1 - \alpha_K\beta)Y_{x,t}$ and investment $K_{x,t+1} = \alpha_K\beta Y_{x,t}$ as we did in the *open economy* problem. Applying these rules to equation (3.59) implies that the left hand side depends on x only through the welfare weight $\upsilon(x)$. The solution to optimal fossil fuel is now completely characterized by the climate dynamics and fossil fuel constraint $\sum_{t=0}^{\infty} q_{x,t} = R_0(x)$. Hence, by solving (3.59) for $q_{x,t}$ and substituting this into the fossil fuel constraint it is easy to show that $\mu_0(x) = \mu_0(x')$ holds whenever $R_0(x)/(\upsilon(x)L_{x,0}) = R_0(x')/(\upsilon(x')L_{x',0})$.

Assume that the cost of transfers across locations is captured by a quadratic cost term affecting the planners resource constraint, which can be written as

$$\int_x (C_{x,t} + K_{x,t+1} - (1-\delta)K_{x,t})\, dx = \int_x Y_{x,t} dx - \frac{\mathcal{C}_0}{2}\int_x \Theta_{x,t} dx, \tag{3.66}$$
$$\Theta_{x,t} \equiv (Y_{x,t} + (1-\delta)K_{x,t} - K_{x,t+1} - C_{x,t})^2$$

If $\mathcal{C}_0 \to 0$ the costs of transfers decline and it is easy to see that the solution converges towards the solution of the *open economy* problem of section 3.3.3. On the other hand if $\mathcal{C}_0 \to \infty$ then each country is forced to live within its own budget constraint since any deviation is infinitely costly which implies that we are back to the *closed economy* problem. We further assume as in section 3.3.3 that the co-albedo function is independent of the iceline $\alpha(x, x_s) = \alpha(x)$ so that the temperature dynamics are given by (3.35). The problem of the planner is thus to maximize the objective function (3.21) subject to (3.3), (3.4), (3.35) and (3.66). In order to see how the solution to this problem can generate a spatially differentiated social price or tax rate it is useful to consider the Lagrangian of the maximization problem

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & \int_x \upsilon\,(x)\, L_{x,t} U(C_{x,t}/L_{x,t}) dx \\
& + \lambda_t^k \big[\int_x \big(Y_{x,t} - C_{x,t} + (1-\delta)K_{x,t} - K_{x,t+1} - \frac{\mathcal{C}_0}{2}\Theta_{x,t} \big) dx \big] \\
& + \lambda_t^m \big[\sigma \int_x q_{x,t} dx + (1-m)M_t - M_{t+1} \big] \\
& + \lambda_t^T \big[-A - BT_{0,t} + \int_x QS(x)\alpha(x) dx + \xi \ln\left(1 + \frac{M_t}{\bar{M}}\right) - cB(T_{0,t+1} - T_{0,t}) \big] \Big\} \\
& + \mu_0 \big[R_0 - \sum_{t=0}^{\infty} \int_x q_{x,t} dx \big]
\end{aligned}$$

where $Y_{x,t} = \mathbb{A}_{x,t}\Omega(T_{x,t})F(K_{x,t}, L_{x,t}, q_{x,t})$ and $T_{x,t} = T_{0,t} + T_{2,t}P_2(x)$. The first order necessary conditions of the maximization problem are

$$\frac{\partial \mathcal{L}}{\partial C_{x,t}} = \upsilon(x)U'(C_{x,t}/L_{x,t}) - \lambda_t^k[1 - \Theta'_{x,t}] = 0 \tag{3.67}$$
$$\frac{\partial \mathcal{L}}{\partial K_{x,t+1}} = -\lambda_t^k[1 - \Theta'_{x,t}] + \beta\lambda_{t+1}^k \alpha_K \frac{Y_{x,t+1}}{K_{x,t+1}}[1 - \Theta'_{x,t+1}] = 0 \tag{3.68}$$
$$\frac{\partial \mathcal{L}}{\partial q_{x,t}} = \beta^t \lambda_t^k \alpha_q \frac{Y_{x,t}}{q_{x,t}}[1 - \Theta'_{x,t}] + \beta^t \lambda_t^m \sigma - \mu_0 = 0 \tag{3.69}$$
$$\frac{\partial \mathcal{L}}{\partial M_{t+1}} = -\lambda_t^m + \beta\lambda_{t+1}^m(1-m) + \lambda_{t+1}^T \xi \left(1 + \frac{M_{t+1}}{\bar{M}}\right)^{-1} \frac{1}{\bar{M}} = 0 \tag{3.70}$$
$$\frac{\partial \mathcal{L}}{\partial T_{0,t+1}} = \beta\lambda_{t+1}^k \frac{\partial Y_{x,t+1}}{\partial T_{0,t+1}}[1 - \Theta'_{x,t+1}] - \lambda_t^T cB + \beta\lambda_{t+1}^T(cB - B) = 0 \tag{3.71}$$

where $\Theta'_{x,t} = \mathcal{C}_0 \left(Y_{x,t} + (1-\delta)K_{x,t} - K_{x,t+1} - C_{x,t}\right)$. In appendix 3.A.3 we show that it is entirely possible to represent this planning problem as a competitive equilibrium with transfers. This requires that both interest rates and fossil fuel prices are latitude dependent. In the following we will however show that if we assume that marginal transfer costs are approximately constant over time we can make the case for both approximately equal rates of return on capital across locations and spatially differentiated optimal tax rates.

To see this we start by observing from the first order condition w.r.t $K_{x,t+1}$, that if $\Theta'_{x,t} \cong \Theta'_{x,t+1}$ then the rates of return on capital will also be approximately equal across latitudes ($R_{x,t+1} \cong R_{t+1}$). Turning to the problem of fossil fuel firms this implies that optimal fossil fuel prices $p_{x,t}$ at each latitude thus grow at the same rate i.e. $p_{x,t+1}/p_{x,t} \cong R_{t+1}$. Combining the first order condition (3.69) with the first order condition w.r.t fossil fuel use of the profit maximizing goods producing firm at latitude x gives us

$$p_{x,t} + \tau_{x,t} = \frac{\mu_0 - \beta^t \lambda_t^m \sigma}{\beta^t \lambda_t^k [1 - \Theta'_{x,t}]} \tag{3.72}$$

Furthermore, comparing social fossil fuel prices between latitude x and 0 we have

$$\frac{p_{x,t} + \tau_{x,t}}{p_{0,t} + \tau_{0,t}} = \frac{[1 - \Theta'_{0,t}]}{[1 - \Theta'_{x,t}]} \tag{3.73}$$

From which we can state the following proposition

Proposition 3.5. *If* $\Theta'_{x,t} > \Theta'_{0,t}$ *then* $[p_{x,t} + \tau_{x,t}] > [p_{0,t} + \tau_{0,t}]$.

Hence, since latitudes around the equator are relatively poorer compared to higher latitude locations, this also implies that marginal transfer costs will likely be lower at the equator i.e. $\Theta'_{x,t} > \Theta'_{0,t}$. Therefore proposition 3.5 suggests that lower latitudes should pay a lower social price for using fossil fuels relative to rich locations, which is similar to the results obtained when no transfers were possible. If we further assumed that $p_{x,t}$ is approximately equal across latitudes then this would imply that latitudes around the equator should pay a lower carbon tax.

Thus the spatially uniform taxes emerge as an optimal solution only under transfers across locations that equalize per capita consumption or when Negishi welfare weights are used and distribution across latitudes does not change. Negishi weights, being the inverse of marginal utility, assign relatively larger welfare weights to locations with higher per capita consumption. For example with a logarithmic utility Negishi weights, assign a welfare weight equal to per capita consumption in each location. Thus the utility of poor locations has a relatively smaller importance, compared to the utility of rich location, in the planners welfare function. The DICE/RICE model adopt Negishi weights and produce spatially uniform carbon taxes keeping at the same time the regional distribution of per capita consumption invariant (e.g. Stanton (2010)). Our result, on the other hand suggest that the spatial structure of the optimal carbon tax is sensitive to the choice of welfare weights and deviations from the Negishi solution will result in spatially differentiated taxes. Thus when intertemporal redistribution is treated as fixed or costly, and when welfare weights are other than the Negishi weights, poor locations could, under plausible assumptions, pay lower carbon taxes.

3.4 Numerical Simulations

In the previous section we derived general results regarding the optimal mitigation policies and their spatial and temporal profiles, and the relative importance of introducing heat transportation into the model. In this section we proceed and show how to solve the *open economy* model numerically. Following, Nordhaus (2007) we calibrate the model based on 10 year time intervals. Assuming ten year time intervals makes assumptions such as for example, full capital depreciation more plausible. However, it should be noted that this is not ideal for policy analysis. Cai *et al.* (2012) demonstrate that many substantive results depend critically on the time step and if one wants to know how carbon prices should react to business cycle shocks, the time period needs to be at most a year. Hence the following calibration exercise and following numerical analysis should not be seen as a serious attempt to give precise answers, but rather as a finger exercise and a first attempt on capturing the value added of spatially dependent EBCMs as opposed to simpler representations of the climate economy interaction.

3.4.1 Calibration

We use the two-mode approximating solution given by (3.8) with the simplification that the co-albedo function does not explicitly depend on the iceline ($\alpha(x, x_s) = \alpha(x)$). We instead assume, following North *et al.* (1981), that the co-albedo instead can be approximated by $a(x) = 0.681 - 0.202P_2(x)$ and $S(x) = 0.5[1 - 0.482P_2(x)]$ from (3.8e) which gives us an approximate globally averaged value for the co-albedo of ≈ 0.7. Using this approximation, the co-albedo is independent of temperature implying that it ignores important feedbacks that follow from perturbations to the climate system. In order to compensate for this crude approximation we calibrate the empirical coefficients A and B so as to match the present climate state and the IPCC (2007) best guess estimate of climate sensitivity of $3°C$ temperature increase to a doubling of atmospheric carbon dioxide. We used data on global carbon dioxide concentrations levels, available from Etheridge *et al.* (1998), based on ice core data for historical observations prior to 1960 and from Tans (2011) for the following years, with a current CO_2 concentration estimate as of December 2011 of $831Gtc$. Temperature data was attained from the Nasa Goddard Space institute, summarized in Hansen *et al.* (2010), which reports temperature anomalies by latitude.

To calibrate the model to the current climate state we proceed as follows. First, assume that temperature dynamics equilibrate fast compared to carbon dioxide dynamics so that (3.8) reaches steady state in every time period for a given given amount of anthropogenic forcing. We believe this to be a not an entirely unreasonable approximation. For example, Held *et al.* (2010) estimate a relaxation or e-folding time for temperature response to a doubling of CO_2 from pre-industrial levels to be less than 5 years, i.e roughly 40% of the equilibrium response to a human induced forcing is obtained within five years.[36] Hence, given the 10 year time steps of our model this equilibrium approximation is perhaps not to far off, from results which would have been attained with a full dynamic model.[37] Second, based on these steady state assumptions we can now solve equation (3.8b) for T_0 and set the empirical coefficients A and B so to match the current global mean temperature. We use North *et al.* (1981) estimate of $Q = 340$, which corresponds to a solar forcing of $1360W/m^2$, and following Nordhaus (2007) assume a pre-industrial concentration of carbon dioxide of $\bar{M} \approx 596Gtc$. These values, together with the above estimates of current carbon dioxide concentration and co-albedo, setting $A = 221.6$ and $B = 1.24$ thus gives us an average global temperature of ≈ 14.4 which is a decent match to temperature levels reported by e.g. IPCC (2007). This value of B also gives us a climate sensitivity estimate $1/B \approx 0.8$ implying that a doubling of CO_2 gives an approximate warming of 3 degrees which is inline with the best guess estimates of IPCC (2007).[38]

Next, given the assumption that temperature dynamics relax to steady state in every time period and by (3.8a) we can now calculate the following expression for latitude specific temperatures

$$T_{x,t} = -\frac{A}{B} + \frac{Q}{B}\int_{-1}^{1} S(x)a(x)dx + \frac{\xi}{B}\ln\left(1 + \frac{M_t}{\bar{M}}\right) + \left(\frac{5Q}{B(1+6D)}\int_{-1}^{1} S(x)a(x)P_2(x)dx\right)P_2(x) \tag{3.74}$$

As can be seen from this expression the heat diffusion parameter D still needs to be calibrated in order to calculate temperature changes by latitude for different levels of atmospheric carbon dioxide. It is worth noting that similar temperature functions have been derived by climate scientists, but without the impact of human activities on climate. In our case this impact is realized by the increase in the concentration of atmospheric carbon dioxide. When $D \to \infty$ the temperature function is spatially homogeneous or flat across latitudes. The distinction between a latitude dependent and a flat temperature field provides a first sign of the impact of thermal transport on the estimation of the temperature function. In order to approximate damages for a given damage function we need to map changes in temperature measured from a benchmark period to numbers indicating the level of damages. Following Nordhaus (2007) we will use year 1900 as a base period. Based on historical climate data it is well known that fluctuations in the global mean temperature are unequally distributed across latitudes

[36] This is also reported by independent studies by Knutti *et al.* (2008) and Hansen *et al.* (2011). However, it should be noted that due to thermal inertia from the slow warming of e.g. the upper and lower oceans a full response is achieved first after several hundred of years.

[37] We caution the reader that we have not investigated the implications of this assumption in detail.

[38] The climate sensitivity relationship is usually depicted as $\Delta T = \lambda F$ where λ denotes the climate sensitivity. Given $\xi = 5.35$ we get an approximate forcing $F \approx 3.7W/M^2$ from a doubling of CO_2 which together with $\lambda = 1/B \approx 0.8$ gives us a warming of approximately 3 degrees for a doubling of atmospheric carbon dioxide.

(Hansen *et al.*, 2010). In order to account for this unequal distribution we calibrate (3.74) for the average temperature year 1880-1900 and 2001-2011. This is done by first calibrating a temperature distribution based on data from Shell and Somerville (2004) (figure 5). Next we define temperature anomaly by latitude as $T^{+}_{x,t} \equiv \hat{T}^{2011}_{x,t} - \hat{T}^{1900}_{x,t}$. Temperature anomalies are then calculated for eight zonal latitudes between the years 1890-1900 and 2001-2011 based on data from the Nasa Goddard Institute for Space Studies.[39] From these anomalies we get average temperatures for 1890-1900 from which we can calibrate a temperature distribution matching the average temperature 1890-1900 which will constitute our base period.

Figure (3.1) depicts temperatures by latitude calculated based on equation (3.74) for the two calibrations. The solid line, represents the current climate, with a atmospheric carbon dioxide level of $831GtC$ and a heat diffusion parameter $D = 0.18$. This line was calibrated to match data reported in Shell and Somerville (2004) (figure 5). Next, the dashed line denotes the climate for the year 1900 based on average CO_2 levels between the year 1890-1900 and the reported average temperature anomalies from Nasa over the same period.[40] The calculated temperature anomalies for each of the eight zonal means are shown in table 3.A.4 in appendix. The results indicate that the temperature has increased more at high altitudes relative to the equator. Between the year 1900 and 2011 this shows that temperature has increased by approximately $0.52°C$ around the equator while at the poles temperature increase has been around $2.4°C$. Hence, the dashed line represents the temperature distribution in the year 1900 given an average carbon dioxide level of $625.8GtC$ for the years 1880-1900 and a heat diffusion parameter of $D_{1900} = 0.166$. Equation (3.74) thus gives us a decent approximation of the temperature anomalies compared to 2011. With a continued increase in temperature the gap between these anomalies will thus continue to grow wider.

Next, considering the atmospheric carbon dioxide dynamics two parameters need to be calibrated. First, the so called airborne fraction σ which is the fraction of anthropogenic carbon dioxide emissions that remain in the atmosphere. We model this fraction as constant hence assuming that there is no trend in the biospheres and oceans ability to absorb human induced emissions. Based on the study by Knorr (2009) we set the airborne fraction $\sigma = 0.43$.[41] Second, the parameter φ captures the rate at which carbon is absorbed by the deep oceans. Archer (2005) claims that "...75% of an excess atmospheric carbon concentration has a mean lifetime of 300 year and the remaining 25% stays there forever". Although our simple carbon cycle is unable to account for the 25% always remaining in the atmosphere by setting $m = 0.05$ this implies that after 300 years (30 periods) approximately 75% of the carbon dioxide has been removed.

For the economic parts of the model, we rely on estimates from Golosov *et al.* (2011)

[39] See Hansen *et al.* (2010). Dataset is public ally available at `http://data.giss.nasa.gov/gistemp/tabledata_v3/ZonAnn.Ts.txt`. For anomalies calculated here see table 3.A.4 in appendix.

[40] The carbon dioxide data was collected from Etheridge *et al.* (1998), available at `http://cdiac.ornl.gov/ftp/trends/co2/lawdome.smoothed.yr20`.

[41] Although there exists several studies have reported an apparent increasing trend in the airborne fraction the study by Knorr (2009) claim that this trend is statistically insignificant.

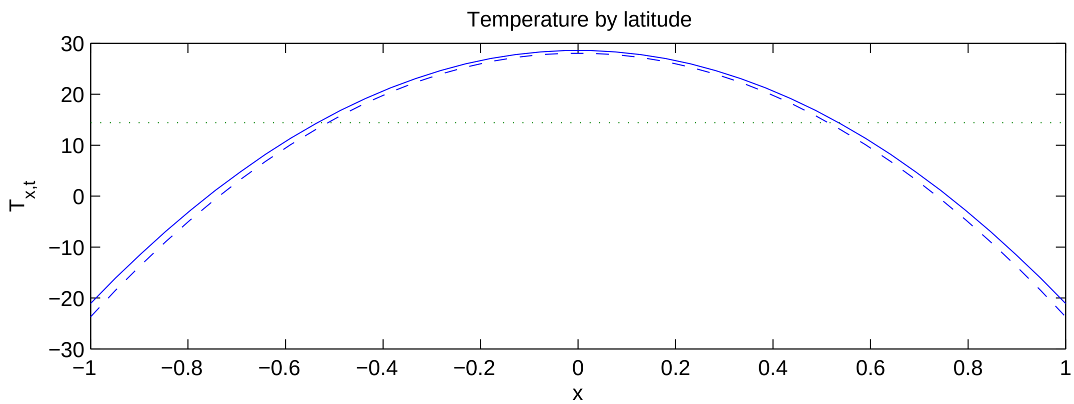

Figure 3.1: Dotted line denotes current global "average" estimate. Parabolas are temperature per latitude given by equation (3.74) for current CO_2 levels and heat diffusion estimate D (solid line) and corresponding estimates for year 1900 (dashed line).

and Nordhaus (2007). As in Golosov *et al.* (2011) we assume a logarithmic utility function, Cobb-Douglas production and full depreciation of capital. These assumptions greatly simplify the numerical analysis since the dynamic investment problem of the model can be separated from decisions on inter temporal fossil fuel use as shown in section 3.3.3. For a discussion regarding how well these assumptions fit historical economic data the reader is referred to the discussion in Hassler *et al.* (2011) and Golosov *et al.* (2011). For the Cobb-Douglas production function the factor shares and discount rate are $\alpha_K = 0.3$, $\alpha_q = 0.03$ and $\beta = 0.985^{10}$. Next, fossil fuel use in our model also requires an estimate of the current stock R_0. We use the global reserve estimate of $5000GtC$ from Rogner (1997) which also accounts for technical progress in extraction. This estimate consists of both coal and oil. It should be noted that the way we model fossil fuel production in this paper is simplistic in the sense that we ignore important extraction costs and refinement costs which are generally much higher for coal than oil. For a detailed model capturing the difference between coal and oil the reader is once again referred to Golosov *et al.* (2011). As we want to specifically focus on the spatial properties inherent in our model we have ignored this distinction in order to keep the model simple.[42]

Concerning the damage function of global warming we rely on the calibration results following Nordhaus (2007). He assumes a quadratic damage function $\Omega(T) = (1 + \theta T^2)^{-1}$ calibrated so that a 2.5 degree warming results in approximately a 1.77% loss of GDP which is consitent with a damage parameter $\theta \approx 0.0028$.[43] There is a large degree of uncertainty regarding the shape of the damage function and its impacts on economic output for different temperatures. Golosov *et al.* (2011) calibrate an ex-

[42] Adding these distinctions at a later stage should however, not affect any of the qualitative results of this paper.

[43] Here T refers to the temperature anomaly from 1900 levels so that a 2.5 degree warming is measured relative to year 1900.

ponential damage function which they find gives results that appproximate well the quadratic damage function used by Nordhaus. As we have shown in section 3.3.3 an exponential damage function is also consistent with our model which due to its multiplicative properties (see eq. (3.37)) implies that the term $\frac{\Omega'(T_{0,t})}{\Omega(T_{0,t})}$ of equation (3.38) becomes a constant equal to γ. The specific properties of the exponential damages thus obscures how heat diffusion impacts on optimal mitigation policies. In this section we will thus follow Nordhaus (2007) and adopt a quadratic damage function as it does not hide any of the spatial characteristics inherent in our model. Concerning, the spatial distribution of damages Nordhaus (2010) presents results from runs of his regional RICE model.[44] Based on the independent damage functions estimated for the 12 regions available from the RICE 2010 Excel version we find that for a warming of one degree the damages are approximately 50% lower in temperate regions surrounding the US and EU relative to equatorial latitudes surrounding the Middle East and Africa. We approximate these regional damages by making the damage parameter latitude dependent. We do this by defining $\theta(x) \equiv 0.75(1-x^2)\theta_0$ and write the damage function as $\Omega(T) = (1+\theta(x)T^2)^{-1}$ with θ_0 set equal to the Nordhaus (2007) damage parameter estimate ≈ 0.0028.[45] This specification makes marginal damages from a given increase in temperature higher around the equator than a corresponding temperature increase around temperate latitudes.

For some of the calculations we also need initial estimates for the production function. For this, we will use estimates from Nordhaus (2007), involving 2005 year's output $Y_{2005} = 55.34$ and capital $K_{2005} = 97.3$ measured in trillion 2005 US dollars. World population in 2005 is estimated to be approximately 6.514 billion people in 2005. Further, in deriving our model we have explicitly assumed that population is immobile across latitudes. Hence, we need to make an assumption regarding the structure of population across latitudes. Kummu and Varis (2011) present charts on population distribution by latitude. They shows that the most densely populated latitude is located at approximately $30°N$ with a distribution that declines in a roughly exponential manner as we move towards the poles with a slightly fatter tail for southern latitudes. We make a rough approximation of this population distribution using a truncated normal distribution on the intervall [-1,1] with a peak slightly below $30°N$ to catch some of the slightly fatter lower tail and set $x = sin(15°N) \approx 0.25$ as the mean of the distribution and 0.2 as the standard deviation. Using the corresponding cumulative distribution function of the truncated normal, we can thus directly get estimates of the amount of the total world population located between specific latitudes. Using these estimates together with 2005 years fossil fuel consumption (IPCC (2007) of $7.2GtC$ we can now calibrate the corresponding values of 2005 years technology levels per latitude.

[44] The new updated DICE and RICE 2010 models are available at `http://nordhaus.econ.yale.edu/RICEmodels.htm`.

