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Economic and Policy Analyses*

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A simple carbon cycle representation for economic and policy analyses

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Abstract Integrated Assessment Models (IAMs) that couple the climate system and the economy require a representation of ocean CO₂ uptake to translate human-produced emissions to atmospheric concentrations and in turn to climate change. The simple linear carbon cycle representations in most IAMs are not however physical at long timescales, since ocean carbonate chemistry makes CO₂ uptake highly nonlinear. No linearized representation can capture the ocean's dual-mode behavior, with initial rapid uptake and then slow equilibration over ~10,000 years. In a business-as-usual scenario followed by cessation of emissions, the carbon cycle in the 2007 version of the most widely used IAM, DICE (Dynamic Integrated model of Climate and the Economy), produces errors of ~2°C by the year 2300 and ~6°C by the year 3500. We suggest here a simple alternative representation that captures the relevant physics and show that it reproduces carbon uptake in several more complex models to within the inter-model spread. The scheme involves little additional complexity over the DICE model, making it a useful tool for economic and policy analyses.

Keywords DICE · carbon cycle · ocean uptake · integrated assessment model · IAM

1 Introduction

All Integrated Assessment Models (IAMs) that couple the climate system and the economy to evaluate the impacts of climate change require some representation of the global carbon cycle. Anthropogenic climate change is driven primarily by CO₂ emissions produced by human economic activity, but CO₂ does not simply accumulate in the atmosphere, and instead evolves according to the balance between emissions and ocean uptake. IAMs must represent this uptake to translate emissions into atmospheric CO₂ concentrations and in turn climate change. Because state-of-the-art climate models are too computationally expensive for use

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in economic analyses, IAMs use simplified representations of the physical climate system. (See e.g. Goodess et al, 2003). Care must be taken however to ensure that simplifications do not produce inaccurate behavior that can affect damage estimates.

Many simple IAMs use a linearized representation of ocean carbon uptake (for review, see Hof et al, 2012; van Vuuren et al, 2011), but linearized representations cannot fully reproduce the ocean response to increased atmospheric CO₂: rapid initial uptake followed by a slow ‘long-tail’ equilibration stage. In state-of-the-art climate models, about half of a CO₂ perturbation is lost in 30 years (Denman et al, 2007) but a significant portion of the remainder persists for thousands of years (Archer et al, 2009). Linearized models that match initial carbon uptake will necessarily produce too-rapid removal of atmospheric CO₂ over the long term and underestimate aggregate climate impacts. In this manuscript we consider the carbon cycle representation in the most widely used simple IAM, DICE (Dynamic Integrated model of Climate and the Economy) (Nordhaus, 1993, 2008, 2010). In a business-as-usual scenario to 2300 followed by cessation of emissions, DICE (2007) produces errors in CO₂ of ~1000 ppm and in temperature of ~6°C on millennial timescales, relative to output of more complex models (Figure 1; see van Vuuren et al (2011) for comparison of many IAMs).

The main cause of nonlinear ocean uptake is carbonate chemistry, well-understood since the 1950’s (Revelle and Suess (1957), or see e.g. Sarmiento and Gruber (2006) for review). After any increase in atmospheric CO₂, uptake will occur until the atmosphere and ocean reach equilibrium. In a simple solution equilibrium, the number of gas-phase molecules leaving the atmosphere would equal the number appearing in solution. In the present-day ocean, about ten times as many CO₂ molecules are removed from the atmosphere as the corresponding increase in aqueous CO₂, because dissolved inorganic carbon species are partitioned between CO₂, bicarbonate (HCO₃⁻), and carbonate (CO₃²⁻) in proportions fixed by the ocean’s acidity. Uptake becomes nonlinear because it increases acidity, slowing further uptake by reducing the ocean’s ability to store carbon. Models without this nonlinear chemistry cannot reproduce removal of atmospheric CO₂ over multi-century timescales.

Deficiencies in simple IAM carbon cycle parameterizations have been frequently discussed (Hof et al, 2012; Joos et al, 1999; Schultz and Kasting, 1997; Warren et al, 2010), and authors have called for improved representations that better capture known physics (e.g. van Vuuren et al, 2011). One suggested carbon cycle approach that is satisfactory for some purposes is representing CO₂ uptake as a combination of multiple exponential decays (Joos et al, 2013; Maier-Reimer and Hasselmann, 1987). However, while multi-exponential representations can reproduce long-term CO₂ uptake for a given emissions scenario, they are not robust across different emissions scenarios. Because CO₂ uptake is a function of acidity, decay timescales depend on the magnitude of CO₂ perturbations (Archer et al, 2009; van Vuuren et al, 2011) and on background concentrations (Joos et al, 2013).

Carbon cycle errors can significantly affect IAM policy recommendations (e.g. Joos et al, 1999). In estimations of the social cost of carbon, the choice of carbon cycle representation can produce differences of a factor of two. (See Section 6 and Online Resources Section 3.) Carbon cycle errors become still more significant if climate damages are assumed larger (e.g. Moyer et al, 2014).

To provide a more robust tool for use in IAMs, we describe here a simplified global atmosphere-ocean carbon cycle representation that we term the ‘Bolin and Eriksson Adjusted Model’ (hereafter ‘BEAM’). The scheme provides a computationally inexpensive means of capturing the known nonlinear chemistry of ocean carbon uptake. In the remainder of the manuscript, we describe the BEAM model and compare it to carbon cycle representations in DICE and in Earth system models of intermediate complexity (EMICs). Equations, parameter values, and initial conditions are listed in full in Appendices A.1, A.2 and A.3.