[45] Note that the properties of the latitude dependent damage parameter are such that $\int_x \theta(x)dx = 1$ and $\theta(-1) = \theta(1) = 0$ and $\theta(0) = 0.75\theta_0$.

For the initial time period equation (3.24) can be written as

$$\tilde{Y}_0 = K_0^{\alpha_K} q_0^{\alpha_q} \left(\int_x (\mathbb{A}_{x,0} L_{x,0}^{\alpha_L} \Omega(T_{x,0}))^{\frac{1}{\alpha_L}} dx \right)^{\alpha_L}$$

Hence, using the initial estimates of output, capital and fossil fuel use in 2005 together with our factor input shares, we can thus calculate $J_0 = \left(\int_x (\mathbb{A}_{x,0} L_{x,0}^{\alpha_L} \Omega(T_{x,0}))^{\frac{1}{\alpha_L}} dx \right)^{\alpha_L}$, where $J_0 \approx 13.2$. Next, assuming that technology is equally spread across latitudes ($\mathbb{A}_{x,0} = \mathbb{A}_{x',0}$) we can substitute $\mathbb{A}_{x,0}$ out of the integral which implies that

$$\mathbb{A}_{x,0} = \frac{C_0}{\left(\int_x L_{x,0} \Omega(T_{x,0})^{\frac{1}{\alpha_L}} dx \right)^{\alpha_L}} \approx 136.9, \quad \forall x$$

where the approximate estimate of $\mathbb{A}_{x,0}$ is arrived to after evaluating the integral in the denominator given our estimates concerning population, damages and temperatures per latitude as given by equation (3.74).

3.4.2 Results

We will now highlight some numerical results of our calibrated model for the ideal planning problem. Given the assumptions of section 3.4.1, in particular the log utility and full depreciation of capital assumption,the externality adjusted Hotelling equation (3.34) can thus be written as

$$\frac{1}{\beta n} = \frac{\alpha_q \frac{1}{q_{t+1}} + \sum_{k=2}^{\infty} (\beta n)^{t+k+1} \frac{1}{\tilde{Y}_{t+k+1}} \frac{\partial \tilde{Y}_{t+k+1}}{\partial T_{x,t+k+1}} \frac{\partial T_{x,t+k+1}}{\partial M_{t+k}} \frac{\partial M_{t+k}}{\partial q_{t+1}}}{\alpha_q \frac{1}{q_t} + \sum_{k=2}^{\infty} (\beta n)^{t+k} \frac{1}{\tilde{Y}_{t+k}} \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t}} \tag{3.75}$$

where $T_{x,t}$ is given by (3.74). Compared to the case of exponential damages the term $\frac{1}{\tilde{Y}_{t+k}} \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}}$ is no longer constant but can be expressed as

$$\begin{aligned} \frac{1}{\tilde{Y}_{t+k}} \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} &= \frac{J'(\{T_{x,t+k}\}_x)}{J(\{T_{x,t+k}\}_x)} \qquad (3.76) \\ &= \left(\int_x \Omega(T_{x,t+k})^{1/a_L} A_{x,0}^{1/a_L} L_{x,0} dx \right)^{-1} \int_x \Omega(T_{x,t+k})^{1/a_L} \Omega(T_{x,t+k}) 2\theta T_{x,t+k} A_{x,0}^{1/a_L} L_{x,0} dx \end{aligned}$$

where the function J is given by (3.25). Finally, $\frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t} = \sigma(1-m)^{k-2} \frac{\xi}{B} \left(1 + \frac{M_{t+k-1}}{\bar{M}}\right)^{-1} \bar{M}^{-1}$.

To solve this numerically we use the solution to the decentralized equilibrium without taxes as an initial guess of the optimal fossil fuel path for the planning solution.[46] This allows us to derive us the corresponding paths of M and T which in turn gives us the sums $\sum_{k=2}^{\infty} (\beta n)^{t+k} \frac{1}{\tilde{Y}_{t+k}} \frac{\partial \tilde{Y}_{t+k}}{\partial T_{x,t+k}} \frac{\partial T_{x,t+k}}{\partial M_{t+k-1}} \frac{\partial M_{t+k-1}}{\partial q_t}$. Using these first guess estimates there is a unique path of fossil fuels that satisfies this expression and $\lim_{t\to\infty} R_t = 0$.

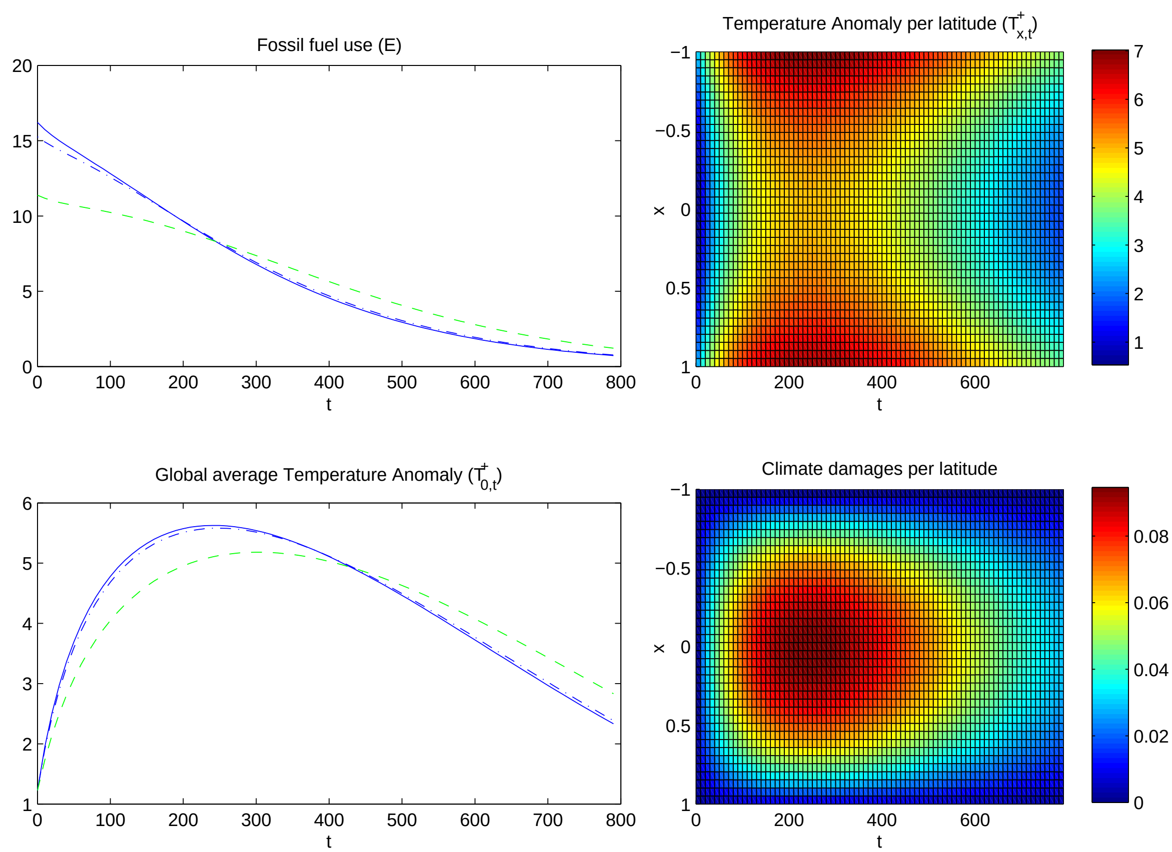

Figure 3.2: Solution to the ideal planning problem for a 800 year time period. The surface plots on the right hand side correspond to the yearly fossil fuel and temperature trajectories depicted by the solid lines on the left hand side. The dashed and dash-dotted lines correspond respectively, to planning scenarios where spatial distribution is fully and partially ignored.

We use this path to update the sum over k for and then keep repeating this procedure until convergence, which gives us the optimal paths for the planner.

Figure 3.2 shows the solution following the ideal planning problem over a 800 year time horizon. The left hand graphs depicts yearly fossil fuel use and the corresponding global temperature anomalies for three different modeling scenarios.[47] The first scenario, given by the solid blue lines of the upper and lower left hand graphs together with the surface plots on the right, are based completely on the assumptions described in section 3.4.1. Here, the spatial distribution of temperatures, population and damages are fully accounted for in the planners spatial and inter-temporal decisions on fossil fuel use . This implies that the planner weighs the social costs of a given temperature increase at a particular location x according to the severity of damages implied by our spatial damage function and the population size inhabiting that specific location. As can be seen optimal fossil fuel use falls over time. Compared to the market outcome without taxes optimal fossil fuel use in the initial time period is substantially lower than the global market outcome without taxes which would have implied a $E_0 = \beta^0(1-\beta)R_0 \approx 70GtC$ per year. The lower left hand graph describes the corresponding temperature anomaly which peaks at $\approx 5.5°C$ above year 1900 levels after about 240 years. The right hand side surface plots show the implications of these decisions for local temperature anomalies and climate damages for this scenario. The upper graph shows the temperature anomalies and as can be seen the temperature anomaly grows larger at the poles as global average temperatures start to rise which is a consequence of differing temperature distributions for the base period compared to the present as depicted in figure 3.1. As can be seen from inspection of the color chart, local temperatures have increased around one degree more at the poles compared to the global average and about one degree less than the global average at the equator after 240 years. Turn now to climate damages in the graph below. Climate damages are calculated as percent of GDP and the color chart displays damages varying from 0 to 9 percent of local GDP. As can be seen, the figure displays a deep red spot in the middle showing that damages in terms of GDP are substantially higher around the equator than at the poles. This is a consequence of the damage parameter $\theta(x)$ giving higher weights to tropical regions but also a consequence of population distribution. Careful inspection of the surface plot shows that damages are actually slightly higher above the equator were the mean of the population distribution lies i.e. in the region below $x = 0$ in the plot. Shifting the population distribution northward will thus increase damages in northern regions.

[46] The solution to the fossil fuel use in decentralized equilibrium without taxes is entirely determined by the discount rate and follows from $E_{t+1} = \beta E_t$ or alternatively $E_t = \beta^t(1-\beta)R_0$.

[47] It should be noted that the results depicted here rely on several stylized assumptions regarding model specifications and parameter choices and the results should thus not be interpreted as saying anything in relation to whether current fossil fuel use is above or below optimal levels. In particular the assumption of freely available international transfers implicit in the ideal planning problem is indeed an extreme assumption. Further, the way fossil fuel is modeled is also very stylized and would also need to be further refined.

Next, concerning the other two scenarios. The left hand graphs also display a dashed and a dash-dotted line. The dash-dotted line concerns the second scenario. This scenario could be seen as our equivalence of what Nordhaus does in his RICE model. Here, we consider an unequal distribution of damages and population across latitudes. However, compared to the first scenario we have set the heat diffusion coefficient D large enough so that temperature distribution across latitudes is essentially flat as was shown in proposition 3.1. Hence, in this scenario we are thus taking space into account from a social perspective while ignoring temperature diffusion implying that we are now in the zero dimensional case which gives us a global average temperature as is also the case in for example the RICE model. As can be seen this gives us a slightly lower initial value of fossil fuel use which also implies that the corresponding temperature trajectory is slightly smoothed compared to the first scenario. Finally, the third scenario, depicted by the dashed line, builds on the case where all latitude specific distributions have been ignored. In terms of Nordhaus models this case corresponds to his DICE model where all global distribution issues are essentially ignored. This gives us an even lower path of initial fossil fuel use and thus smooths out the temperature increase over time even more.

3.5 Concluding Remarks

In this paper we develop a model of climate change consisting of a one- dimensional energy balance climate model which is coupled with a model of economic growth. We believe that modeling heat transport in the coupled model is the main contribution of our paper since it allows, for first time as far as we know, the derivation of latitude dependent temperature and dam- age functions, as well as optimal mitigation policies, in the form of optimal carbon taxes, which are all determined endogenously through the interac- tion of climate spatiotemporal dynamics with optimizing forward looking economic agents.

We derive Pareto optimal solutions for a social planner who seeks to implement optimal allocations with taxes on fossil fuels and we show the links between welfare weights and international transfers across locations and the spatial structure of optimal taxes. Our results suggest when per capita consumption across latitudes can be adjusted through costless transfers for any set of non negative welfare weights, so that marginal valuations across latitudes are equated, or alternatively that transfers are zero due to Negishi weighting, then optimal carbon taxes will be spatially homogeneous. On the other hand when marginal valuations across latitudes are not equated, due to institutional/political constraints or the cost of transfers, optimal carbon taxes will instead be spatially differentiated. We show that if international transfers are costly and the planner is not constrained to by Negishi weighting, then taxes on fossil fuels could be lower in relatively poor geographical areas. The degree of geographical tax differentiation will depend on the heat transport across latitudes. Without appropriate implementation of international transfers, and without Negishi weights that keep the existing international distribution invariant, carbon taxes will be latitude specific and their sizes will depend on the heat transfer across locations. We also show how to derive numerical results for the optimization problem of the unconstrained planning problem with open markets. These results show how the optimization model can generate surface plots of temperature and damages across latitudes and over time.

The climate module of our model is sometimes referred to as a surface EBCM where the impact of oceans is reflected in the carbon decay parameter m, with no further modeling of the deep ocean component is undertaken. Further extensions of our simple one-dimensional model to richer climate models (e.g. Kim and North (1992), Wu and North (2007)) with a ocean and with simple atmospheric layers added and where tipping phenomena are possible may help understand results like those of Challenor *et al.* (2006), which found higher probabilities of extreme climate change than expected. Challenor *et al.* (2006) suggest several reasons for their findings including "The most probable reason for this is the simplicity of the climate model, but the possibility exists that we might be at greater risk than we believed". We emphasize that we are still doing what economists call a "finger exercise" in this paper where one deliberately posits an over simplified "cartoon" model in order to illustrate forces that shape, for example, an object of interest like, a socially optimal fossil fuel tax structure over time and space that might be somewhat robust to introduction of more realism into the toy model. For example, we believe that the interaction of spatial heat transport phenomena and difficulties in implementing income transfers (or their equivalent, e.g. allocations of tradeable carbon permits) will play an important role in determining the shape of the socially optimal tax schedule over different parts of space in more complex and more realistic models. Our simple model is useful in making this type of point under institutions where income transfers are possible and where they are politically infeasible, i.e. essentially impossible.

The one-dimensional model allows the exploration of issues which cannot be fully analyzed in conventional zero-dimensional models. In particular one-dimensional models with spatially dependent co-albedo allow the introduction of latitude depended damage reservoirs like endogenous ice-lines and permafrost. Since reservoir damages are expected to arrive relatively early and diminish in the distant future, because the reservoir will be exhausted and sufficient adaptation would have taken place, the temporal profile of the policy ramp could be declining, enforcing the result obtained for profit taxes, or even U-shaped.[48] A U-shaped policy ramp could explained by the fact that as high initial damages due to the reservoir will start declining as the reservoir is exhausted, giving rice to a declining policy ramp, damages from the increase of the overall temperature will dominate causing the policy ramp to become increasing. This is another potentially interesting and important area of further research.

[48] Judd and Lontzek (2011) have formulated a dynamic stochastic version of DICE, the SDICE - which includes stochastic tipping points possibilities. They show that this complexity affects the optimal policy results in comparison to RICE. Some of these issues are also explored in an accompanying paper of ours Brock *et al.* (2012a).

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3.A Appendix

3.A.1 The two mode solution

In this appendix we show how to derive the two mode solution (3.8a-(3.8e). We start with the basic PDE with temperature as the state variable which is defined using ((3.7)) with $S(x) = \frac{1}{2}[1 + S_2 P_2(x)]$.

$$cB(T_{x,t+1} - T_{x,t}) = QS(x)\alpha(x, x_s) - [(A + BT_{x,t}) - h_t] + DB\frac{\partial}{\partial x}\left[(1 - x^2)\frac{\partial T_{x,t}}{\partial x}\right]$$

The two mode solution is defined, after dropping D to ease notation, as:

$$\hat{T}_{x,t} = T_{0,t} + T_{2,t}P_2(x), \quad P_2(x) = \frac{(3x^2 - 1)}{2} \tag{3.77}$$

then

$$\hat{T}_{x,t+1} - \hat{T}_{x,t} = T_{0,t+1} - T_{0,t} + (T_{2,t+1} - T_{2,t})P_2(x) \tag{3.78}$$

$$\frac{\partial T_{x,t}}{\partial x} = T_{2,t}\frac{dP_2(x)}{dx} = T_{2,t}3x \tag{3.79}$$

Substitute the above expressions into (3.77) to obtain:

$$\begin{aligned} cB(T_{0,t+1} - T_{0,t}) + cB(T_{2,t+1} - T_{2,t})P_2(x) = \\ QS(x)\alpha(x, x_s) - [(A + B(T_{0,t} + T_{2,t}P_2(x)) - h_t] - BD\frac{\partial}{\partial x}\left[(1 - x^2)T_{2,t}3x\right] \end{aligned} \tag{3.80}$$

or

$$\begin{aligned} cB(T_{0,t+1} - T_{0,t}) + cB((T_{2,t+1} - T_{2,t})P_2(x)) = \\ QS(x,t)\alpha(x, x_s) - A - BT_{0,t} - BT_{2,t}P_2(x) + h_t - 6DBT_{2,t}P_2(x) \end{aligned} \tag{3.81}$$

Note the following properties of legendre polynomials:

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2\delta_{nm}}{2n + 1} \tag{3.82}$$

$$\delta_{nm} = 0 \text{ for } n \neq m, \delta_{nm} = 1 \text{ for } n = 1$$

where $P_0(x) = 1$, $P_2(x) = \frac{(3x^2-1)}{2}$.

Multiply (3.81) by $P_0(x)$ and integrate from -1 to 1, which after applying the properties of defined in (3.82) noting that $\int_{-1}^{1} P_2(x)dx = 0$, becomes

$$cB(T_{0,t+1} - T_{0,t}) = \int_{-1}^{1} [QS(x,t)\alpha(x, x_s) + h_t]\, dx - [A + BT_{0,t}] \tag{3.83}$$

Next, multiply (3.81) by $P_2(x)$ and integrate from -1 to 1, which after applying the properties of defined in (3.82) noting that $\langle P_2(x), P_2(x)\rangle = \frac{1}{5}$ we obtain

$$cB(T_{2,t+1} - T_{2,t}) = -B(1+6D)T_{2,t}(t) + 5\int_{-1}^{1} QS(x,t)\alpha(x,x_s)P_2(x)dx \tag{3.84}$$

The ODEs (3.83) and (3.84) are the ODEs of the two mode solution which together with (3.77) determine the two-mode solution. Substituting $h_t = \frac{\xi}{2}\ln\left(1+\frac{M_t}{M}\right)$ into the above gives us (3.8). □

3.A.2 Static equilibrium

Start by defining $\Psi_x(T_{x,t}) \equiv \mathbb{A}_{x,0}L_{x,0}^{\alpha_L}\Omega(T_{x,t})$ so that

$$Y_{x,t} = (an^{\alpha_L})^{t-1}\mathbb{A}_{x,0}L_{x,0}^{\alpha_L}\Omega(T_{x,t})K_{x,t}^{\alpha_K}q_{x,t}^{\alpha_q} = (an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})K_{x,t}^{\alpha_K}q_{x,t}^{\alpha_q}$$

The lagrangian

$$\begin{aligned}\mathcal{L} = &\int_x (an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})(S_{x,t}^K)^{\alpha_K}(S_{x,t}^q)^{\alpha_q}dx \\ &+ \lambda_K(1 - \int_x S_{x,t}^K dx) + \lambda_q(1 - \int_x S_{x,t}^q dx)\end{aligned}$$

The first order conditions are

$$\alpha_K(an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})(S_{x,t}^K)^{\alpha_K-1}(S_{x,t}^q)^{\alpha_q} = \lambda_K \tag{3.85}$$

$$\alpha_q(an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})(S_{x,t}^K)^{\alpha_K}(S_{x,t}^q)^{\alpha_q-1} = \lambda_q \tag{3.86}$$

Combining the first order conditions we can after some algebra solve for $S_{x,t}^K$ and $S_{x,t}^K$

$$S_{x,t}^K = \frac{\lambda_K}{\alpha_K(an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})}^{\frac{\alpha_q-1}{\alpha_L}}\frac{\lambda_q}{\alpha_q(an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})}^{\frac{-\alpha_q}{\alpha_L}}$$

$$S_{x,t}^q = \frac{\lambda_q}{\alpha_q(an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})}^{\frac{\alpha_K-1}{\alpha_L}}\frac{\lambda_K}{\alpha_K(an^{\alpha_L})^{t-1}\Psi_x(T_{x,t})}^{\frac{-\alpha_K}{\alpha_L}}$$

Next, substitute these expressions into their respective constraints we can write

$$1 = \int_x S_{x,t}^K dx = \frac{\lambda_K}{\alpha_K(an^{\alpha_L})^{t-1}}^{\frac{\alpha_q-1}{\alpha_L}}\frac{\lambda_q}{\alpha_q(an^{\alpha_L})^{t-1}}^{\frac{-\alpha_q}{\alpha_L}}\int_x \Psi_x(T_{x,t})^{\frac{1}{\alpha_L}}dx$$

$$1 = \int_x S_{x,t}^q dx = \frac{\lambda_q}{\alpha_q(an^{\alpha_L})^{t-1}}^{\frac{\alpha_K-1}{\alpha_L}}\frac{\lambda_K}{\alpha_K(an^{\alpha_L})^{t-1}}^{\frac{-\alpha_K}{\alpha_L}}\int_x \Psi_x(T_{x,t})^{\frac{1}{\alpha_L}}dx$$

after substituting back in the conditions (3.85) and (3.86) into these expressions we have that

$$S_{x,t}^K = S_{x,t}^q = \Psi_x(T_{x,t})^{\frac{1}{\alpha_L}} / \int_x \Psi_x(T_{x,t})^{\frac{1}{\alpha_L}}dx$$

Next, after substituting these optimality shares back into the objective function it follows that

$$\begin{aligned} F_{total}(K_t, q_t, \{T_{x,t}\}_{x=-1}^{x=1}; t) &= (an^{\alpha_L})^{t-1} K_t^{\alpha_K} q_t^{\alpha_q} \int_x \Psi_x(T_{x,t}) \frac{\Psi_x(T_{x,t})^{\frac{1}{\alpha_L}}}{\int_x \Psi_x(T_{x,t})^{\frac{1}{\alpha_L}} dx}^{\alpha_K+\alpha_q} dx \\ &= (an^{\alpha_L})^{t-1} K_t^{\alpha_K} q_t^{\alpha_q} \left(\int_x \Psi_x(T_{x,t})^{\frac{1}{\alpha_L}} dx \right)^{\alpha_L} \end{aligned}$$

3.A.3 Proof that the planning problem with costly transfers can be represented by a competitive equilibrium.

For the case of costly transfers we for simplicity ignore bonds and modify the consumers problem as follows

$$\max \sum_{t=0}^{\infty} \beta^t L_{x,t} U(C_{x,t}/L_{x,t}) \tag{3.87}$$

subject to

$$\sum_{t=0}^{\infty} p_t^k (C_{x,t} + K_{x,t+1}) = \sum_{t=0}^{\infty} p_t^k (R_{x,t} K_{x,t} + I_{x,t} - \Theta_{x,t}) \tag{3.88}$$

$$\Theta_{x,t} \equiv \frac{\mathcal{C}_0}{2} \left(R_{x,t} K_{x,t} + I_{x,t} - K_{x,t+1} - C_{x,t}\right)^2 \tag{3.89}$$

$$I_{x,t} \equiv w_{x,t} L_{x,t} + p_t q_{x,t} + \tau_{x,t} q_{x,t} \tag{3.90}$$

From the first order necessary condition for consumption and capital we have

$$U'(C_{x,t}/L_{x,t}) = \Lambda_x^k p_t^k [1 - \Theta'_{x,t}] \tag{3.91}$$

$$R_{x,t+1} = \frac{p_t^k [1 - \Theta'_{x,t}]}{\beta p_{t+1}^k [1 - \Theta'_{x,t+1}]} \tag{3.92}$$

The assumptions regarding the consumption goods producing firms stay the same as in section 3.3.4 with the exception that fossil fuel prices are now due to the adjustment costs, latitude dependent, and given by $p_{x,t}$ so that the first order condition w.r.t $q_{x,t}$ instead is given by $\mathbb{A}_{x,t}\Omega(T_{x,t})F'_q(K_{x,t}, L_{x,t}, q_{x,t}) = p_{x,t} + \tau_{x,t}$. The fossil firms also face latitude specific prices $p_{x,t}$ and interest rates $R_{x,t}$ which implies that their first order conditions now satisfy $p_{x,t+1} = R_{x,t+1} p_{x,t}$. Based on these revised assumptions of the competitive equilibrium it is clear that both the Euler equation and the externality adjusted Hotelling rule of the planning problem can be produced by the first order conditions of the competitive equilibrium given that taxes are set to

$$\tau_{x,t} = \frac{\sigma \lambda_t^m}{\lambda_t^k [1 - \Theta'_{x,t}]} \tag{3.93}$$

Next, we note that by inserting the first order conditions of the consumption good producers into the adjustment cost function and budget constraint of the representative consumer these conditions become equivalent to the planners adjustment cost function and budget constraint. Hence, following the same logic as we did in the open and closed market solutions we note that the planners multiplier must equal consumption good prices $\lambda_t^k = p_t^k$.[49] Hence, it once again becomes clear that welfare weights must be set according to $v(x) = 1/\Lambda_x^k$, for the first welfare theorem to apply providing a match between the first order condition (3.67) of the planner and condition (3.91) of the representative consumer. In order to show correspondence with other arbitrary set of welfare weights the second welfare theorem is required showing how any income distribution can be achieved via the appropriate set of transfers.

3.A.4 Data and calibrations

Zone	90-64N	64-44N	44-24N	24N-EQ	EQ-24S	24-44S	44-64S
Avg.(1890-1900)	-0.76	-0.3	-0.25	-0.15	-0.14	-0.23	-0.30
Avg.(2001-2011)	1.51	0.96	0.57	0.45	0.43	0.42	0.18
Temp.Anom. (°C)	2.27	1.27	0.82	0.6	0.57	0.65	0.48

Table 3.1: Mean temperature anomalies for 8 zonal regions relative to base period 1951-1980, calculated based on data from Hansen et.al (2010).

[49] See section 3.3.5 or 3.3.6 for details.

Chapter 4

Energy Balance Climate Models, Damage Reservoirs and the Time Profile of Climate Change Policy*

4.1 Introduction

Energy balance climate models (EBCMs) have been extensively used to study Earth's climate (e.g. Budyko (1969), Sellers (1969), North (1975a), North (1975b) , North *et al.* (1981)), and Wu and North (2007)). The basic components of these models are incoming solar radiation, outgoing infrared radiation, transportation of heat across the globe and the presence of an endogenous ice line where latitudes north (south) of the ice line are solid ice and latitudes south (north) of the ice line are ice free. The ice line has the important property of regulating the planets energy heat budget where the location of the ice line determines how much of the incoming solar radiation is reflected back out to space. Ice covered areas have a higher albedo implying that they absorb less of the incoming solar radiation thus contributing less to planetary warming.

In the economics literature, climate change is often studied in the framework of integrated assessment models (IAMs) featuring carbon cycles and temperature dynamics (e.g. Nordhaus (1994), Tol (1997) Nordhaus and Boyer (2000), Hope (2006), Nordhaus (2007)). These models typically feature empirically calibrated, to the most part linear, climate modules capturing global average estimates of for example atmospheric temperature levels. This approach tends to ignore the complexities associated with heat transportation across latitudes and ice-albedo feedback effects which lie at the heart of the EBCM literature.[1] The importance of ice-albedo feedback effects and latitudinal heat transportation in regulating the climate was recognized early in efforts to represent the earthŠs climate with EBCM's which uncovered the disconcerting possibility that a relatively small decrease in the solar input could lead to catastrophic global

*This paper has been co-authored with William A. Brock and Anastasios Xepapadeas

[1] In this context, the ice-albedo feedback refers to a process allowing the surface albedo to vary with the climate state.

glaciation, the result of a runaway ice albedo feedback (North, 1984). Similarly it was also shown that the ice-albedo feedback effect could have an equally strong amplifying effect on the climate when driven by increasing concentrations of atmospheric carbon dioxide (Wang and Stone, 1980). This showed how something happening at one particular latitude, the albedo changing due to ice line movements, could act to affect the global mean climate. Such feedback effects have also been associated with the notion of climate "tipping points" defined as points where a small forcing is enough to set of a chain of interactions causing a major change in behavior of the system (Roe and Baker, 2010). The potential threats associated with such tipping points has raised much concern within the climate science community in recent years (see e.g. Kerr (2008); Lenton *et al.* (2008); Smith *et al.* (2009)).

In the present paper we couple a latitude dependent EBCM with an endogenous ice line based on the model by North (1975a,b), with a simplified economic growth model. This allows us to investigate what new insights might be gained regarding the time profile of mitigation policy and distribution of damages when accounting for increased complexitity in terms of the ice-albedo feedback and latitudinal heat transport. The explicit presence of a spatial dimension and an ice line, whose latitude is determined endogenously, also suggests a different damage profile for sources of damages connected to the movement of the ice line. This does not appear in traditional IAMs. In particular, we differentiate between two types of damages from climate change, traditional gradually increasing damages, and a damage reservoir type, where the later represents a finite source of economic damage associated with the movement of the ice line. Damage reservoirs in the context of climate change can be regarded as sources of damage which eventually will cease to exist when the source of the damage has been depleted. We identify, ice caps and permafrost as typical damage reservoirs, where the state of the reservoir is connected to the latitudinal position of the ice line.

Concerning the ice caps, the movement of the ice line closer to the poles is clearly connected to shrinking ice caps. We consider the implied damages caused by sea level rise due to the release of water from melting glacial ice sheets. We might expect that marginal damages from melting ice caps will increase slowly at first, accelerating to a peak but then eventually diminish as the ice line approaches the Poles. When there is no ice left on the Poles this damage reservoir will have been exhausted. The exact shape of an ice cap specific damage function is of course unknown, it might as well be that damages are proportional to the size of the ice caps so that marginal damages are initially high but diminish as more ice is melted.[2] However, regardless

[2] Damages due to sea level rise also depend on the shape of the shoreline which will determine the amount of land to be covered by water due to melting ice caps (see e.g. the study by Li *et al.* (2009)). Further sea level rise can also be caused by thermal expansion of warming oceans, as a direct result of a rising global temperature. Which of these effects dominate depends upon the time scale studied. For example, the Intergovernmental Panel on Climate Change's Fourth Assessment Report (IPCC (2007)) concluded that thermal expansion can explain about 25 percent of observed sea-level rise for 1961-2003 and 50 percent for 1993-2003, but with considerable uncertainty. There may of course also be other damages caused by the increasing loss of the ice caps and their role in regulating the climate.

of the intermediate behavior, claiming that marginal damages due to ice melting must eventually be zero when all ice has melted is hardly controversial. Thus as human activities move the ice line towards the North Pole the ice area lost diminishes and marginal damages diminish also. The presence of an endogenous ice line in the EBCM allows us to model this type of damages explicitly given the relevant information.[3]

Permafrost is also related to damage reservoirs. Permafrost or permafrost soil is soil at or below the freezing point of water (0° C or 32° F) for two or more years. Permafrost regions occupy approximately 22.79 million square kilometers (about 24 percent of the exposed land surface) of the Northern Hemisphere (Zhang *et al.* (2003)). Permafrost occurs as far north as 84° N in northern Greenland, and as far south as 26° N in the Himalayas, but most permafrost in the Northern Hemisphere occurs between latitudes of 60° N and 68° N. (North of 67° N, permafrost declines sharply, as the exposed land surface gives way to the Arctic Ocean.) Recent work investigating the permafrost carbon pool size estimates that 1400-1700 Gt of carbon is stored in permafrost soils worldwide. This large carbon pool represents more carbon than currently exists in all living things and twice as much carbon as exists in the atmosphere (Tarnocai *et al.* (2009)). The thawing of permafrost as high latitudes become warmer can also be modeled in this context. Thawing of permafrost is expected to bring widespread changes in ecosystems, increase erosion, harm subsistence livelihoods, and damage buildings, roads, and other infrastructure. Loss of permafrost will also cause release of greenhouse gases, methane in wetter areas and CO_2 in dryer areas. Furthermore, permafrost damages are related to damage reservoirs since when permafrost is gone they will vanish provided appropriate adaptation has been implemented.[4] Once again the exact shape of the damage function in the intermediate is unknown, but it is clear that damages must eventually diminish once all greenhouse gases trapped in the soil has been released. The permafrost feedback also suggests that permafrost carbon emissions could affect long-term projections of future temperature change. An increase in Arctic temperatures could release a large fraction of the carbon stored in permafrost soils. Studies indicate that up to 22% of permafrost could be thawed already by 2100. Once unlocked under strong warming, thawing and decomposition of permafrost can release amounts of carbon until 2300

[3]Scientific evidence seems to support the argument that ice sheets might be seriously affected by relatively low increases in temperature. Oppenheimer (2005) reports a number of results suggesting that both the Greenland Ice Sheet (GIS) and the West Antarctic Ice Sheet (WAIS) could be highly vulnerable to temperature rise within the range studied by the current IAMs. Oppenheimer and Alley (2004) report that a 2-4°C global mean warming could be justified for WAIS. Carlson *et al.* (2008) conclude that geologic evidence for a rapid retreat of the Laurentide ice sheet, which is the most recent (early Holocene epoch) and best documented disappearance of a large ice sheet in the Northern Hemisphere, may describe a prehistoric precedent for mass balance changes of the Greenland Ice Sheet over the coming century. In a recent report from the European Energy Agency Agency (2010), it was stated that one of the potential large-scale changes likely to affect Europe is the deglaciation of the WAIS and the GIS and that there is already evidence of accelerated melting of the GIS. Further, a sustained global warming in the range of 1-5°C above 1990 temperatures, could generate tipping points leading to at least partial deglaciation of the GIS and WAIS, thus implying a significant rise in sea levels. See also Lindsay and Zhang (2005).

[4]For more details see for example Zhang *et al.* (2003), Zimov *et al.* (2006), Schaefer *et al.* (2011).

comparable to the historical anthropogenic emissions up to 2000 (approximately 440 GtC) (von Deimling *et al.* (2011)).

To the best of our knowledge, we believe this to be the first attempt at introducing an explicit spatial dimension to the climate module of a climate-economy model which also connects the spatial aspects to the temporal profile of climate damages. This helps in understanding how latitude dependent damages might affect decision-making related to climate change. To be more precise, by allowing for damage reservoirs, as described above, we explicitly introduce two types of damage functions having different temporal profiles. These are the traditional damage function used in most IAMs, in which damages increase monotonically with temperature, and a damage function associated with damage reservoirs. The damage reservoir function is given a similar form as the traditional damage function with the exception that there exist a point where marginal damages will start to decline and eventually become zero implying that damages are bounded from above. This is related to the idea that once the ice caps are gone and the thawed permafrost has released most of its carbon, then reservoir damages will be exhausted. Our results suggest that endogenous ice lines and damage reservoirs introduce non-convexities which induce multiple steady states and Skiba points. The policy implication of these results is that when damage reservoirs are ignored we have a unique steady state and the policy ramp is monotonically increasing. That is, carbon taxes start at low levels and increase with time, which is the 'gradualist approach' to climate policy Nordhaus (2007), Nordhaus (2010), Nordhaus (2011). On the other hand the existence of damage reservoirs and multiple steady states induced by endogenous ice lines results in policy ramps which suggest increased mitigation now, the opposite of what is advocated by the gradualist approach. Furthermore by incorporating damage reservoirs into a DICE type model, our simulations suggest a u-shaped policy ramp with high mitigation now.[5]

The rest of the paper is structured as follows. Since EBCMs are new in economics we proceed in steps that we believe make this methodology accessible to economists. In section 4.2 we present a basic energy balance climate model[6] which incorporates human impacts on climate which result from carbon dioxide emissions that eventually block outgoing radiation. In developing the model we follow (North (1975a), North (1975b)) and use his notation. Section 4.3 couples the spatial EBCM with an economic growth model characterized by both traditional and reservoir damages. We show that nonlinearities induced by endogenous ice lines and reservoir damages result in multiple steady states and Skiba points. Section 4.4 derives similar results in a model more similar in structure to most IAM's. Finally, in section 4.5 we simulate the well known

[5]Multiple equilibria and high current mitigation are also suggested by models incorporating uncertain climate thresholds into DICE (Keller *et al.* (2004), Lempert *et al.* (2006)). See also Nævdal (2006) for an optimal control version featuring uncertain thresholds. More recently Judd and Lontzek (2011) have formulated a dynamic stochastic version of DICE which they call DSICE. They also extend their model to include stochastic tipping point possibilities. They show how this additional real world complexity substantially affects the optimal policy results in comparison to DICE.

[6]For more on EBCMs, see for example Pierrehumbert (2011).

DICE model allowing for damage reservoirs and derive a u-shaped policy ramp. The last section concludes.

4.2 A Simplified One-dimensional Energy Balance Climate Model

In this section we present a simplified integrated model of economy and climate, with the climate part motivated by the one-dimensional energy balance models described in the introduction. The term "one-dimensional" means that there is an explicit spatial dimension in the model, measured in terms of latitudes. The important feature of these models is that they allow for heat diffusion or transportation across latitudes which increases the relevance of the models in describing climate dynamics. Let $T(x,t)$ denote the surface temperature at location (or latitude) x and time t measured in °C. Climate dynamics in the context of the ECBM (e.g. North (1975a,b), North *et al.* (1981)) are defined as:

$$
\begin{aligned}
B\frac{\partial T(x,t)}{\partial t} &= QS(x)\alpha(x,x_s) - [A + BT(x,t) - g(M(t))] \\
&\quad + D\frac{\partial}{\partial x}\left[(1-x^2)B\frac{\partial T(x,t)}{\partial x}\right] \qquad (4.1) \\
T_s &= T(x_s(t),t) \qquad (4.2)
\end{aligned}
$$

where x denotes the *sine* of the latitude "x", where units of x are chosen so that $x = 0$ denotes the Equator, $x = 1$ denotes the North Pole[7] and to simplify we just refer to x as "latitude". A and B are constants which are used to relate the outgoing long wave infrared radiation flux $I(x,t)$ measured in W/m^2 at latitude x at time t with the corresponding surface temperature $T(x,t)$ through the empirical formula,[8]

$$I(x,t) = A + BT(x,t) \qquad (4.3)$$

$g(M(t)$ denotes forcing induced by the atmospheric CO_2 concentration given by $M(t)$. A common form for $g(M(t)$ is a logarithmic form identifying the amount of global warming that can be induced from a doubling CO_2 levels.[9] For the qualitative exercise we pursue in this paper we will however assume a simple linear form in order to keep technicalities to a minimum. More about this below. Q is the solar constant divided by 4.[10] As pointed out by North (1975b), in equilibrium at a given latitude the incoming

[7]Symmetry for the part $x \in [-1, 0]$ is assumed. This assumption is common in EBCMs.

[8]It is important to note that the original Budyko (1969) formulation cited by North parameterizes A, B as functions of fraction cloud cover and other parameters of the climate system. North (1975b) points out that due to non-homogeneous cloudiness A and B should be functions of x. There is apparently a lot of uncertainty involving the impact of cloud dynamics (e.g. Trenberth *et al.* (2010) versus Lindzen and Choi (2009)). Hence robust control in which A, B are treated as uncertain may be called for but this is left for further research. Example, of values used by North (1975a) are $A = 201.4W/m^2$, $B = 1.45W/m^2$.

[9]See for example table 6.2 of the IPCC (2001) report.

[10]The solar constant includes all types of solar radiation, not just the visible light. It is measured by satellite to be roughly 1.366 kilowatts per square meter (kW/m^2).

absorbed radiant heat is not matched by the net outgoing radiation and the difference is made up by the meridional divergence of heat flux which is modeled by the term $D\frac{\partial}{\partial x}\left[(1-x^2)B\frac{\partial T(x,t)}{\partial x}\right]$. Several forms are possible here, the seminal contributions by Budyko (1969) and Sellers (1969) both differ in their parametrizations and structure of heat diffusion. Our form follows that of North (1975a,b) featuring a single thermal diffusion coefficient D which is a calibration parameter determining both heat diffusion and temperature anomalies across latitudes.[11] $S(x,t)$ is the mean annual meridional distribution of solar radiation which is determined from astronomical calculations and ban be uniformly approximated within 2% by

$$S(x) = 1 + S_2 P_2(x) \tag{4.4}$$

with $S_2 = -0.482$ and where $P_2(x) = (3x^2-1)/2$ is the second Legendre polynomial (North, 1975a). Note that $S(x)$ has been normalized so that its integral from 0 to 1 is unity which implies that the integral of incoming radiation reaching the earth is given by Q. $\alpha(x, x_s(t))$ is the absorption coefficient which equals one minus the albedo of the earth-atmosphere system, with $x_s(t)$ being the latitude of the ice line at time t. In equation (4.5) below the ice line absorption drops discontinuously because the albedo jumps discontinuously. North (1975b), page 2034, equation (3) specifies this co-albedo function as:[12]

$$\alpha(x,x_s) = \begin{cases} b_0 = 0.38 & x > x_s \\ \alpha_0 + \alpha_2 P_2(x) & x < x_s \end{cases}, \quad \begin{matrix} \alpha_0 = 0.697 \\ \alpha_2 = -0.0779. \end{matrix} \tag{4.5}$$

where $P_2(x) = (3x^2-1)/2$ represents the second Legendre polynomial. In this set-up the ice line is determined dynamically by the following condition from (Budyko (1969), North (1975a,b)):

$$\begin{matrix} T > -10°\text{C} & \text{no ice line present} \\ T < -10°\text{C} & \text{ice present} \end{matrix} \tag{4.6}$$

finally equation (4.2) determines the location of the ice line ($x_s(t)$). Given the above specification the temperature (T_s) constitutes a break even temperature where temperatures below this level are assumed to be ice covered over the whole year and vice versa. Hence, by setting $T_s = -10$ as in Budyko and North we can solve the equation $T_s = T(x_s(t), t)$ for $x_s(t)$ which is needed in order to determine the solution to (4.1) for given levels of atmospheric carbon dioxide $M(t)$. Equation (4.1) thus states that the temperature at any given latitude is determined by the difference in incoming solar radiation $QS(x)\alpha(x,x_s)$ and outgoing radiation heat radiation $I(x,t)$ adjusted for latitudinal heat flux $D\frac{\partial}{\partial x}[\ldots]$.

Although the introduction of heat diffusion adds extra complexity, since it defined through the use of partial differential equations, a more simplified approach is available

[11] As an example of the magnitude of D, North *et al.* (1981) pick a value of $D = 0.649$.

[12] A smoothed version of a co-albedo function is equation (38) of North *et al.* (1981).

through the use of Legendre approximation methods as introduced by (North (1975b)). The solution can then be approximated by

$$T(x,t) = \sum_{nEven} T_n(t)P_n(x) \tag{4.7}$$

where $T_n(t)$ are solutions to appropriately defined ODEs and $P_n(x)$ are even numbered Legendre polynomials. A satisfactory approximation of the solution for (4.1)-(4.2) within a few percent, can be obtained by the so called two mode solution where $n = \{0,2\}$ (North, 1975b).[13] The two-mode approximation is thus defined as $T(x,t) = T_0(t) + T_2(t)P_2(x)$ where $T_0(t)$, is the first mode, and $T_2(t)$, the second mode. Hence, a two-mode approximation to the system (4.1)-(4.2) can be obtained from the solution to the following system of differential algebraic equations:

$$\frac{BdT_0}{dt} = -(A + BT_0(t)) + \int_0^1 QS(x)\alpha(x,x_s(t))dx + g\left(M\left(t\right)\right) \tag{4.8}$$

$$\frac{BdT_2}{dt} = -(1+6D)BT_2(t) + 5\int_0^1 QS(x)\alpha(x,x_s(t))P_2(x)dx \tag{4.9}$$

$$T(x,t) = T_0(t) + T_2(t)P_2(x) \tag{4.10}$$

$$T(x_s,t) = T_s \tag{4.11}$$

where $P_2(x) = (3x^2-1)/2$ is the second Legendre polynomial that provides the spatial dimension to the solution. Note, that the constant ice line temperature $T_s = -10$ is needed in order to determine the position of the ice line x_s and hence the co-albedo $\alpha(x,x_s(t))$ of (4.8) and (4.9).

From the two-mode approximation of the temperature, we obtain the global mean temperature $T_0(t)$, which is just the integral of $T(x,t)$ over x from zero to one. The variance of the temperature can be defined as

$$V_T = \int_0^1 \left[T(x,t) - T_0(t)\right]^2 dx = \int_0^1 (T_2(t)P_2(x))^2 dx = \frac{(T_2(t))^2}{5} \tag{4.12}$$

Local temperature means at latitudes $(x, x+dx)$ and the mean temperature over a set of latitudes, $Z = [a,b]$, are defined by

$$\left[T_0(t) + T_2(t)P_2(x)\right]dx,\ m\left[a,b\right] = \int_a^b \left[T_0(t) + T_2(t)P_2(x)\right]dx \tag{4.13}$$

while the variance of temperature over the set of latitudes $Z = [a,b]$ is

$$V\left[a,b\right] = \int_a^b \left[T_0(t) + T_2(t)P_2(x) - m\left[a,b;t\right]\right]^2 dx. \tag{4.14}$$

[13]The complete derivation of the two-mode solution is provided in Appendix 4.A. For more details regarding the use of approximation methods see chapter 6 of Judd (1998).

When the area $Z = [a,b]$ is introduced, it is plausible to assume that utility in each area $[a,b]$ depends upon both the mean temperature and the variance of temperature in that area. For example we may expect increases in mean temperature and variance to have negative impacts on output in any area Z, if it is located in tropical latitudes. In contrast mean temperature increases in some areas Z (e.g. Siberia) may increase rather than decrease utility.[14] Existing dynamic IAMs cannot deal with these kinds of spatial elements, such as impacts of changes in temperature variance, generated by climate dynamics over an area Z.

In the climate model $M(t)$ is the stock of the atmospheric carbon dioxide. This stock affects the evolution of the temperature through the function g, and evolves through time under the forcing of human inputs in the form of emissions of green house gases (GHGs) $h(x,t)$ emitted at latitude x and time t.

For the human input we assume that emissions $h(x,t)$ relate to $S(t)$ by the simple equation

$$\dot{M}(t) = \int_0^1 h(x,t)dx - mM(t) = h(t) - mM(t) \tag{4.15}$$

where m is the carbon decay rate. To simplify the exposition we reduce the number of state variables in the problem by assuming that $M(t)$ has relaxed to a steady state and it relates to $h(t)$ through the simple linear relation $M(t) = (1/m)h(t)$. Thus we approximate $g(M(t))$ by a simple linear relation $\gamma h(t)$.[15]In this model the latitude of the ice line can move in time in response to changes in human input since the ice line solution depends on $h(t)$. Moving of the ice line towards the poles generates the damages related to damage reservoirs.

The climate model (4.8)-(4.11) that incorporates human input, which affects the evolution of temperature can be further simplified by following simplifications proposed by Wang and Stone (1980) which suggest that an approximation for the solution equation $T(x,t) = T_0(t) + T_2(t)P_2(x)$ can be achieved by replacing $T_2(t)$ by an appropriate constant. Then $dT(x,t)/dt = dT_0(t)/dt$, where $T_0(t)$, is global mean surface (sea level) temperature. Writing $T(t) = T_0(t)$ the evolution of the global mean temperature can be approximated by:

$$\frac{dT(t)}{dt} = -\frac{A}{B} - T(t) + \frac{1}{B}\int_0^1 \left[QS(x)\alpha(x, x_s(t))\right]dx + g(M(t)). \tag{4.16}$$

Thus the Wang and Stone (1980) approximation reduces the model to one whose evolution is described by (4.16). Wang and Stone (1980) (equation 3) calibrate the

[14]Here, we are referring to variance across latitudes. In a stochastic generalization of our model, we could introduce a stochastic process to represent "weather," i.e. very high frequency fluctuations relative to the time scales we are modeling here. Here the "local variance" of high frequency phenomena like "weather" may change with changes in lower frequency phenomena such as mean area Z temperature and area Z temperature variance. See North *et al.* (1981) for an example of how stochastic forcing can be modeled in an EBM framework. We leave this task to future research.

[15]More complicated and probably more realistic approximations will not affect our qualitative results regarding the multiplicity of steady states and the emergence of Skiba points.

model to get a simple equation for the ice line

$$x_s(t) = (a_{ice} + b_{ice}T(t))^{1/2}, a_{ice} = 0.6035, b_{ice} = 0.02078. \tag{4.17}$$

4.3 The Economic-Climate Model: Damage Reservoirs and Multiple Steady States

We introduce the two types of damages due to climate change mentioned earlier. Let us define these damages by two functions $D_1(T(t))$ and $D_2(x_s(t))$, where 1 denotes the traditional damages due to temperature rise, and 2 denotes damages due to reservoir damages from movement of the ice line towards the north and permafrost melting. A simplified integrated EBCM can be developed along the following lines.

We consider a simplified economy with aggregate capital stock K. An amount K_2 from this capital stock is diverted to alternative "clean technologies". Output in the economy is produced by capital and emissions h according to a standard production function $F(K - K_2, h + \phi K_2)$, where ϕ is an efficiency parameter for clean technologies.[16] The cost of using a unit of h is $C_h(h)$,with $C_h(0) = 0$, $C_h^{'} > 0, C_h^{''} > 0$. The use of emissions can be reduced by employing clean technologies at an effective rate ϕK_2. Denoting consumption by C, net capital formation in our simplified economy is described by

$$\frac{dK}{dt} = F(K - K_2, h + \phi K_2) - C - C_h(h) - \delta K \tag{4.18}$$

where δ is the depreciation rate on the capital stock. Assuming a linear utility function or $U(C) = C$, we consider the problem of a social planner that seeks to maximize discounted life time consumption less damages from climate change subject to (4.16), (4.17), and (4.18).

In this set-up the problem of the social planner can be described, in terms of the

[16] See Xepapadeas (2005) for different ways in which emissions and environment can be modeled as production factors.

following Most Rapid Approach Problem (MRAP) problem,[17]

$$V(T(0)) = \max \int_0^\infty \mathrm{e}^{-\rho t} \left[F(K - K_2, h + \phi K_2) - C_h(h) - (\delta + \rho) K \right. \tag{4.19}$$

$$\left. - D_1(T(t)) - D_2(x_s(t)) \right] dt$$

subject to (4.17) and

$$\frac{dT(t)}{dt} = -\frac{A}{B} - T(t) + \frac{\gamma}{B} h(t) + \frac{1}{B} \Psi(T(t)), \tag{4.20}$$

$$\Psi(T(t)) = \int_0^1 \left[Q S_2(x) \alpha(x, x_s(t)) \right] dx \ , T(0) = T_0, \tag{4.21}$$

where $V(T(0))$ is the current value state valuation function, ρ is the subjective rate of discount on future utility,and the nonlinear function $\Psi(T(t))$ is an increasing function of T (North (1975a)). Problem (4.19)-(4.21), after the successive approximations have been made, has practically been reduced, regarding the climate part, to a zero-dimensional model as found in North *et al.* (1981). We believe that this exercise is of value because it outlines a pathway to extensions to one-dimensional models and is even suggestive via the Legendre basis method of how one might potentially extend the work to two-dimensional models on the sphere.[18] Problem (4.19)-(4.21) is in principle tractable to phase diagram methods with the costate variable on the vertical axis and the state variable on the horizontal axis.

At this point, it should be noted that technical change and population growth could also have been introduced in the form of Harrod neutral (labor augmenting) technical change, a formulation which is required for consistency with balanced growth in the neoclassical context. Balanced growth formulations allow us to conduct phase diagram analysis as in the text below. In this case the production function might be written as $F(K - K_2, h + \phi K_2, AL)$, where F is a constant returns to scale production function and $dA/dt = gA$, $dL/dt = nL$, where g is the rate of exogenous labor augmenting technical change and n is the population rate of growth. Output, capital, consumption, emissions and the capital accumulation equation (4.18) can thus be defined in per effective worker (AL) terms. However the temperature dynamics (4.21) and (4.23) now have a non-autonomous term due to exponentially growing emissions. Dealing with this problem while staying within a framework of autonomous dynamics, requires

[17] The assumption of linear utility allows the capital accumulation problem too be written as a MRAP problem. Problem (4.19) is an approximation of the MRAP problem for very large B and $-B \leq \frac{dK}{dt} \leq B$. In problem (4.19) capital, K, can thus be eliminated as a state variable. It should also be noted that in this section, damages are modeled using an additive functional form as explained in Weitzman (2010). In section 4.4 we will revert to the more common multiplicative form. The main qualitative results hold for both these forms.

[18] Research in progress Brock and Judd (2010) focuses on the development of a two-dimensional spherical coupled climate/economic dynamics model by using a basis of spherical harmonics as in Wu and North (2007). This approach, as well as the Legendre basis approach we are using in this paper for one-dimensional models, fits in nicely with the general approach to approximation methods in (Judd (1998), Chapter 6).

introduction of emission reducing technological progress at an appropriate rate in order to be able to transform the temperature dynamics into a stationary form so that phase diagram techniques of analysis of autonomous systems can still be applied. However, this is beyond the scope of the current paper. In the current paper we wish to show how spatial EBCMs can be integrated with capital accumulation models in economics while preserving analytical tractability. The time stationary analysis developed here indicates that a full analysis of more realistic non-stationary systems is potentially tractable now that we have pointed the way in this paper.

Returning to our time stationary framework, we feel that insights are gained more rapidly by analyzing the following qualitatively similar problem that is strongly motivated by the problem (4.19)-(4.21):

$$V(T(0)) = \max \int_0^\infty \mathrm{e}^{-\rho t} \left[F(K - K_2, h + \phi K_2) - C_h(h) - (\delta + \rho) K \right. \tag{4.22}$$
$$\left. - D_1(T) - D_2(T) \right] dt$$
$$\text{s.t. } \frac{dT}{dt} = a_T - b_T T + c_T h \ , (a_T, b_T, c_T) > (0,0,0) \tag{4.23}$$

where $D_1^{'}(T) = a_1 T$, implying increasing marginal damages due to temperature increase, while $D_2^{'}(T)$ is a function increasing at low T reaching a maximum and then decreasing gradually to zero. The shape of $D_2(T)$ is intended to capture initially increasing marginal damages associated with damage reservoirs which reach a maximum as temperature increases, and eventually vanish once the polar ice caps are gone.

The exposition of a number of issues related to damages functions is useful at this point. Assuming a quadratic or a higher degree power function for damages $D_1(T)$ due to temperature increase is consistent with damages related to falling crop yields or reduction to ecosystem services, and this has been the shape adopted in many IAMs. To consider a plausible shape for $D_2(T)$ we have argued in the introduction that as the ice line moves towards the north, marginal damages must eventually tend to zero when the ice cap disappears. Similar behavior is expected by permafrost. Once permafrost is gone further damages associated with permafrost thawing should vanish. A potential damage function invoking these properties is the s-shaped function used in Brock and Starrett (2003) to describe internal loading of phosphorous in a lake system. This functional form has similar qualitative properties as the traditional damage function up to a certain point where marginal damages starts to decline eventually approaching zero. Furthermore, we argue that the combination of these two damage functions, $D_1(T)$ and $D_2(T)$, each one associated with climate change impacts having different time profiles and being disciplined by scientific evidence, provides a more comprehensive description of the problem.

To further analyze the economic part of the problem, define

$$\pi(h) = \max_{K \geq 0, K_2 \geq 0} \left\{ F(K - K_2, h + \phi K_2) - (\delta + \rho) K \right\}. \tag{4.24}$$

Since we assume that $F(\cdot,\cdot)$ is concave increasing, $\pi(h)$ is an increasing concave function of h.[19] We may now write down the current value Hamiltonian and the first order necessary conditions for an optimum,

$$\mathcal{H}(h,T,\lambda_T) = \pi(h) - C_h(h) - D_1(T) - D_2(T) + \lambda_T(a_T - b_T T + c_T h) \tag{4.25}$$
$$\pi'(h) = C_h' - \lambda_T c_T \Rightarrow h = h^*(\lambda_T)\ ,\ h^{*'}(\lambda_T) > 0, \tag{4.26}$$

where it is understood in (4.26) that the inequality conditions of boundary solutions are included, and

$$\frac{dT}{dt} = a_T - b_T T + c_T h^*(\lambda_T)\ ,\ T(0) = T_0 \tag{4.27}$$
$$\frac{d\lambda_T}{dt} = (\rho + b_T)\lambda_T + a_1 T + D_2'(T). \tag{4.28}$$

We know that since $\lambda_T(t) = \frac{\partial V(T(t))}{\partial T(t)} := V'(T(t)) < 0$, the costate variable can be interpreted as the shadow cost of temperature. We also know that if a decentralized representative firm pays an emission tax, then the path of the optimal emission tax is $-\lambda_T(t)$. We can study properties of steady states of the problem (4.19)-(4.21) by analyzing the phase portrait implied by (4.27)-(4.28). The isocline $dT/dt = 0$ is easy to draw for (4.27). Along this isocline we have $\frac{d\lambda_T}{dT} = \frac{b_T}{c_T h^{*'}} > 0$, by using (4.26), thus along this isocline λ_T is increasing in T. There is a value λ_{Tc} such that if $\lambda_T(t) < \lambda_{Tc}$ then $h^* = 0$ and $a_T/b_T = T$. If there are no ice line damages, the $d\lambda_T/dt$ isocline is just a linear decreasing function of T that is zero at $T = 0$, or $\lambda_T = -\frac{a_1}{(\rho+b_T)}T$, which implies that $\lambda_T < 0$ for all $T > 0$. Now add the damages emerging from the damage reservoir to this function. The isocline is defined as

$$\lambda_T|_{\frac{d\lambda_T}{dt}=0} = -\frac{a_1 T + D_2'(T)}{(\rho + b_T)}, \quad \frac{d\lambda_T}{dT} = -\frac{a_1 + D_2''(T)}{(\rho + b_T)}$$

With an s-shaped function representation of $D_2(T)$, $D_2''(T)$ is positive and decreasing, it becomes negative, reaches a minimum, increases and then approaches zero. This induces a nonlinearity to the $d\lambda_T/dt = 0$ isocline. In general it is expected that this isocline will have an inverted N-shape, which means that with an increasing $dT/dt = 0$ isocline if a steady state $(\bar{T}, \bar{\lambda}_T)$ exists, there will be either one or three steady states. To study the stability properties of these steady states we form the Jacobian matrix of (4.27)-(4.28),

$$J(\bar{T}, \bar{\lambda}_T) = \begin{bmatrix} -b_T & c_T h^{*'}(\bar{\lambda}_T) \\ a_1 + D_2''(\bar{T}) & b_T + \rho \end{bmatrix}. \tag{4.29}$$

If at a steady state $a_1 + D_2''(\bar{T}) > 0$ so that the $d\lambda_T/dt = 0$ isocline is decreasing then $\det J(\bar{T}, \bar{\lambda}_T) < 0$ and the steady state is a local saddle point. If $a_1 + D_2''(\bar{T}) < 0$

[19] Note that $\pi'(0) < \infty$ if $\phi > 0$ for the alternative "clean" technology.

so that the $d\lambda_T/dt = 0$ isocline is increasing, the steady state is an unstable spiral.[20] Thus when a unique steady state exists it will be a saddle point. The case of three candidate optimal steady states $\bar{T}_1 < \bar{T}_2 < \bar{T}_3$ is of particular interest. In this case given the shapes of the two isoclines the smallest one and the largest one are saddles and the middle one is an unstable spiral. Thus we have a problem much like the lake problem analyzed by Brock and Starrett (2003), and following a similar argument, it can be shown (under modest regularity conditions so that the Hamiltonian is concave-convex in T) that there are two value functions, call them, $V_{mitigate}(T)$ and $V_{adapt}(T)$, and a "Skiba" point $T_s \in (\bar{T}_1, \bar{T}_3)$ such that $V_{mitigate}(T_s) = V_{adapt}(T_s)$. For $T_0 < T_s$, it is optimal to follow the costate/state equations associated with $V_{mitigate}(T)$ and converge to $\bar{T}_1$, while for $T_0 > T_s$ it is optimal to follow the costate/state equations associated with $V_{adapt}(T)$ and converge to $\bar{T}_3$. In Figure 1 we present this situation for an appropriate choice of functional forms and parameters.[21] Besides the solution path the figure also plots the isoclines both with and without ice line damages. Without ice line damages we have the case when the $\dot{\lambda}_T$-isocline is a linear decreasing function of T, implying that we get a unique global saddle point at the crossing of the $\dot{\lambda}_T = 0$, $\dot{T} = 0$ isoclines denoted by $\bar{T}_n$. For the case with ice line damages on the other hand, we get the inverted N-shaped $\dot{\lambda}_T$, isocline giving us a "Skiba" point T_s lying just between the unstable spiral $\bar{T}_2$ and the local saddle point $\bar{T}_3$.

Hence, for low initial $T_0 < \bar{T}_1$, it will be optimal to levy a low initial carbon tax even though there is a polar ice cap threat and then gradually increasing the carbon tax along a gradualist policy ramp. However, if $T_0 \in (\bar{T}_1, T_s)$, it is optimal to tax carbon higher at T_0 and let the tax gradually fall. But if the initial temperature is large enough, the ice caps are essentially already gone and damage reservoirs have been exhausted. Then the optimal thing to do is to tax carbon initially quite modestly but along an increasing schedule through time to deal with the rising marginal damages due to temperature rise. Figure 1 thus shows how the qualitative picture changes completely when a different shape for the ice line damage function is considered. In particular, the area $T \in (\bar{T}_1, T_s)$ is of interest since, if ice line damages go unaccounted for, the optimal strategy will be to levy a low carbon tax which eventually will raise temperature to $\bar{T}_n$, while in a model with ice line damages included the exact opposite will be true, implying a decrease in temperature to $\bar{T}_1$.

It is important to note that this stationary model is not rich enough to capture the eventual rather sharp increase along the "gradualist" policy ramp of (Nordhaus (2007), Nordhaus (2010)) because in Nordhaus's case the Business as Usual (BAU) emissions path would be growing because of economic growth. Thus the damages from temperature rise alone, growing quadratically as the quantity of emissions grows, would

[20] The eigenvalues of J are: $\frac{1}{2}(\rho \pm \sqrt{\Delta})$, where $\Delta = \rho^2 + 4\left[(a_1 + D_2^{''}(\bar{T}))c_T h^{*'} + b_T(b_T + \rho)\right]$. When $a_1 + D_2^{''}(\bar{T}) > 0$ then $\Delta < 0$ and we have two complex eigenvalues with positive real parts which implies an unstable spiral.

[21] The assumed functions, parameters and calculations used in figure 4.1 are provided in Appendix 4.B.

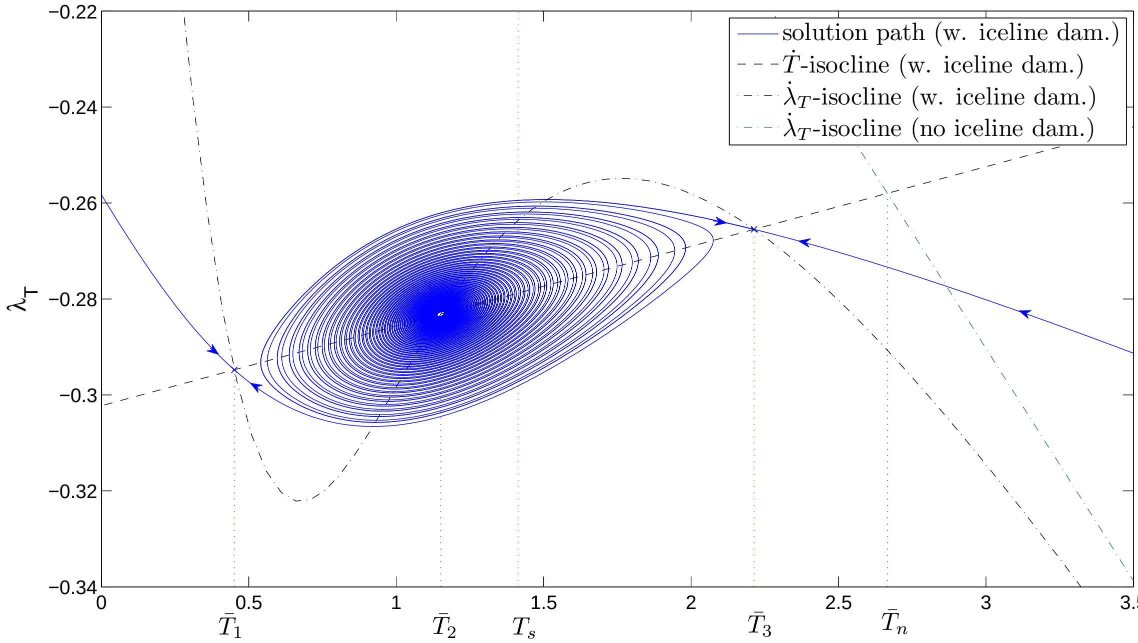

Figure 4.1: Phase diagram for the system (4.27)-(4.28). See appendix for details on the numerical procedure.

lead to the gradualist path of carbon taxes "taking off" in the future. However, this simple stationary model does expose the new behavior of a higher initial carbon tax for $T_0 \in (\bar{T}_1, T_s)$.

4.4 Energy balance - integrated assessment models with damage reservoirs

In this section we incorporate the framework of the simplified energy balance models developed above into a framework similar to well established IAMs such as the DICE/RICE models proposed by Nordhaus. We use notation close to that of Nordhaus for the DICE/RICE part of the model. Consider the continuous time spatial analog of Nordhaus's equations (Nordhaus (2007) Appendix 1 or Nordhaus (2010), A.1-A.20) where we have made some changes to be consistent with our notation and have suppressed (x, t) arguments to ease typing, unless (x, t) is needed for clarity,

$$W = \int_0^\infty \mathrm{e}^{-\rho t} \int_0^1 \phi(x) U(C) dx dt, \tag{4.30}$$

where $U(C)$ is utility and C is aggregate consumption at (x, t), and $\phi(x)$ is a Negishi weight function.[22] Furthermore,

$$Y_n = C + \frac{dK}{dt} + \delta K \tag{4.31}$$

$$Y_n = \Omega(1 - \Lambda)Y, \; Y = F(K) \tag{4.32}$$

where, $Y_n(x, t)$ is output of goods and services at latitude x and time t, net of abatement and damages; $\Omega(T(x, t))$ is the damage function (climate damages as fraction of output) as a function of temperature at (x, t); $\Lambda(x, t)$ is the abatement cost function (abatement costs as fraction of output)[23] at (x, t); and $F(K(x, t))$ is a concave production function of capital. δ is the usual depreciation rate of capital. As explained in the previous section, technology and labor have been removed from the production function in order to avoid problems of non-stationarity in the temperature equation.

Aggregate emissions at time t are defined as:

$$E(t) = \int_0^1 \sigma(1 - \mu(x, t))Y(x, t)dx \tag{4.33}$$

where σ is ratio of industrial emissions to output (metric tons carbon per output at a base year prices), and $\mu(x, t)$ is the emissions-control rate at (x, t). Climate dynamics in the context of the ECBM are given by (4.1) and (4.2). Notice that we have replaced Nordhaus's climate equations Nordhaus (2010), equations A.14-A.20) with the spatial climate dynamics, (4.1) and (4.2).

Maximization of objective (4.30) subject to the constraints above is a very complicated and difficult optimal control problem of the PDE (4.1) on an infinite dimensional space $x \in [0, 1]$. We reduce this problem to a much simpler approximate problem of the optimal control of a finite number of "modes" using the two-mode approach described earlier.

For the two-mode approximation equations $T(x, t) = T_0(t) + T_2(t)P_2(x)$, (4.1) and (4.2) reduce to the pair of differential algebraic

$$\frac{dT_0}{dt} = \frac{1}{B}\left[-(A + BT_0) + \int_0^1 QS(x)\alpha(x, x_s(t))dx + \gamma E\,(t)\right] \tag{4.34}$$

$$\frac{dT_2}{dt} = \frac{1}{B}\left[-(1 + 6D)BT_2 + 5\int_0^1 QS(x)\alpha(x, x_s(t))P_2(x)dx\right] \tag{4.35}$$

$$T_0(t) + T_2(t)P_2(x_s(t)) = T_s, \; T_s = -10°\text{C}. \tag{4.36}$$

[22]The maximization of objective (4.30) with the "Negishi" $\phi(x)$weighting function is a way of computing a Pareto Optimum competitive equilibrium allocation across latitudes as in Nordhaus (2010) discrete time non-spatial formalization. For a presentation of the use of the Negishi weights in IAMs, see Stanton (2010).

[23]With our spatial approach abatement costs could be made site specific which would enable a more comprehensive analysis of issues concerning, e.g., geoengineering. However this goes beyond the scope of the current paper and is left for future research.

Once again we have assumed emissions affect temperature in a linear fashion which is sufficient for the qualitative exercise we are pursuing here. A more accurate representation can be found in table 6.2 of the IPCC (2001) report. Further, since γ adds nothing qualitatively we set $\gamma = 1$ and interpret σ as the product of these two parameters in what follows.

Before continuing notice that North's two-mode approximation has reduced a problem with a continuum of state variables indexed by $x \in [0, 1]$ to a problem where the climate part has only two state variables. We can make yet a further simplification by assuming, as in section 4.3, that the utility function is linear, i.e. $U(C) = C$. This will allow us to write (4.30) as the MRAP problem:

$$W = \int_0^\infty \mathrm{e}^{-\rho t} \int_0^1 \phi C dx dt = \int_0^\infty \mathrm{e}^{-\rho t} \int_0^1 \phi \left[\Omega(1-\Lambda)F - (\rho+\delta)K\right] dx dt. \tag{4.37}$$

Note that for the two mode approximation, the damage function should be defined as:

$$\Omega(T(x,t)) = \Omega(T_0(t) + T_2(t)P_2(x)). \tag{4.38}$$

To ease notation we introduce the inner product notation $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$. We may now write the current value Hamiltonian for the optimal control problem (4.37) and show how we have drastically simplified the problem by using a two-mode approximation,[24]

$$\begin{aligned} \mathcal{H} = \int_0^1 \phi \left[\Omega(1-\Lambda)F - (\rho+\delta)K + \frac{\lambda_0}{B}\sigma(1-\mu)F\right] dx \\ \frac{\lambda_0}{B}\left[\langle QS\alpha, 1\rangle - A - BT_0\right] + \frac{\lambda_2}{B}\left[5\langle QS\alpha, P_2\rangle - (1+6D)BT_2\right]. \end{aligned} \tag{4.39}$$

For the simplified problem (4.37), the capital stock and the emissions control rate $K^*(x,t), \mu^*(x,t)$ are chosen to maximize $\mathcal{H}$ for each (x,t), which is a relatively simple problem. However there is one complication to be addressed. The absorption function $\alpha(x, x_s(t))$ depends upon the ice line $x_s(t)$ where the ice line is given by a solution of (4.36), i.e.

$$x_s(t) = P_+^{-1}\left(\frac{T_s - T_0(t)}{T_2(t)}\right) \tag{4.40}$$

where the subscript "+" denotes the largest inverse function of the quadratic function $P_2(x) = (1/2)(3x^2 - 1)$. Notice that the inverse function is unique and is the largest one on the set of latitudes $[0, 1]$. Equation (4.40) induces a nonlinear dependence of equations (4.34) and (4.35) through the absorption function, but no new state variables are

[24] The important thing to note about this Hamiltonian compared to the Hamiltonian of the original problem (4.30) is this. The original problem would generate a Hamiltonian with a continuum of costate variables, one for each $x \in [0, 1]$. The two-mode approximation approach developed could be quite easily extended to an n-mode approximation approach. Since however North argues that a two-mode approximation is quite good, we continue with a two-mode approximation here.

introduced by this dependence. An additional dependence induced by equations (4.34) and (4.35) as well as equation (4.40) is on the damage function which we parameterize as:

$$\Omega = \Omega(T_0(t), T_2^2(t)P_2^2(x); x_s(t), x) \tag{4.41}$$

The first term in (4.41) represents damages to output at latitude x as a function of average planetary temperature as in (Nordhaus (2007), Nordhaus (2010)) and the second term is an attempt to capture extra damages due to climate "variance". Note that the component $P_2^2(x)$ is larger at $x = 0$ and $x = 1$ than it is at the "temperate" latitude $x = (1/3)^{1/2}$ where $P_2^2(x) = 0$. This is an admittedly crude attempt to capture the component of damages due to "wetter places getting wetter" and "dryer places getting dryer" as well as damages to arctic latitudes compared to temperate latitudes. But some of this dependence can be captured also in the "x" term in the parameterization (4.41). Finally the impact on damages at latitude x due to shifts in the ice line is captured by inclusion of the ice line in (4.41). This is a fairly flexible parameterization of spatial effects (i.e. latitude specific effects) that are not captured in the traditional non-spatial formulations of integrated assessment models.

4.4.1 Optimal mitigation and location specific policy ramp

Let us first illustrate optimal mitigation using our two-mode simplification of our original "infinite mode" problem with linear utility by considering a version of the problem where the impact of policy $\{\mu(x,t)\}$ on the location of the ice line $x_s(t)$ is ignored. That is there is no ice line dependence of any functions of the problem including the absorption function. In this simplified case the albedo function depends only upon x and thus the terms $\langle QS\alpha, 1\rangle, \langle QS\alpha, P_2\rangle$ do not depend upon $T_0(t), T_2(t)$ in (4.34) and (4.35). We also start of by assuming that abatement costs are linear and given by $\Lambda = \psi\mu,\ \psi > 0$, implying that the solution is of the bang-bang type. In section 4.4.2 we will consider a nonlinear version of abatement costs. Hence the two costate differential equations become

$$\frac{d\lambda_0}{dt} = (\rho+1)\lambda_0 - \frac{\partial \mathcal{H}}{\partial T_0} = (\rho+1)\lambda_0 - \int_0^1 \phi \frac{\partial \Omega}{\partial T_0}(1-\Lambda)F dx \tag{4.42}$$

$$\frac{d\lambda_2}{dt} = (\rho+1+6D)\lambda_2 - \frac{\partial \mathcal{H}}{\partial T_2} = (\rho+1+6D)\lambda_2 - \int_0^1 \phi \frac{\partial \Omega}{\partial T_2}(1-\Lambda)F dx$$

Wang and Stone (1980) argue that one can even get a fairly good approximation of T_2 by exploiting how fast mode 2 converges relative to mode zero in equation (4.35) as compared to (4.34). Hence we can further simplify the problem by assuming that T_2 has already converged to:

$$T_2 = \frac{5\langle QS\alpha, P_2\rangle}{(1+6D)B} \tag{4.43}$$

for each $T(t)$.[25] The Hamiltonian (4.39) for the case when the absorption function and T_2 are constant can thus be written as[26]

$$\mathcal{H} = \int_0^1 \left[\phi(\Omega(1-\psi\mu)F - (\rho+\delta)K) + \frac{\lambda_0}{B}\sigma(1-\mu)F \right] dx \tag{4.44}$$

$$+ \frac{\lambda_0}{B}\left[Q\alpha - A - BT_0\right]. \tag{4.45}$$

In this case we obtain the following switching decision rule for $\mu^*(x,t)$

$$\mu^*(x,t) \begin{Bmatrix} = 0 \\ \in [0,1] \\ = 1 \end{Bmatrix} \text{ for } -\lambda_0(t) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \frac{\phi(x)\psi B}{\sigma}\Omega \tag{4.46}$$

$$\Omega = \Omega(T_0(t), (T_2P_2(x))^2, x) \tag{4.47}$$

$$\lambda_0(t) = \int_{s=t}^{\infty} e^{-(\rho+1)(s-t)} \left[\int_0^1 \phi(x)\Omega(1-\psi\mu^*)F\frac{\partial\Omega}{\partial T_0}dx \right] ds. \tag{4.48}$$

Suppose some type of institution wanted to implement this social optimum. One way to do it would be to impose a tax $\tau(\lambda) \equiv \frac{-\lambda_0(t)}{B}$ on emissions when individual agents solve the static problems

$$\max_{\{\mu\in[0,1],K\geq 0\}} \left\{\Omega(1-\psi\mu)F - (\rho+\delta)K - \tau(\lambda)\sigma(1-\mu)F\right\}. \tag{4.49}$$

We see right away that the first order necessary conditions for the problem (4.49) are the same with those resulting from the Hamiltonian function (4.44). Since $F(K)$ is a concave increasing function, then setting $\tau(\lambda) = \frac{-\lambda_0(t)}{B}$ implements the social optimum. Note that the socially optimal emissions tax is uniform across all locations as one would expect from (Nordhaus (2007), Nordhaus (2010)).

An important question arises at this point: What substantive difference does the spatial climate model coupled to the economic model add that is not already captured by non-spatial climate models? There are several important differences regarding policy implications.

The emission reduction policy ramp $\mu^*(x,t)$ is location specific and dictates $\mu^*(x,t) = 1$ for all (x,t) where the relative Negishi weight $\phi(x)$ on welfare at that location is small (recall that $\int_0^1 \phi(x)dx = 1$ by normalization). Assume that the damage function $\Omega = \Omega(T_0(t), (T_2P_2(x))^2, x) = \Omega(T_0(t), (T_2P_2(x))^2)$ is decreasing in both arguments.[27] This crudely captures the idea that damages increase at each latitude as average planetary temperature, $T_0(t)$, increases and as a measure of local climate "variance" $(T_2P_2(x))^2$ increases. Let R denote a set of "at risk latitudes" with low values

[25] Note that in the case where the absorption function does not depend upon $x_s(t)$ the RHS of (4.43) is constant.

[26] Note that with a constant absorption function, $\langle QS\alpha, 1\rangle = \langle Q(1+S_2P_2(x))\alpha, 1\rangle = \langle Q\alpha + QS_2\alpha P_2(x), 1\rangle = \langle Q\alpha, 1\rangle = Q\alpha$, since $\langle QS_2\alpha P_2(x), 1\rangle = 0$.

[27] $(T_2P_2(x))^2$ denotes the variance of the average temperature at location x.

of $\Omega(T_0(t), (T_2P_2(x))^2)$, i.e. with high values of the arguments. The set R is a crude attempt to capture latitudes that would be relatively most damaged by climate change. A plausible type of objective would be to solve the social problem above but with $\phi(x) > 0, x \in R$, $\phi(x) \simeq 0, x \notin R$. We see right away that this social problem would require all xs not in R to reduce all emissions immediately. In general we have,

$$\mu^*(x,t) = 1, \; for \; -\lambda_0(t) > \frac{\phi(x)\psi B}{\sigma}\Omega \tag{4.50}$$

and vice versa. This makes good economic sense. The marginal social burden on the planet as a whole of a unit of emissions at date t, no matter from which x it emanates is, $-\lambda_0(t)$. Locations x where the Negishi weight on the location is small, where emissions per unit of output are relatively large (relatively large $\sigma(x)$), and that are already relatively heavily damaged ($\Omega(T_0(t), (T_2P_2(x))^2, x)$ is high) are ordered to stop emitting. Thus our modeling allows plausible specifications of the economic justice argument stemming from geography to shape policy rules.

In the following section, we use this framework to extend our results in the presence of an discontinuous absorption function that changes at the ice line. This is a more realistic model which introduces ice line damages which we develop in the context of a DICE/RICE-type integrated assessment model.

4.4.2 Optimal mitigation in an IAM-type model with damage reservoirs

We now introduce as the absorption function the version proposed in North (1975a) where

$$\alpha(x, x_s) = 1 - \alpha(x) = \begin{cases} \alpha_1 = 0.38\, x > x_s \\ \alpha_0 = 0.68\, x < x_s \end{cases}, \tag{4.51}$$

where $\alpha(x)$ is the albedo. With this absorption function, the dynamics $T_0(t)$ in (4.34) and the T_2 approximation in (4.43) become respectively

$$\frac{dT_0}{dt} = \frac{1}{B}\left[-(A + BT_0) + Q(\alpha_0 - \alpha_1)\int_{x=0}^{x=x_s(t)} (1 + S_2P_2(x))dx + E + Q\alpha_1\right] \tag{4.52}$$

$$T_2 = \frac{1}{(1+6D)B}\left[5Q(\alpha_0 - \alpha_1)\int_{x=0}^{x=x_s(t)} (1 + S_2P_2(x))P_2(x)dx + Q\alpha_1 S_2\right], \tag{4.53}$$

where the equation for the ice line is, using (4.40),

$$x_s(t) = \left[\frac{2}{3}\frac{T_s - T_0(t)}{T_2} + \frac{1}{3}\right]^{\frac{1}{2}}. \tag{4.54}$$

The objective (4.30) and the constraints (4.51)-(4.54) determine optimal mitigation over time and latitude. The discontinuous absorption function can create a strong non-linearity where a small change in T_0 can cause a large change in damages at some latitudes. However this nonlinearity makes it difficult to proceed with analytical solutions.

To obtain a qualitative idea of the impact of the nonlinearity due to the absorption function and the ice line, we use the climate parametrization used by North (1975a) ($\alpha_0 = 0.68, \alpha_1 = 0.38, A = 201.4, B = 1.45, S_2 = -0.483, T_s = -10, Q = 334.4$). The heat transport coefficient D is found to be approximately 0.2214 by calibrating the ice line function to the current ice line estimate ($x_s = 0.95$).[28]

The system (4.52)-(4.54) is highly nonlinear and can be simplified by deriving a polynomial approximation of x_s as a function of $T_0(t)$. We proceed in the following way. If we substitute $x_s(t)$ from (4.54) into (4.53), then T_2 is a fixed point of (4.53). We solve numerically the fixed point problem (4.53) for values of $T_0 \in \left[-\bar{T}_0, \bar{T}_0\right]$, obtaining the solution $\hat{T}_2(T_0)$. Substituting this back into equation (4.54) gives us the $\hat{x}_s(\hat{T}_2(T_0), T_0)$ which is then used to fit a quadratic curve on $(T_0, \hat{x}_s)$ by using least squares. Thus $\hat{x}_s$ is approximated by a convex curve $\hat{x}_s = \zeta_0 + \zeta_1 T_0 + \zeta_2 T_0^2 = \zeta(T_0)$, $(\zeta_0, \zeta_1, \zeta_2) > 0$.[29] Making use of this approximation, the system (4.52)-(4.54) can thus be written as:

$$\frac{dT_0}{dt} = \frac{1}{B}\left[-(A + BT_0) + Q(\alpha_0 - \alpha_1)\theta(T_0) + E + Q\alpha_1\right] \tag{4.55}$$
$$\text{where } \theta(T_0) := \left[\hat{x}_s + \frac{S_2}{2}(\hat{x}_s^3 - \hat{x}_s)\right] \text{ with } \hat{x}_s := \zeta_0 + \zeta_1 T_0 + \zeta_2 T_0^2$$

Assuming linear utility once again, the Hamiltonian can be written as:

$$\mathcal{H} = \int_0^1 \left[\phi[K^\beta \Omega(T_0)(1-\Lambda) - (\rho+\delta)K] + \frac{\lambda_0}{B}\sigma(1-\mu)K^\beta\right] dx \tag{4.56}$$
$$+ \frac{\lambda_0}{B}\left[-A - BT_0 + Q(\alpha_0 - \alpha_1)\theta(T_0) + Q\alpha_1\right].$$

We now assume that abatement costs are increasing in abatement activities, $\Lambda = \psi\mu^2$. The optimal μ and K will thus be defined as:

$$\mu^*(x,t) = -\frac{\lambda_0 \sigma}{2B\phi\psi\Omega(T_0)}, \forall x \in [0,1] \tag{4.57}$$
$$K^*(x,t) = \left(\frac{\rho+\delta}{\beta}\right)^{\frac{1}{\beta-1}} \left[\Omega(T_0)(1-\psi\mu^{*2}) - \frac{\lambda_0}{\phi B}\sigma(1-\mu^*)\right]^{\frac{-1}{\beta-1}}. \tag{4.58}$$

and the canonical system becomes:

$$\frac{dT_0}{dt} = \left[-A - BT_0 + Q(\alpha_0 - \alpha_1)\theta(T_0) + \int_0^1 \sigma(1-\mu^*)K^{*\beta} dx\right] \tag{4.59}$$
$$\frac{d\lambda_0}{dt} = (\rho + 1 - \frac{Q}{B}(\alpha_0 - \alpha_1)\theta'(T_0))\lambda_0 - \int_0^1 \phi\left[K^{*\beta}\Omega'(T_0)(1-\psi\mu^{*2})\right] dx \tag{4.60}$$

[28] The calibration procedure is explained in detail by North (1975b) (p.2035-2037).

[29] The estimated quadratic function was

$$\hat{x}_s = 0.7126 + 0.0098T_0 + 0.0003T_0^2\ ,\ R^2 = 0.99.$$

which can be solved numerically given a specific shape of $\phi(x)$.

To proceed further we need a more detailed specification for the damage function, which as explained above should contain a temperature component denoted by $D_1(T_0)$ and an ice line component, denoted by $D_2(T_0)$. We specify the damage function in the following way. Lost output from temperature induced damages is: $Y - \frac{Y}{1+D_1(T_0)} = \frac{YD_1(T_0)}{1+D_1(T_0)} := Yd_1(T_0)$. Lost output from ice line movement towards the poles written as a function of T_0 is: $Y - \frac{Y}{1+D_2(T_0)} = \frac{YD_2(T_0)}{1+D_2(T_0)} := Yd_2(T_0)$. The sum of lost output from both sources is: Lost$Y = Yd_1(T_0) + Yd_2(T_0)$. Thus net output available for consumption and mitigation is: $Y - \text{Lost}Y = (1 - d_1(T_0) - d_2(T_0))Y$.

If we define $\Omega_i(T_0) = \frac{1}{1+D_i(T_0)}, i = 1, 2$, then the term $(1 - d_1(T_0) - d_2(T_0))$ can be written as the damage function Ω of the system (4.57)-(4.60) in the form

$$\Omega(T_0) = \Omega_1(T_0) + \Omega_2(T_0) - 1. \tag{4.61}$$

As the global warming problem concerns damages resulting from temperature increases rather than decreases, we restrict the state space to include only temperatures $T_0 >$ 15°C, i.e. in the vicinity of the present average global temperature level.[30] In the spatial model used in this section, this temperature level is found by setting $E = 0$ and solving (4.55), which gives us $T_0 \approx 15.27$. Hence, 15°C can be viewed as a rough ballpark estimate of the preindustrial global temperature average. Damages are assumed to start at 15°C and we thus write our normalized damage function as $\Omega(T_0 - 15)$. Furthermore, we will use the same functional forms for the damage functions as used in section 4.3 (see appendix 4.B).[31]

The EBCM that we presented in this section, resulting from the concepts developed in the earlier part of the paper, has many similarities to the traditional IAMs but also two potentially important differences. The first is the discontinuous absorption function and the second is an alternative shape for ice line damages as opposed to other temperature related damages. Together they introduce complex nonlinearities into the temperature dynamics. The question of whether these differences imply significant deviations from the model's predictions, cannot be answered analytically due to the high complexity of the models. So we resort to numerical simulations.

Figure 4.2 shows the results for the spatial climate model presented in this section. As in section 4.2 this model also gives us 3 candidate optimal steady states, $\bar{T}_{01} < \bar{T}_{02} < \bar{T}_{03}$, where the largest and the smallest ones are saddles while the middle one is an unstable spiral.[32] Between the unstable spiral $\bar{T}_2$ and the saddle $\bar{T}_1$ we have a Skiba

[30]During the development of many energy balance models in the 1960s and 1970s the main concern was usually not that of global warming, but rather that of drastic global cooling that could result due to a slight decrease in the solar constant. This hypothesis was later coined "Snowball earth" by Kirschvink (1992).

[31]The parameters estimates used in deriving figure 4.2 can be found in table 4.B in the appendix.

[32]The corresponding eigenvalues are approximated numerically as $e_{01} = [-0.7037, 0.7237]$, $e_{02} = [0.01 \pm 0.3302i]$ and $e_{03} = [-0.2355, 0.2555]$.

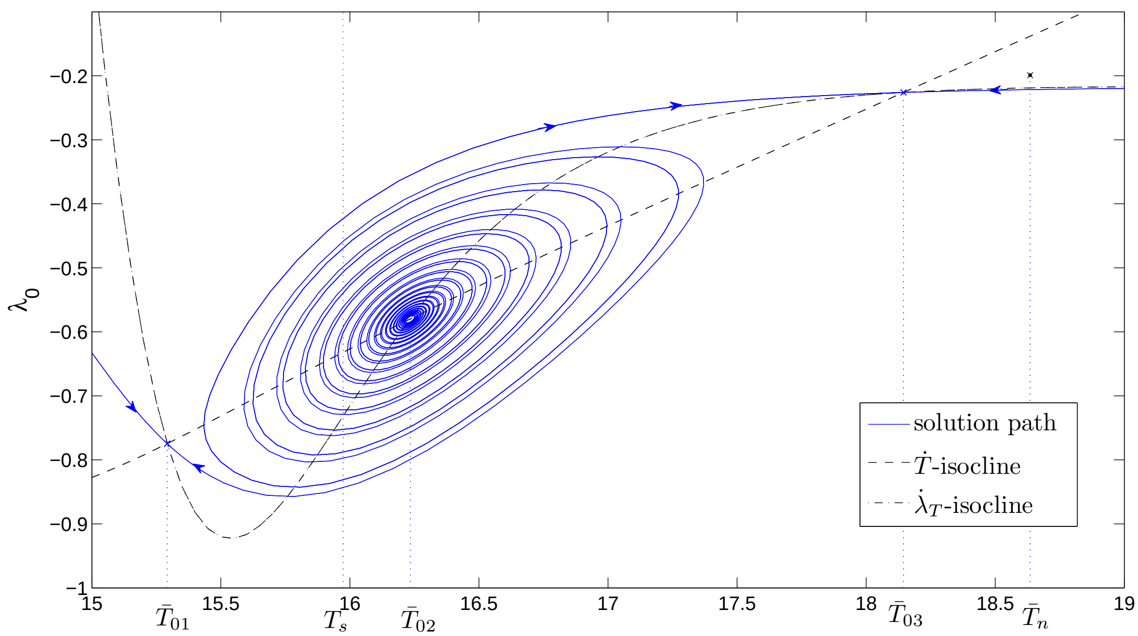

Figure 4.2: Phase diagram for the system (4.59)-(4.60).

point $\bar{T}_s$ similar to that of section 4.2.[33] Hence, defining the carbon tax as above i.e. $\tau = -\lambda_0(t)/B$, for low initial temperatures $T_{00} < \bar{T}_1$ a low but gradually increasing carbon tax will be optimal, while for $\bar{T}_1 < T_{00} < T_s$ the optimal carbon tax an inverted u-shape and is increasing close to T_s but starts decreasing as $\bar{T}_1$ is approached. In the region $T_s < T_{00} < \bar{T}_3$ on the other hand optimal tax policy is u-shaped where initially in the vicinity of T_s it is optimal to levy a high carbon tax which then gradually will decrease. Furthermore, figure 2 also depicts the case when ice line damages are omitted, $\bar{T}_n$. In contrast to section 4.2, both of the isoclines are now affected and in order to keep the figure from becoming too messy, we have chosen to plot only the single equilibrium at the crossing of these isoclines, which is denoted by the black dot at $\bar{T}_n$ in figure 3. The qualitative behavior is however the same as in section 4.2, i.e. the "damage reservoir - no ice line damage equilibrium" is a saddle having a positive slope for the $\dot{T}$-isocline and a negative slope for the $\dot{\lambda}$-isocline.

[33]Greiner *et al.* (2009) find multiple equilibria in a zero-dimensional EBCM, where albedo is modeled by a continuous s-shaped function of temperature. The derived multiple-equilibria and Skiba planes however, only apply for fixed levels of abatement, i.e. there is just a single control variable (consumption). If, however, the social planner can control both consumption and abatement then there exists only a single stable saddle. Our approach, apart from explicitly addressing the more appropriate one-dimensional model also differs in the sense that we obtain multiple equilibria and Skiba points when controlling both consumption and abatement.

4.5 The DICE Model with Damage Reservoirs

Both the relatively simple model of section 4.2 and the more complex model of section 4.4 strongly suggest that the explicit modeling of ice line damages shows the need for strong mitigation now. In order to further demonstrate that this result is robust to the choice of model, we now turn to the DICE model. The purpose of this exercise is to show how the introduction of ice line damages into the damage function, along the lines suggested by the EBCMs, will affect the optimal emission policy implied by DICE. The DICE model, probably the most well known of the IAMs, assumes that all damages to the economy evolve according to the quadratic equation (A.5) in Nordhaus (2007). The calibrated version of this damage function is plotted on page 51 of Nordhaus (2007). Based on this calibration we can see that a 4 degree warming results in approximately a 5% loss of output. We proceed by calibrating our disaggregated damage function in the following way. First, in order to separate out the ice line component from the total amount of damages, we follow the procedure shown in section 4.4.2. We thus replace (A.5) of Nordhaus (2007) with equation (4.61) from section 4.4.2. Hence, we have two separate damage components, $D_1(T)$ and $D_2(T)$, which can be calibrated independently. Next, we use the Nordhaus (2007) impact estimate of 5% loss of output for a 4 degree warming and make a rough assumption that exactly half of these damages should be attributed to the melting of ice sheets causing sea level rises, flooding, changes in ocean currents, etc. Finally, using the same shapes for the temperature and ice line specific components as in previous sections, i.e. $D_1(T) = \frac{a_1}{2}T^2$ and $D_2(T) = a_2\frac{T^{\xi}}{\varphi+T^{\xi}}$, we proceed by calibrating the damage parameters a_1 and a_2 so that $D_1(4) = D_2(4) = 0.025$. In this way our new damage function produces an amount of damage at a 4 degree warming which is equivalent to that of the original model but with differing damage estimates for other temperature levels. For $D_1(T)$ this gives us an estimate of $a_1 = 0.0007813$. In order to calibrate $D_2(T)$ we however, also need to know the values of ξ and φ. The s-shaped function is usually used in models trying to capture thresholds or tipping points. Here, the parameters ξ and φ will have an effect on the steepness and level at which temperature crosses such a threshold. We provide estimates for 3 different assumptions regarding these parameters in order to highlight how they impact on optimal trajectories.

Figure 4.3 plots the optimal emission control rate and carbon price resulting from the DICE-2007 model for three different sets of estimates for ξ and φ. First, for comparison we provide the trajectories for the original model without iceline damages i.e. $a_2 = 0$, which are depicted as solid lines in both graphs. These trajectories are thus based on the Nordhaus (2007) quadratic damage function calibrated as $D(4) = 0.05$ thus yielding a 5% drop in output at a warming of 4 degrees. This provides a good benchmark for comparison since both simulations with and without iceline specific damages in this way yield the same damage estimate for a 4 degree rise in temperature.

As can be seen from this graph, the separation of different damage structures gives us u-shaped type policies where it is optimal to mitigate more initially as opposed to the normal gradualist policy ramp. Look first at the dashed lines which depart

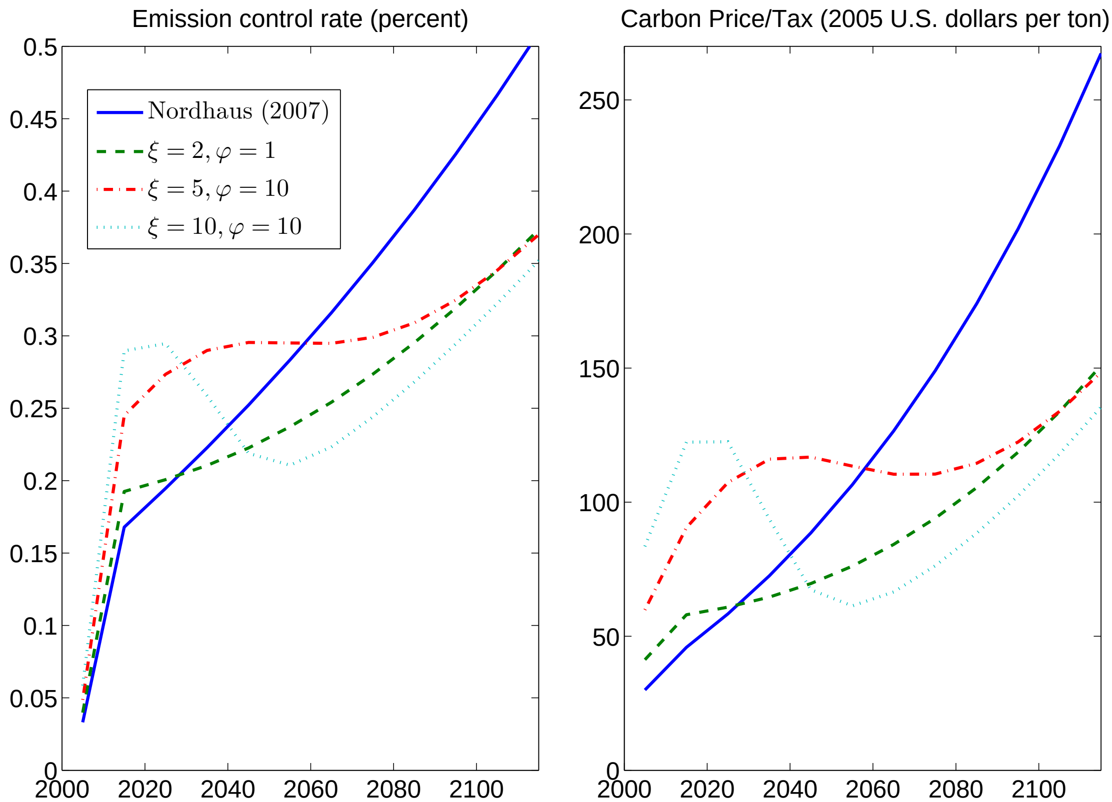

Figure 4.3: Optimal emission control rate and carbon prices *without* iceline damages (solid lines) corresponding to the Nordhaus (2007) and *with* iceline damages (dashed/dash-dotted/dotted) for the three sets of iceline damage coefficients with corresponding damage functions found in figure 4.4 of the appendix between the years 2000-2115.

the least from the original quadratic damage function of Nordhaus. These paths were produced analogous to the calibration in the previous sections with $\xi = 2$ and $\varphi = 1$. The effect that ξ has on the shape of the damage function is that it increases the steepness of the function creating an almost discontinuous jump for very large values while φ is more of a shift parameter moving the location of the threshold. Figure 4.4 in the appendix depicts the iceline damage functions for the three sets of estimates we considered when generating the paths corresponding to figure 4.3.[34] As can be seen for the case when $\xi = 2$ and $\varphi = 1$ this produces only a modest increase in the slope of the damage function when temperature is increased and thereby also logically generates

[34] As we mention in the introduction, Oppenheimer and Alley (2004) report that a 2-4°C global mean warming could be justified for destabilization of the WAIS. Hence, if one confides in this study, the iceline damage function should be calibrated so that marginal damages become zero for temperatures above 4 degrees. This is met for varying degrees of approximation of the damage function parametrization adopted here as can be seen by inspection of figure 4.4 in appendix.

paths similar to those of the original Nordhaus simulation. For higher values however, $\xi = 5$ and $\varphi = 10$, we begin to see an increasingly clear u-shape depicted by the dash-dotted lines in figure 4.3. The steepness of the iceline damage function thus seems to be have a large effect on the emission policy calling for more mitigation now. Finally, the dotted line depicts the most extreme case when ξ is raised to 10. As can be seen from figure 4.4 in appendix this gives us a steep threshold type function for iceline damages where damages remain small up to a little over 1 degree and then increase rapidly. This produces a clear u-shaped tax and emission policy as can be seen in figure 4.3.

The results above thus show off how a u-shaped policy might arise with heavy mitigation now and less later when damages from climate change arrive in a more threshold specific manner as opposed to the more gradual increase, common in contemporary damage functions. Although these results remain specific to our assumptions regarding the shape of the damage function for the ice line as well as the temperature component, we still believe they are valuable since they show off the sensitivity of climate-economy models to structural changes in the damage function.

4.6 Summary, Conclusions, and Suggestions for Future Research

In this paper we introduce the economics profession to spatial Energy Balance Climate Models (EBCMs) and show how to couple them to economic models while deriving analytical results of interest to economists and policy makers. While we believe this contribution is of importance in its own right, we also show how introduction of the spatial dimension incorporated into the EBCMs leads to new ways of looking at climate policy.

In particular, by accounting for an endogenous ice line and paying attention to the associated damage reservoirs and albedo effects we show that due to nonlinearities even simple economic-EBCMs generated multiple steady states and policy ramps which do not in general follow the "gradualist" predictions. These results carry over to more complex models where the economic module has an IAM structure. The interesting issue from the emergence of multiple steady states, is that when the endogenous ice line and discontinuous albedo are ignored, as in traditional IAMs, the policy prescription of these models could be the opposite of the policy dictated by the economic-EBCMs. Furthermore the spatial aspect of the EBCMs allows arguments associated with the spatial structure of climate change damages to shape policy rules. When we applied the damage function implied by the EBCMs and calibrated appropriately simulations in the DICE model gave results interpretable as a u-shaped policy ramp indicating an important deviation from the gradualist policy ramp derived from the standard DICE model. Thus a rapid mitigation policy can be justified on the new insights obtained by coupling the economy with the EBCMs.

Areas for further research could range from making the economics more sophisticated by abandoning the simplifying assumption of linear utility; allowing for technical change and knowledge spillovers across latitudes; or introducing strategic interactions among regions and extensions of the EBCMs. It is thus also of importance to extend our Skiba type analysis to include (exogenous) growth. This could give rise to a dynamic set of Skiba points with a value function of both state and time, thus determining the optimal policy seperatly at each given point in time.[35] Other extensions might also consider how emissions arise more explicitly from the use of fossil fuels (see e.g. Golosov *et al.* (2011)). Future work also needs to be done regarding the extension of EBCMs to a two-dimensional spherical EBCM, because Earth is a sphere, not a line. Brock and Judd (2010) are attempting to make a dent in this problem. They frame the problem as a recursive dynamic programming problem where the state vector includes a number of "spherical modes" that are analogs of the modes in this paper as well as economic state variables. Another possible extension could be the consideration of new policy instruments. Emissions reduction acts on the outgoing radiation in the sense that by reducing emissions the outgoing radiation increases through the second term of the right hand side of (4.1). Another kind of policy could act on the first term of the right hand side of (4.1) in the sense of reducing the incoming radiation. This type of policy might be associated with geoengineering options. Finally a policy which acts on the damage function in the sense of reducing damages for any given level of temperature and radiation balance might be associated with adaptations options. Unified economic-EBCMs might be a useful vehicle for analyzing the structure and the trade offs among these different policy options.

[35] These extensions will undoubtedly increase the complexity and the computational needs for solving the economic-EBCMs.

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4.A Appendix: The two mode solution

In this appendix we show how to derive the two mode solution (4.8) - (4.11). We start with the basic PDE

$$B\frac{\partial T(x,t)}{\partial t} = QS(x)\alpha(x,x_s) - [A + BT(x,t) - g(M(t))] + D\frac{\partial}{\partial x}\left[(1-x^2)B\frac{\partial T(x,t)}{\partial x}\right] \tag{4.62}$$

The two mode solution is defined as:

$$T(x,t) = T_0(t) + T_2(t)P_2(x), \quad P_2(x) = \frac{(3x^2-1)}{2} \tag{4.63}$$

then

$$\frac{\partial T(x,t)}{\partial t} = \frac{dT_0(t)}{dt} + \frac{dT_2(t)}{dt}P_2(x) \tag{4.64}$$

$$\frac{\partial T(x,t)}{\partial x} = T_2(t)\frac{dP_2(x)}{dx} = T_2(t)3x \tag{4.65}$$

Substitute the above derivatives into (4.62) to obtain:

$$B\frac{dT_0(t)}{dt} + B\frac{dT_2(t)}{dt}P_2(x) = QS(x)\alpha(x,x_s(t)) - [A + B(T_0(t) + T_2(t)P_2(x)) - g(M(t))] + D\frac{\partial}{\partial x}\left[(1-x^2)T_2(t)3x\right]$$

or

$$B\frac{dT_0(t)}{dt} + B\frac{dT_2(t)}{dt}P_2(x) = QS(x)\alpha(x,x_s(t)) - A - BT_0(t) - BT_2(t)P_2(x) - g(M(t)) - 6T_2(t)P_2(x) \tag{4.66}$$

Use:

$$\int_0^1 P_n(x)P_m(x)dx = \langle P_n(x), P_m(x)\rangle = \frac{\delta_{nm}}{2n+1} \tag{4.67}$$

$$\delta_{nm} = 0 \text{ for } n \neq m, \delta_{nm} = 1 \text{ for } n = 1$$

and note that $P_0(x) = 1$, $P_2(x) = \frac{(3x^2-1)}{2}$

Multiply (4.66) by $P_0(x)$ and integrate from 0 to 1 to obtain

$$B\frac{dT_0(t)}{dt} = A - BT_0(t) + \int_0^1 \left[QS(x)\alpha(x,x_s(t))\right]dx + g(M(t)) \tag{4.68}$$

Multiply (4.66) by $P_2(x)$ and integrate from 0 to 1 noting that $\int_0^1 P_2(x)dx = 0$, and $\int_0^1 P_2(x)P_2(x)dx = \frac{1}{5}$ to obtain

$$B\frac{dT_2(t)}{dt} = 5\int_0^1 \left[QS(x)\alpha(x,x_s(t))P_2(x)\right]dx - BT_2(t) - 6BDT_2(t) \tag{4.69}$$

where (4.68) and (4.69) are the ODEs of the two mode approximation given by (4.8) - (4.11).

4.B Appendix: analytics and calibration results for section 4.3 and 4.4

The production function in (4.24) is assumed to take the following form:

$$F(K-K_2, h+\phi K_2) = (K-K_2)^{\beta_1}(h+\phi K_2))^{\beta_2} \tag{4.70}$$

with $\beta_1 > 0, \beta_2 > 0$. The solution to problem (4.24) is derived from the first order conditions:

$$\frac{\partial F}{\partial K} = \beta_1(K-K_2)^{\beta_1-1}(h+\phi K_2))^{\beta_2} - (\delta+\rho) = 0 \tag{4.71}$$

$$\frac{\partial F}{\partial K_2} = -\beta_1(K-K_2)^{\beta_1-1}(h+\phi K_2))^{\beta_2} + \beta_2\phi(K-K_2)^{\beta_1}(h+\phi K_2))^{\beta_2-1} = 0 \tag{4.72}$$

Solving the system (4.71) and (4.72) for K and K_2 gives the solution to problem (4.24).

$$K_2^*(h) = \frac{1}{\phi}\left(\frac{(\delta+\rho)}{\beta_1}\left(\frac{\beta_1}{\phi\beta_2}\right)^{1-\beta_1}\right)^{\frac{1}{\beta_1-1+\beta_2}} - \frac{h}{\phi}$$

$$K^*(h) = \frac{\beta_1}{\phi\beta_2}h + \left(1+\frac{\beta_1}{\beta_2}\right)K_2^*(h)$$

Plugging these values back into (4.24) allows us to write $\pi(h)$ as a linear function of h, i.e. $\pi(h) = \tilde{A} + \tilde{B}h$ with

$$\tilde{A} := \left(\frac{\beta_1}{\phi\beta_2}\right)^{\beta_1}\left(\frac{(\delta+\rho)}{\beta_1}\left(\frac{\beta_1}{\phi\beta_2}\right)^{1-\beta_1}\right)^{\frac{\beta_1+\beta_2}{\beta_1-1+\beta_2}}$$

$$-(\delta+\rho)\frac{(1+\phi)}{\phi}\left(\frac{(\delta+\rho)}{\beta_1}\left(\frac{\beta_1}{\phi\beta_2}\right)^{1-\beta_1}\right)^{\frac{1}{\beta_1-1+\beta_2}}$$

$$\tilde{B} := -(\delta+\rho)\left(\frac{\beta_1}{\phi\beta_2} - \frac{(1+\phi)}{\phi}\right)$$

which is increasing in h given that $\beta_1/\beta_2 < (1+\phi)$. Assuming also that $D_1(T) = \frac{a_1}{2}T^2$, $D_2(T) = a_2\frac{T^\xi}{\varphi+T^\xi}$ and $C_h(h) = c_h h^2$. [36] Substituting this into (4.25), using the first order condition we can thus derive the function specific canonical system corresponding to (4.28)-(4.27) as:

$$\frac{dT}{dt} = a_T - b_T T + c_T\frac{\tilde{B}+\lambda_T c_T}{2c_h}\ ,\ T(0) = T_0 \tag{4.73}$$

$$\frac{d\lambda_T}{dt} = (\rho+b_T)\lambda_T + a_1 T - 2a_2 e^{-2T}(T-1)T \tag{4.74}$$

[36] The shape of $D_1(T)$ has become fairly standard in the literature. Still, in a recent review by Ackerman *et al.* (2009), they uncovered no rationale, whether empirical or theoretical, for adopting a quadratic form for the damage function. $D_2(T)$ follows the s-shape found in e.g. Brock and Starrett (2003).

From (4.73) and (4.74) it is easy to confirm the shape of the isoclines depicted in figure 1. For the numerical calculations of the solution paths and the Skiba point we used numerical methods described in Grass *et al.* (2008), Grass (2010). The parameter values used for the numerical calculations are

Parameter	Value	Description
ρ	0.02	discount rate
β_1	0.3	capital income share
β_2	0.5	energy income share
δ	0.1	depreciation rate of capital
ϕ	0.42	efficiency parameter of clean energy
a_1	0.06	damage parameter of $D_1(T)$
a_2	0.25	damage parameter of $D_2(T)$
a_T	0.8	parameter of temperature equation
b_T	0.6	parameter of temperature equation
c_T	0.85	parameter of temperature equation
c_h	0.01	parameter of cost function
ξ	2	parameter of $D_2(T)$ function
φ	1	parameter of $D_2(T)$ function

Table 4.1: The parameter estimates used to generate figure 1.

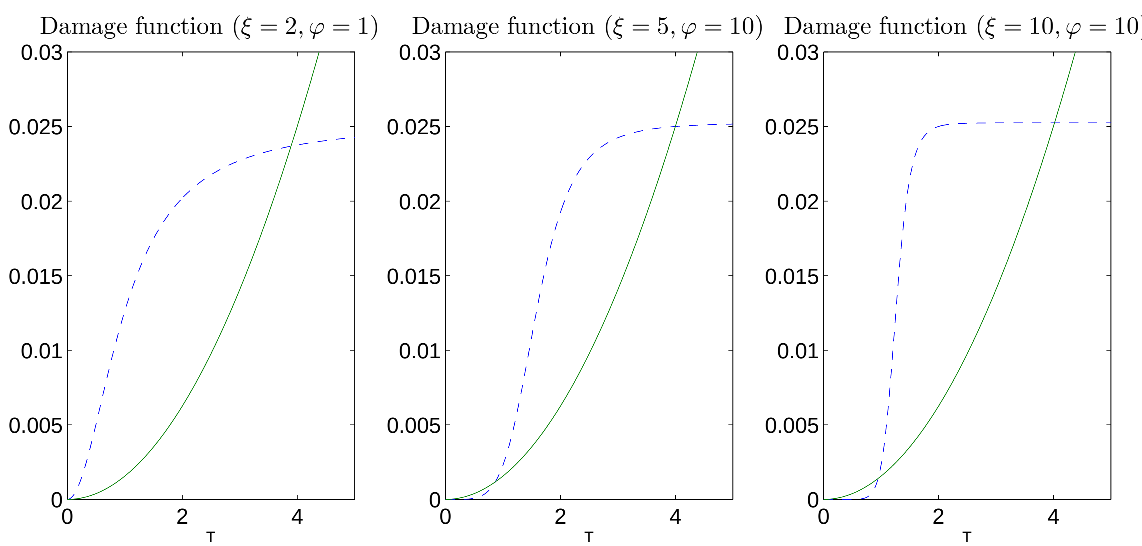

Figure 4.4: Calibrated damage functions D_1 (solid) and D_2 (dashed) for the 3 sets of estimates for ξ and φ.

Parameter	Value	Description
ρ	0.02	discount rate
A	201.4	capital income share
B	1.45	energy income share
α_0	0.68	solar absorbation coefficient for $x < x_s$
α_1	0.38	solar absorbation coefficient for $x > x_s$
ζ_1	0.7126	estimated coefficient of iceline function
ζ_2	0.0098	estimated coefficient of iceline function
ζ_3	0.0003	estimated coefficient of iceline function
Q	334.4	incoming solar radiation divided by 4
S_2	-0.482	temperature distribution parameter
σ	0.01	parameter of $D_2(T)$ function
T_0	15	parameter of $D_2(T)$ function
δ	0.1	depreciation rate of capital
β	0.5	capital income share
$\phi(x)$	1	equal welfare weights for all x
a_1	0.002	damage function coefficient function
a_2	0.1	damage function coefficient function
ξ	2	parameter of $D_2(T)$ function
φ	1	parameter of $D_2(T)$ function

Table 4.2: The parameter estimates used to generate figure 2.

Chapter 5

Assessing Sustainable Development in a DICE World

5.1 Introduction

Integrated assessment models have become well used tools among researchers when trying to estimate the costs associated with climate change (Cline, 1992; Nordhaus, 1994; Manne *et al.*, 1995; Hope, 2006). These general models describe the climate-economy inter linkages in terms of dynamic, global macroeconomic growth models that are coupled with a climate model, describing the effects of increasing greenhouse gas concentrations on temperature. The models typically assume rationality among economic agents and therefore take the normative approach of deriving optimal policies for the most efficient way of slowing climate change. This has resulted in varying recommendations and much debate regarding the urgency for climate change mitigation e.g. ranging from postponing mitigation for several more decades (Nordhaus, 2007), to spending roughly 2% of GDP on mitigation efforts as proposed in the *Stern Review* (Stern, 2007). The debate surrounding these models have had large policy implications, one example being the worldŠs first long term legally binding framework to tackle the dangers of climate change, known as the *Climate Change Act 2008*[1]. As, these models are much referred to in the climate change debate it is troublesome that there is such a divergence in modelling outcomes. The purpose of this paper is therefore to shed some light on the matter by looking at what optimal consumption paths imply for the social-wellbeing of future generations and how these outcomes are affected by different policy regimes and the social-ethical assumptions undertaken. This is done by applying the concept of sustainable development to a neoclassical growth context.

Sustainable development is one of many ways in which we can compare social-wellbeing between different generations. The term became popular after a publication by the World Commission on Environment and Development, which commonly became

[1] The *Climate Change Act 2008* is a legislation forcing a 80% reduction in greenhouse gas emissions in the UK by the year 2050. In the background documents several referrals are made to the *Stern Review*. `http://www.opsi.gov.uk/acts/acts2008/pdf/ukpga_20080027_en.pdf`

known as the Bruntland Commission Report. In this report sustainable development was defined as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs".[2] Since then there have been many attempts to make this term operational in conventional theoretical models of the economy. Pezzey and Toman (2005) provide a good overview of some of these different economic interpretations of sustainable development. In this paper I will define sustainable development as non-declining social welfare. This definition relates to the idea that each generation should leave behind at least as large a *productive base* as they were given by their ancestors, meaning that each generation will have the same possibility to generate welfare as the generation before them.[3] The definition is mainly due to Hamilton and Clemens (1999), Dasgupta and Mäler (2000) and Arrow *et al.* (2003) and is formulated in terms of the Ramsey-Koopmans social welfare functional, where a consumption path is denoted as sustainable if the time derivative of the social welfare functional is greater than or equal to zero.[4] As emphasized in Arrow *et al.* (2003) the concept of optimality differs from sustainability. Even under an optimal policy plan the realized consumption path could be rendered unsustainable if for example the pure rate of time preference, also commonly referred to as the utility discount rate, was set too high.[5] Therefore it is of interest to evaluate if the policy recommendations given by the integrated assessment models also imply consumption paths that are consistent with sustainable development i.e. that they do not exhaust the economy's *productive base*. If this were the case, the policy recommendations given by the model would be steering us in a direction of impoverishment, without informing us that this was actually happening.

This paper makes a first pass at evaluating one of these integrated assessment model in terms of sustainability, namely the DICE-2007 model (Nordhaus, 2007).[6] The results show that although the DICE-2007 model proves to be *productive base* sustainable, this result remains highly sensitive to the specific discounting assumptions employed within the model. When the discounting assumptions used in the DICE-2007 model are compared to the alternative discounting approach, of the DICE-(1994,1999) model, it is only the former that manages to maintain a sustainable *productive base*. This finding implies that even though the parameters of the social welfare function are chosen consistently (i.e. to match historical rates of return on capital), with small implications for the saving rate, the social cost of carbon or the optimal carbon tax, the relative choices of these parameters still affect other aspects of the model. The robustness of these

[2] World Commission on Environment and Devolopement (1987)

[3] The term *productive base* refers not only to all sorts of physical, human and natural capital, but also to institutional arrangements and knowledge. Arrow *et al.* (2003) show that the maintenance of a productive base implies and is implied by non-declining social welfare.

[4] The equivalent notion for the discrete time case, is that social welfare should be non-declining between two periods of time.

[5] Arrow *et al.* (2003) also claim that it is possible to find that along an optimum path social welfare declines for a period and then increases thereafter, in which case the optimum programme does not correspond to a sustainable path locally, but does so in the long run.

[6] DICE is an acronym for the "Dynamic Integrated model of Climate and the Economy".

results are evaluated by introducing uncertainty regarding the most model sensitive parameter values, which to a large extent shape the dynamic structure of the model. The conclusions from the uncertainty analysis is that the most important parameter estimates determining whether the model will turn out to be *productive base* sustainable or not, are the social welfare parameters (i.e. the utility discount rate and the elasticity of marginal utility) along with the total factor productivity (TFP) growth rate. For example, if the growth rate of TFP is set to zero, while leaving everything else unchanged, this would induce an unsustainable path in the DICE-2007 model. As argued by Dasgupta *et al.* (1999) and recently shown in Vouvaki and Xepapadeas (2008b) this assumption not implausible if natural capital is taken into account.

To my knowledge, this is the only study that has evaluated sustainability in terms of changing production possibilities within an integrated assessment model. Previous empirical analysis of *productive base* sustainability within theoretical frameworks include Vouvaki and Xepapadeas (2008a, 2009). Vouvaki and Xepapadeas (2008a) analyze the behavior of the sustainability criterion by empirically parameterizing a standard Solow model subject to a flow pollutant (SO_2) with estimates from the Greek economy. In Vouvaki and Xepapadeas (2009) they instead evaluate sustainability in a global macroeconomic growth model subject to the influence of a stock pollutant (CO_2) for 44 different countries. Their main empirical finding from the later paper is that a business as usual increase in CO_2 emissions will produce a negative measure of sustainability for most countries, while a constant level will yield a positive measure. Although, there are many structural differences between the model used in their analysis compared to the DICE model, the results from this paper still indicates that the major differences in outcomes could be due to different assumptions regarding key parameter estimates. For example, when implementing the choices of social welfare parameters used by Vouvaki and Xepapadeas (2009) in the DICE-2007 model, this produces an unsustainable production path.

The contribution of this paper is threefold. First, it shows how issues regarding sustainable development can be evaluated within an integrated assessment model framework. Second, it highlights which parameter estimates have the greatest effect on the evolution of the productive base. Third, it shows that model projections are sensitive to the relative choices of social welfare parameters, regardless whether these are calibrated to match historical rates of return on capital.

The rest of this paper is structured as follows: Section 5.2 describes the basic structure of the DICE-2007 model and how sustainable development can be evaluate within this framework. Section 5.3 presents result regarding sustainability for the different policy scenarios analyzed by Nordhaus (2007) and how these results are altered when the discounting assumptions of the DICE-(1994,1999) model are implemented. Section 5.4 evaluates how uncertainty regarding key parameter estimates effect the results with respect to sustainability. The final section concludes.

5.2 Sustainability Criterion in the DICE-model

The DICE model is one of the most accepted and widely known integrated assessment models for analyzing links and feedbacks between economic and climatic system. The DICE model was originally developed by Nordhaus (1994) and has since then been updated twice (Nordhaus and Boyer, 2000; Nordhaus, 2007); it is highly transparent and well documented with a freely available program code developed in the GAMS software.[7] The purpose of the model is to combine knowledge from economic and climate sciences in order to derive insights into the costs and benefits of alternative policies for slowing climate change.

5.2.1 DICE model structure

The model assumes that economic and climate policies should be constructed in a way that maximizes the discounted population-weighted utility of per capita consumption over a 600 year time period. This optimization problem is solved by choosing the level of investment and emission control rate that maximizes the sum of discounted future utility (see equation A.1 of appendix), subject to economic and climatic conditions (A.2-A.21).[8] The model involves production of a single commodity, which can be used for either consumption or investment (A.7).

The aggregate output (A.3) of the model is produced using a Cobb-Douglas production function with an exogenous population growth (A.15) and Hicks-Neutral technological change (A.16). The environmental damages and abatement costs (A.5-A.6) are assumed to be proportional to world output and the accumulation of capital (A.4), depends on investment decisions and the natural depreciation rate. The model incorporates a simple carbon cycle system, where carbon flows between three adjacent reservoirs consisting of the atmosphere, upper and lower oceans. The accumulation of greenhouse gases in the atmosphere leads to a warming of the surface through an increase in radiative forcing, which in turn leads to a rise in average global temperature levels (A.9-A.14).[9] The increase in global temperature levels results in physical damages that hinder future production possibilities e.g., through damages on agriculture, migration due to sea-level rise, adverse impacts on health, non-market damages and potential catastrophic impacts. It is also assumed that damages from small and gradual temperature increases are low, but that damages rise in a non-linear fashion (A.5).

Output is produced using energy from either a carbon based or a non-carbon based energy source. Energy use produces an externality in form of CO_2 emissions that together with the emissions from land use (A.19) change accumulates into the atmosphere (A.8-A.9) The amount of emissions originating from output depends on the level

[7] As of August, 2010, there also exist a newly updated beta version for Microsoft Excel. Due to lack of documentation for these models I have chosen to stick with the 2007 model.

[8] The level of investment I_τ and emission control rate μ_τ constitute the control variables of this optimization problem.

[9] Other greenhouse gases are also included in the model as exogenous trends in radiative forcing (A.21).

of carbon-saving technological change, which is modelled as exogenous (A.17). Production decisions are also based on the size of the abatement costs, which depends on the carbon-saving technology, the price of carbon-fuel replacements (backstop technology) (A.18) and the global participation rate in mitigation efforts (A.6).

The amount of available fossil fuels is limited, which implies that there is an upper bound on emission levels (A.20). This assumption generates hotelling or scarcity rents for carbon based energy sources that can be of use in describing the market path for emission reduction. This also implies that even in a laissez-faire economy there will be some amount of mitigation effort due to fossil fuel scarcity.

By modifying this general framework Nordhaus (2007) attempts to replicate and compare alternative environmental policies for tackling climate change. In total, 16 alternative policies are analyzed and compared with respect to the resulting economic and climatic outcomes that they produce.[10]

5.2.2 Sustainability

In order to evaluate whether the economic programmes analyzed within the DICE model framework are consistent with sustainable development, I will use a similar criteria for assessing sustainable development as was adopted by Dasgupta and Mäler (2000).[11] As opposed to optimality, which focuses on achieving a maximum present value flow of consumption over time, the sustainability criterion instead aims at evaluating whether the production possibility set is growing or declining. The main advantage of this method is that it can be incorporated into a general framework which is independent of whether the economies exhibit optimizing or non-optimizing behavior.[12] In mathematical terms the theory basically says that, a consumption path corresponds to a sustainable path at time t, if social welfare at time $t+1$ is not smaller than at time t, i.e. if $V(t+1) \geq V(t)$, where social welfare $V(t)$ can be defined as in expression (A.1):[13]

$$V(t) = \sum_{\tau=t}^{T} (1+\rho)^{-(\tau-t)} U[c(\tau)] L(\tau) \tag{5.1}$$

The value function $V(t)$ reflects social welfare as a function of a consumption and population growth at each moment in time. Our consumption possibilities will further depend on our production possibilities, which in turn is determined by our original endowment of manufactured capital, human capital, natural capital and knowledge, and also by the institutions governing the economy. All these factors put together can be said to constitute society's *productive base* (Dasgupta, 2001). If the institutional

[10] A brief description of these policies is given in section 3, for further details see Nordhaus (2007).

[11] Dasgupta and Mäler (2000) developed their analysis in continuous time. Mäler (2001) provides a translation of basic theory into a discrete time setting.

[12] Dasgupta and Mäler (2000) defined a non-optimizing economy in the following way: a non-optimizing economy is an economy where the government whether by design or incompetence does not choose policies that maximize intergenerational welfare.

[13] See also Dasgupta (2001).

structure of the economy remains unchanged over time, the change in social welfare from one time period to the next, will be entirely determined by the changes in the *productive base*. Under these circumstances sustainable development at time t can be described as a pathway in which

$$V(\Phi(t+1)) \geq V(\Phi(t)), \ \ \textit{at a given time } t \tag{5.2}$$

where $\Phi(t)$ denotes the state of the *productive base* at time t.[14] As shown in Arrow *et al.* (2003), this implies that at each period in time t the change in social welfare will equal genuine investment, which is defined as the accounting value of the rate of change in the productive assets of the economy. All assets which make up the economy's *productive base* will thus carry accounting prices, which are the weights determining, the direction and magnitude, resulting from increases in specific productive assets, such as for example manufactured capital, human capital, natural capital or technology. The sustainability criterion will thus also be a measure of how the economy's wealth, in terms of assets, are evolving.

5.2.3 Sustainability in the DICE model

Returning to the DICE model, a given set of parameter values, along with the emission control and saving rate associated with an optimal consumption path at time t and $t+1$, will imply that the value of the social welfare function $V(t)$ will be entirely determined by the state of its *productive base*. The *productive base* of the DICE model consists of initial values for all variables that determine the state of the system at any given moment of time. The vector of elements included in the *productive base* of the DICE-model, are presented in Table 5.4 of the Appendix. These are the variables that, as opposed to the parameters and constants of the model, evolve over time and therefore implicitly determine the level of feasible consumption at each point in time. The DICE model runs over a time horizon of 600 years which is broken down into fixed 10 year time intevals starting in 2005. Letting $\Phi(\cdot)$ denote the vector of elements given in Table 5.4, the sustainability criterion can thus be formulated as:

$$V(\Phi(2015)) \geq V(\Phi(2005)) \tag{5.3}$$

This means that sustainable development in the year 2005 is determined by the differences in the value of the sums of the maximized discounted future utility streams evaluated in the years 2015 and 2005, respectively. This evaluation is done for two sets of estimates for the pure rate of time preference and elasticity of marginal utility, used in the DICE-2007 model (section 3) and the DICE-(1994,1999) model (section 4), respectively.

[14] It is also possible to formulate the much more demanding requirement for sustainable development, that this condition should be satisfied for all t.

5.3 Sustainability of policy scenarios and the role of the social welfare parameters

One of the major novelties with the DICE-2007 model is that the social welfare function has been revised with updated values for the utility discount rate (ρ) and the elasticity of marginal utility (η). In the new model, the level of utility discounting has been revised from an initial value of 3%, used in the DICE-(1994,1999) model to a lower value of 1.5%, while simultaneously raising the value for the elasticity of marginal utility from 1 (logarithmic) to a new value of 2. As explained in Nordhaus (2007): "this revision moves the model closer to one that displays intergenerational neutrality while maintaining the calibration of the modelŠs rate of return on capital with empirical estimates". This argument stems from the famous Ramsey Rule which states that along an efficient and optimal economic programme the social rate of return on investment r will be given by:

$$r = \rho + \eta \frac{dc/dt}{c} \tag{5.4}$$

With perfectly functioning capital markets, no taxes and lack of divergence between private and social benefits, the social rate of return on investment will equal the private rate, implying that the market interest rate (return on capital) can serve as an appropriate proxy for social discounting.[15] Hence, these two parameters work in dissimilar ways; raising the value of η produces a more egalitarian outcome with increased intergenerational consumption smoothing while a lower value of ρ raises the value of future consumption streams. By recalibrating the model in this way Nordhaus finds a way to lower the value for the pure rate of time preferences (thus pleasing his critics that have pointed out the ethical dilemmas for high rates of utility discounting), while still maintaining the general model results.

5.3.1 General results

Table 1a presents sustainability measures based on criteria (5.3) for different policy scenarios and calibrations of the social welfare parameters ρ and η following the DICE-(1994,1999) and the DICE-2007 model where the policy scenarios are identical to the ones analyzed by Nordhaus (2007). Table 1b shows the results of a separate run using the discounting assumptions following the *Stern Review*.[16] The numbers in parenthesis are the exact sustainability measures defined in social welfare terms where a relatively high (low) value is an indication of a higher (lower) level of sustainability. The numbers proceeding these are index numbers indicating policy performance relative to the *business as usual* (*250-year delay*) policy run. The first column contains the social welfare parameters used in the 2007 model with a utility discount rate (ρ) of 1.5% and a elasticity of marginal utility (η) of 2. The second column contains the social welfare parameters used in the 1994 and 1999 versions of the model.

[15] See for example Arrow and Kurz (1970), Dasgupta and Heal (1979)

[16] The differences between this run and the last policy proposal in table 1a are explained in section 3.6.

Table 1a: Sustainability criteria for alternative policy scenarios

Discounting assumptions	DICE-2007 $\rho = 1.5\%,\ \eta = 2$	DICE-(1994,1999) $\rho = 3\%,\ \eta = 1$
Reference Scenarios		
250-year delay (BAU)	100 (21245)	100 (-107.14)
50-year delay	100.22 (21292)	100.43 (-106.68
Optimal	100.32 (21313)	100.66 (-106.43
Kyoto Protocol		
With United States	100.11 (21269)	100.23 (-106.89)
Without United States	100.02 (21249)	100.05 (-107.09)
Strengthened	100.24 (21295)	100.21 (-106.92)
Climatic constraints		
Concentration limits		
Limit to $1.5xCO_2$	92.06 (19559)	84.59 (-123.65)
Limit to $2xCO_2$	100.23 (21294)	100.49 (-106.62)
Limit to $2.5xCO_2$	100.32 (21313)	100.66 (-106.43)
Temperature limits		
Limit to $1.5°C$	97.23 (20656)	93.10 (-114.53)
Limit to $2°C$	99.52 (21144)	98.77 (-108.46)
Limit to $2.5°C$	100.14 (21275)	100.26 (-106.86)
Limit to $3°C$	100.25 (21300)	100.59 (-106.51)
Ambitious proposals		
Gore Proposal	98.82 (20995)	94.81 (-112.7)
Optimal (low-cost backstop)	101.92 (21653)	104.76 (-102.04)
Stern Proposal (dual-discount)*	101.98 (21453)	(98.92) -108.22

*See section 3.6 for details.

Table 1b: Sustainability criteria for the Stern Review

	$\rho = 0.1\%,\ \eta = 1$
Stern Review	1731

An immediate observation from table 1a is that all alternative policy scenarios analyzed in the 2007 model (column 1) have large positive sustainability measures while the social welfare parameters associated with the 1994 and 1999 model (column 2) generate negative sustainability measures. This is an interesting result which indicates that the choices of these two key parameter estimates are very important in determining the model outcome with respect to sustainability and that a calibration approach which only focuses on maintaining a certain level of return on capital does not insure against possibly large impacts on the economyťs *productive base*. Further, as will be shown in the sensitivity analysis, the sustainability measure is more sensitive to the choice of elasticity of marginal utility compared to the utility discount rate.

5.3.2 Reference scenarios

The first policy run analyzed is one where governments take no policy measures to internalize the costs of damages associated with greenhouse warming. In this policy run the market path for allocating carbon fuels is followed for 250 years, after which the world wakes up and optimizes its emission trajectory in light of climate damages.[17] This run corresponds to the *business as usual* (BAU) scenario which is also referred to as the *baseline* or *no controls* case. The baseline scenario can be compared to the *optimal* policy scenario where an optimal path for emission reduction is followed in order to maximize the value of net economic consumption. Comparing the sustainability measures of these two we find that the *optimal* policy scenario produces a more sustainable consumption path in both columns. This is inline with what we would expect since an *optimal* policy run corresponds to the case where the best possible policy path is followed with regards to the economic, technological and geophysical constraints for the entire time horizon. Logically this should improve on the case where the market path is followed for 250 years with no consideration for damages inflicted by climate change. The reason for this is that when the market path is followed, one of the control variables (the emission control rate) is fixed, which leads to an inferior solution implying that the *optimal* run is more sustainable. The same is true to that of the *50-year delay* run which is equivalent to the *250-year delay* except that the world wakes up earlier, which implies that it lies closer to the projections of the *optimal* policy run.

5.3.3 Kyoto Protocol

Concerning the three versions of the Kyoto Protocol runs they all produce sustainable results which supersede that of BAU. This is because the emission control rate chosen in this run corresponds better to that of the *optimal* policy run. We can also see that the version with the United States included generates a higher sustainability measure than the one without. This is a logical result indicating that a higher world participation rate in climate policies is more effective and hence increases sustainability. In the strengthened version of the protocol more countries are added gradually; they begin with a 10 percent emission reduction and reduce further with 10 percent every 25 years, which generates a more sustainable path than the other versions of the protocol. However, as can be seen in column 2 the equivalent result does not hold for the social welfare parameters used in the DICE-(1994,199) model. It is difficult to assess the exact reasons for this, but the result in itself shows that the choice of time discounting and elasticity of marginal utility will matter when comparing different policy scenarios.

5.3.4 Climatic thresholds

The next two policy categories impose different concentration and temperature limit constraints which comply to quantitative regulation forms such as quotas, targets or commands. Computationally, these are similar to the *optimal* run but with an upper limit climate constraint imposed. The first is an upper atmospheric CO_2-concentration level constraint which is set in relation to preindustrial levels. The second is an upper limit on global temperature increase compared to 1900 levels. The economic intuition

[17] The market path implies that the optimal consumption path is chosen without consideration for the externalities associated with production. A more detailed description of the policy runs can be found in Nordhaus (2007), the time period 250 years is arbitrary and chosen for computational reasons, for example increasing the time span to 350 years has a very marginal effect.

behind these upper limit constraints is to consider them as threshold levels after which damages become infinitely large. For both of these constraints we see that as the limits become increasingly generous, the sustainability increase in both columns. This pattern may at first glance seem counterintuitive, but is simply a consequence of modelling assumptions. The likely reason for this behavior is that the production possibility set will become increasingly limited as the emissions associated with production must be constrained to correspond with stricter upper limit climate constraints. Hence, the costs of the restrains on capital gains resulting from stricter climate constraints seem to outweigh the benefits associated with reduced damages from global warming. However, compared to the BAU scenario the upper limits of $2 \times CO_2$-concentration levels and 2.5-temperature rise produce more sustainable results for both columns. The restrictions imposed are all binding in the optimization problem except in the case of $2.5 \times CO_2$-concentration level which therefore takes on the same value as in the *optimal* policy run.

5.3.5 Ambitious proposals

The last three policy runs constitute ambitious proposals which call for sharp emission reductions.

The *Gore proposal* run implies a rise in the emission control rate from 15 percent in 2010 to 90 percent in 2050, further it is assumed that participation increases from an initial 50 percent to a 100 percent by 2050. This policy run does not improve sustainability compared to BAU regardless of the choice of social welfare parameters. The reason for this is that with these climatic constraint policies, a high emission control rate reduces production possibilities to a greater extent than the benefits received from damage reduction.

The *Optimal (low-cost backstop)* policy analyzes the implications of the development of a new energy source that could replace fossil fuels at a cost that is competitive with today's technologies. This scenario corresponds to the *optimal* run except that the price for the backstop technology is reduced, which in turn lowers abatement costs. This means that more abatement can be done for the same amount of money; leading to a higher sustainabilty measure.

5.3.6 The Stern Review

In order to assess the results of applying the discounting assumptions adopted in the Stern Review Nordhaus uses, what he terms a dual-discounting approach. Here the emission control rates corresponding to the discounting assumptions used in the Stern Review are set as constraints on the *Optimal* policy scenario using the discounting assumptions of the DICE-2007 model. The reason for carrying out this exercise is that a meaningful comparison of policy scenarios requires that social preference structures are the same.

Even though compelling, I find a certain awkwardness to this approach. In the simulation run Nordhaus takes market preferences to be the correct proxy for social preferences. He then tries to assess the impacts of forcing emission standards corresponding to governmental preferences upon this society. However, if a government, discounting in a Stern like fashion, were to adopt an optimal emission control rate the optimal emission path would be optimal only conditional upon the corresponding optimal savings rate. This is also captured in the DICE model since governments choice of emission control rate is intertwined with its choice of saving rate. In sum, the sustainability measure becomes difficult to interpret. However for sake of completeness the result of this policy run is presented in the last row of table 1a.

The results of a policy scenario adopting the simple discounting assumptions of the Stern Review is presented in table 1b. This policy run corresponds to the *Optimal* run in table 1a but with a utility discount rate of 0.1% and a elasticity of marginal utility of 1. This results in a sustainability measure of approximately 1731 units of social welfare. As previously pointed out this measure is not directly comparable to that of table 1a since different preference structures are applied. However, it can be noted that a lower utility discount rate implies that future well-being is valued higher, while a lower elasticity of marginal utility implies that present value of a marginal increase in consumption will rise compared to an equivalent marginal increase in the future. Therefore, as indicated by the relatively lower estimate of sustainability, the effect of lowering the elasticity of marginal utility seems to dominate the sustainability impact of a decrease in the utility discount rate.

5.4 Sensitivity analysis

As was shown in the previous section, the projections of sustainability within the DICE-model rely heavily upon the assumptions made regarding the estimates of social welfare parameters. The other parameter estimates are based on statistical analysis and scientific predictions regarding the evolution the relevant climatic and socioeconomic variables. Usual problems of incomplete knowledge regarding for example measurement and structural issues generate uncertainty. In this section I will address uncertainty regarding the estimated values for some of the key parameter estimates. I will restrict the analysis to the *business as usual* policy scenario and optimal policy scenarios.[18] I will further use the basic references regarding parameter uncertainty provided by Nordhaus (2007) with just a few exceptions. Nordhaus selected eight major parameters of uncertainty, including: the growth rate of total factor productivity, the rate of decarbonization, temperature sensitivity with regard to a doubling of CO_2-concentration levels, damage to output from greenhouse warming, the cost of a backstop technology, population growth, the atmospheric retention fraction of CO_2, and the total availability of fossil fuels.[19] To this list, which is presented in Table 5.1, I have also added uncertainty regarding the choice of utility discount rate and elasticity of marginal utility. I have chosen a normal distribution for both these model parameters, with an expected

[18]Uncertainty has also been addressed within other policy scenarios. These simulations have led to similar conclusions to that of the BAU scenario and are therefore left out.

[19]In Nordhaus (2007) a technical background is given regarding the estimation processes underlying some of the most model-sensitive parameters.

value of 1.5% and a standard deviation of 0.4% for the utility discount rate and an expected value of 2 and a standard deviation of 0.15 for the elasticity of marginal utility.[20] This means that roughly 99.7% of all parameter draws fall in the range [0.3, 2.7] for the utility discount rate and in the range [1.55, 2.45] for the elasticity of marginal utility.[21] Further, I assume that future utility is valued less than current utility and therefore discard all negative values for the utility discount rate. Considering, asymptotic population, this parameter is also drawn from the normal distribution. All values drawn above or below four standard deviations from the mean are discarded. This implies that world population will never decline below 1850 year levels. All other estimates with exception for the temperature sensitivity parameter are drawn from the normal probability distributions with means and variances provided in Table 5.1.

Table 5.1: Uncertainty regarding specific parameter assumptions.

Parameter	Definition	Mean	S.D.
ρ	Pure rate of time preference (time discount rate)	0.015*	0.004
η	Elasticity of marginal utility	2.0*	0.15
g_a	Growth rate of total factor productivity	0.092	0.04
g_σ	Growth rate of CO2-emission to output ratio	-0.073	0.02
$T2 \times CO_2$	Temperature sensitivity (řC) to CO_2 doubling	3.06	1.01
π_2	Damage parameter	0.0028	0.0013
P_{back}	Cost of backstop technology ($)	1170	468
$PopAsym$	Asymptotic global population	8600	1892
$CarCyc$	Transfer coefficient in carbon cycle	0.189	0.017
$Fosslim$	Total amount fossil fuels (Billion tons of carbon)	6000	1200

*The monte-carlo analysis of section 4.3 uses mean values of 3% and 1 respectively for the simulations of the DICE-(1994,1999) model.

The effect on atmospheric temperature associated with a doubling of CO_2 ($T2 \times CO_2$) is set to 3 in the DICE-model, however many estimates from models and observations yield broad and asymmetric probability distributions. Roe and Baker (2007) derive a theoretical probability distribution for temperature sensitivity which, in a satisfactory way manages to approximate, several other published probability distributions using a simple system of linear physical feedback processes. I model uncertainty regarding temperature sensitivity using the theoretical probability distribution derived in their article. Inline with an example from this article, I assume a normal feedback distribution with a mean and a standard deviation of 0.1 and leave out upper tail values exceeding three standard deviations in order to avoid catastrophic runaway feedback effects. This means that 99.9% of the temperature sensitivity values will fall in the

[20] Expected values of 3% and 1 where used for the DICE-94 simulation in section 4.3

[21] The utility discount rate was set as large as possible without risking the possibility of drawing negative values in the monte carlo simulation conducted in section 4.3. Concerning the elasticity of marginal utility the GAMS software had problems solving the model for values above 2.7 so the standard deviation was set in order to not risk that values above 2.7 were drawn.

region [1.7, 12]. The theoretical distribution derived by Roe and Baker (2007) is generated by passing the draws from the selected feedback distribution through the equation given in Figure 1, of their article. Given the expected value and standard deviation of the feedback parameter, this generates a highly skewed distribution for temperature sensitivity with a pronounced upper tail that carries an average expected value of 3.06 and a standard deviation of 1.01.

5.4.1 Individual effects

In figure 5.1 I have outlined the effects on the sustainability criterion under the BAU scenario, when varying some of the most sensitive model parameters. Each line in the graphs represents the sensitivity of the sustainability criterion when varying that specific parameter. The upper graph shows the sensitivity of the sustainability criterion to the assumptions underlying key economic parameters, while the lower graph displays sensitivity to important climate parameters. An immediate observation when studying these two graphs is that the sustainability criterion proves to be much more sensitive to the parameter assumptions underlying the economic part of the model compared to that of the climate part. This is interesting because it means that even if we were to choose climate parameter estimates that lie far out in the tail of their respective probability distributions this would still not matter much with respect to sustainability in comparison to just a marginal change in either one of the social welfare parameters.

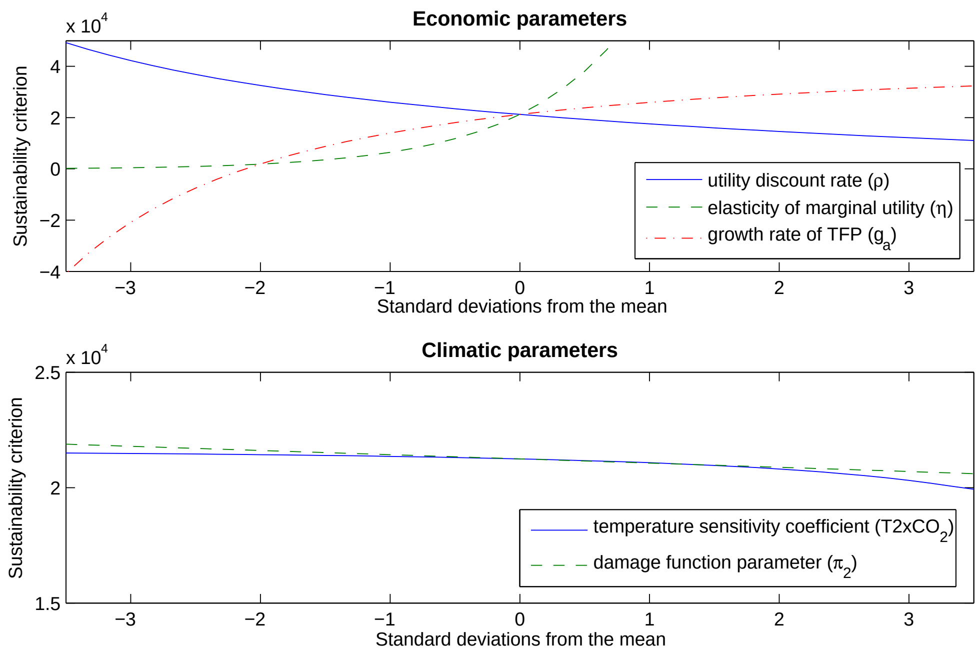

Figure 5.1: Sustainability effects from varying parameter values for the *business as usual* scenario.

From the upper graph we can further see that the elasticity of marginal utility has the largest impact on sustainability in the sense that the variation in the sustainability criterion is the largest for this variable.[22] The sustainability criterion increases exponentially as this estimate increases in value. Although not shown in the figure, values below 0.9 for this parameter will generate negative values for the sustainability criterion.

As previously mentioned we cannot compare the sustainability measure for different values of η, in the sense of saying that one is more sustainable than the other. The reason for this is that social welfare is being measured using a cardinal utility function where preferences are assumed to be identical among all individuals. By definition this utility function allows only for ranking of different consumption bundles conditional on the assumed preference structure. For the exact same reason the definition of sustainability used in this paper, has to be conditioned on the assumed social preferences and cannot be compared to other potential societies having different preferences. However, the sensitivity analysis points out the importance of choosing the estimate of η with great care since the selection can greatly alter the model outcome with respect to sustainability.

The growth rate of total factor productivity is the second most model sensitive parameter. Judging from the variation in this parameter, the model becomes unsustainable at about -2 standard deviations from the mean, i.e. the model has about a 2.5% probability of becoming unsustainable, cetris paribus. The utility discount rate is the third most model sensitive parameter. The sustainability criterion increases in an exponential fashion when the utility discount rate approaches zero and declines slowly when the discount rate increases. The model becomes unsustainable first when the utility discount rate is roughly 6.2% i.e at about 12 standard deviations from the mean. This means that if the model predictions were to hold with certainty we could allow ourself to discount at a rate as high as 6.2% before the *productive base* we leave behind is smaller than the one we started with.

5.4.2 Synergistic effects

Although the above analysis provides an indication of how each specific parameter value effects the sustainability criterion, it does not capture the possibility of interacting effects amongst them. The histograms in figure 5.2 shows the empirical probability distributions of the sustainability criterion, for the BAU scenario under the two different discounting assumptions used in the DICE-(1994,1999) and DICE-2007 model.[23] The results were generated in a Monte Carlo simulation based on 10000 draws using the uncertainty assumptions depicted in table 5.1. The model was thus optimized 10000 times for each set of parameter estimates that were randomly drawn from their respective probability distributions. By recording the sustainability criterion for each set of parameter estimates the empirical probability distributions depicted in figure 5.2 were

[22] That is the difference in sustainability values attained at 3.5 and -3.5 standard deviations from the mean.

[23] The mean values for ρ and η in table 5.1 were thus set to correspond to the DICE-(1994,1999) model.

generated. The first of these two distributions was generated using the discounting assumptions from the DICE-2007 model.Í

In comparison to the DICE-(1994,1999) model these assumptions generate a histogram with a wider distribution and a more pronounced right skew. By subsequently fixing the investigated parameters, one at a time, It is found that the asymmetry of this distribution is generated due to the variation in η. When the value of η is pushed above 2, the sustainability criterion increases in an exponential fashion. This is illustrated in figure 5.1. In particular this generates a long right tail but it also works to spreads out the distribution as a whole. However, an increase in η can also result in decreased sus-

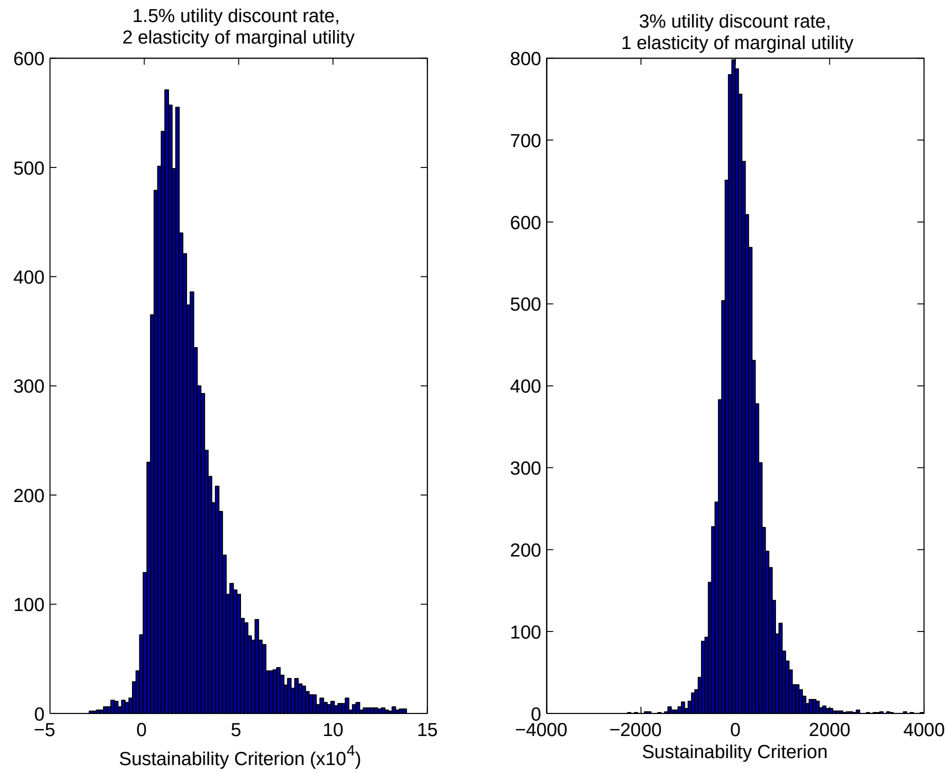

Figure 5.2: Distribution of the sustainability criterion for the BAU scenario.

tainability. An example of this could arise when occasional negative values are drawn for the growth rate of total factor productivity.[24]

Based on these empirical probability distributions, the probability of arriving at an unsustainable path is approximately 2.85% in the DICE-2007 model and about 41.33% for the DICE-(1994,1999). These probabilities are represented by the lower tail of their respective histograms depicted in figure 5.2. Monte Carlo simulations were also performed for the other scenarios. These histograms had similar shapes to the ones showed in figure 5.2 with their respective means and medians distributed in a fashion consistent with the results displayed for the mean values used in section 3 of this paper.

[24] For arguments and evidence regarding negative growth rates see Dasgupta *et al.* (1999) and Vouvaki and Xepapadeas (2008b).

5.5 Conclusions

This paper explores how sustainable development in terms of society's future production possibilities can be analyzed within an integrated assessment model of climate change. The study shows that the DICE-2007 model is production base sustainable in its current form but that the degree of sustainability is highly dependent on the values of key parameter estimates. When revising the social welfare function using estimates for the utility discount rate and elasticity of marginal utility originally put forth in the DICE-(1994,1999) model, this results in an unsustainable productive base. This finding implies when evaluating production base sustainability as defined in this article, the conclusions might differ vastly depending on how the parameters of the social welfare function are chosen. It is not enough to calibrate to an assumed rate of return on capital since sustainability is affected differently depending on the choice of elasticity of marginal utility and discount rate even when the different choices lead to the same rate of return on capital via the Ramsey rule. Instead it might be better to calibrate them independently, considering each parameter estimates individual effect on social welfare. Further, when assessing uncertainty regarding parameter estimates it is found that total factor productivity growth along with the social welfare parameters are those that to the greatest extent influence whether or not the model will maintain a sustainable productive base. This analysis shows that the economic parameters completely dominate important climatic parameters such as the damage parameter or temperature sensitivity coefficient, in the sense that small incremental changes in the economic parameters have a much larger effect on sustainability than corresponding large changes in the climatic parameters. Even when very extreme estimates are assumed for the climate change parameters, this still does not effect the model outcome w.r.t. sustainability as much as a slight reduction in the utility discount rate would have. The sensitivity of the model with respect to the social welfare parameters calls for careful evaluations of the projections and that before using it for policy recommendation purposes it is important to highlight these aspects to policy makers.

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5.A Appendix

Table 5.2: Endogenous relationships

(A.1)	$V(t) = \sum_{\tau=t}^{T}(1+\rho)^{-(\tau-t)}U[c(\tau)]L(\tau)$
(A.2)	$U[c(\tau)] = c(\tau)^{(1-\eta)}/(1-\eta), \;\; c(\tau) = C(\tau)/L(\tau)$
(A.3)	$Q(\tau) = \Omega(\tau)[1-\Lambda(\tau)]A(\tau)K(\tau)^{\gamma}L(\tau)^{(1-\gamma)}$
(A.4)	$K(\tau+1) = K(\tau)(1-\delta)^{10} + 10I(t)$
(A.5)	$\Omega(\tau) = 1/[1+\pi_1 T_{AT}(\tau) + \pi_2 T_{AT}(\tau)^2]$
(A.6)	$\Lambda(\tau) = \pi(\tau)\theta_1(\tau)\mu(\tau)^{\theta_2}, \;\; \pi(\tau) = \varphi(\tau)^{1-\theta_2}$
(A.7)	$Q(\tau) = C(\tau) + I(\tau)$
(A.8)	$E(\tau) = \sigma(\tau)(1-\mu(\tau))A(\tau)K(\tau)^{\gamma}L(\tau)^{(1-\gamma)} + E_{land}(\tau)$
(A.9)	$M_{AT}(\tau) = E(\tau) + \phi_{11}M_{AT}(\tau-1) + \phi_{21}M_{UP}(\tau-1)$
(A.10)	$M_{UP}(\tau) = \phi_{12}M_{AT}(\tau-1) + \phi_{22}M_{UP}(\tau-1) + \phi_{32}M_{LO}(\tau-1)$
(A.11)	$M_{LO}(\tau) = \phi_{23}M_{UP}(\tau-1) + \phi_{33}M_{LO}(\tau-1)$
(A.12)	$F(\tau) = F_{2\times co_2}\{log[M_{AT}(\tau)/M_{AT}(1750)]\} + F_{ex}(\tau)$
(A.13)	$T_{AT}(\tau) = T_{AT}(\tau-1) + \xi_1\{F(\tau) - \frac{F_{2\times co_2}}{T_{2\times co_2}}T_{AT}(\tau-1) - \xi_2[T_{AT}(\tau-1) - T_{LO}(\tau-1)]\}$
(A.14)	$T_{LO}(\tau) = T_{LO}(\tau-1) + \xi_3[T_{AT}(\tau-1) - T_{LO}(\tau-1)]$

Table 5.3: Exogenous relationships

(A.15)	$L(\tau) = L(t)[1-(e^{n(\tau-t)}-1)/e^{n(\tau-t)}] + L_{max}[(e^{n(\tau-t)}-1)/e^{n(\tau-t)}]$
(A.16)	$A(\tau+1) = A(\tau)/(1-g_a(\tau)), \;\; g_a(\tau) = g_a(t)e^{-d(\tau-t)}$
(A.17)	$\sigma(\tau+1) = \sigma(\tau)/(1-g_\sigma(\tau)), \;\; g_\sigma(\tau) = g_\sigma(t)e^{-d_{sig}(\tau-t)-d_{sig2}(\tau-t)}$
(A.18)	$\theta_1(\tau) = P_{back}(t)\sigma(\tau)[(\zeta - 1 + e^{-g_{back}(\tau-t)})/(\zeta\vartheta)]$
(A.19)	$E_{land}(\tau) = E_{land}(t) * 0.9^{(\tau-t)}$
(A.20)	$CCum(t) \geq \sum_{\tau=t}^{T}E(\tau)$
(A.21)	$F_{EX}(\tau) = \begin{cases} F_{EX}(t) + 0.1(F_{EX}(t+10) - F_{EX}(t))(\tau-t), & (\tau-t) \leq 11; \\ F_{EX}(t) + \Upsilon(t), & (\tau-t) > 11; \end{cases}$

Table 5.4: Productive Base (Nordhaus, 2007)

	$\tau = 2005$	$\tau = 2015$	
$K(\tau)$	137	*	U.S.D. trillions
$L(\tau)$	6514	7130	World pop. millions
$A(\tau)$	0.027	0.030	Initial level of total factor productivity
$\sigma(\tau)$	0.134	0.125	CO2-equivalent emissions-GNP ratio 2005
$M_{AT}(\tau)$	808.9	*	Concentration in atmosphere (GtC)
$M_{UP}(\tau)$	1255	*	Concentration in upper strata (GtC)
$M_{LO}(\tau)$	18365	*	Concentration in lower strata (GtC)
$T_{AT}(\tau)$	.0068	*	2000 atmospheric temp change (C)from 1900
$T_{LO}(\tau)$	.7307	*	2000 lower strat. temp change (C) from 1900
$E_{land}(\tau)$	11	9.9	Carbon emissions from land (GtC)
$F_{EX}(\tau)$	-0.060	-0.024	Exogenous forcing for other greenhouse gases
$CCum(\tau)$	6000	5914	Maximum cumulative extraction fossil fuels
$g_a(\tau)$	0.092	0.091	Growth rate for total factor productivity
$g_\sigma(\tau)$	-.0730	-0.071	Growth rate for sigma
$\theta_1(\tau)$	0.056	*	Adjusted cost for backstop technology

Values are dependent on the choice of policy scenario *

Table 5.5: Endogenous variables

$V(t)$	Social welfare at initial time t
$U(\cdot)$	Utility function
$c(\tau)$	Per capita consumption
$Q(\tau)$	Output net damages and abatement (trillion 2005 U.S. dollars)
$K(\tau)$	Capital stock (trillion 2005 U.S. dollars)
$E(\tau)$	Total emission of tons CO_2 (billion tons)
$F(\tau)$	Total amount of radiative forcing (watts per square meter)
$I(\tau)$	Investment (trillion 2005 U.S. dollars)
$C(\tau)$	Consumption (trillion 2005 U.S. dollars)
$\Omega(\tau)$	Damage function
$\Lambda(\tau)$	Abatement-cost function
$\pi(\tau)$	Participation cost markup
$\mu(\tau)$	Emission-control rate
$\varphi(\tau)$	Participation rate
$M_{AT}(\tau), M_{UP}(\tau), M_{LO}(\tau)$	CO_2 in atmosphere, upper and lower oceans (billion tons)
$T_{AT}(\tau), T_{LO}(\tau)$	Atmospheric and lower ocean temp. (^{o}c incr. from 1900)

Table 5.6: Exogenous variables and parameters

τ, t, T	Variable, initial and final time periods respectively (decades)
$L(\tau)$	Population (millions)
ρ	Pure rate of social time preference
η	Elasticity of marginal utility
γ	Elasticity of output w.r.t. capital
δ	Depreciation rate of capital
$A(\tau)$	Total factor productivity
π_1, π_2	Parameters of damage function
$\theta_1(\tau)$	Adjusted cost for backstop technology
θ_2	Parameter of abatement-cost function
$\sigma(\tau)$	Emission to output ratio
$E_{land}(\tau)$	Emission of carbon from land use (billion tons)
$\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}, \phi_{23}, \phi_{32}, \phi_{33}$	Parameters of carbon cycle (flows per period)
ξ_1, ξ_2, ξ_3	Parameters of temperature equations (flows per period)
$F_{2\times co_2}$	Estimated forcings of equilibrium to a doubling of CO_2 mass
$T_{2\times co_2}$	Equilibrium temp. sensitivity to a doubling of CO_2 mass
n	Growth rate of population (decade)
L_{max}	Asymptotic population
g_a	Growth rate for total factor productivity (decade)
g_σ	Growth rate for sigma (decade)
$\zeta, g_{back}, \vartheta$	Parameters of the abatement-cost function
P_{back}	Price of backstop technology
$CCum(\tau)$	Maximum cumulative extraction fossil fuels
$F_{EX}(\tau)$	Exogenous forcing of other greenhouse gases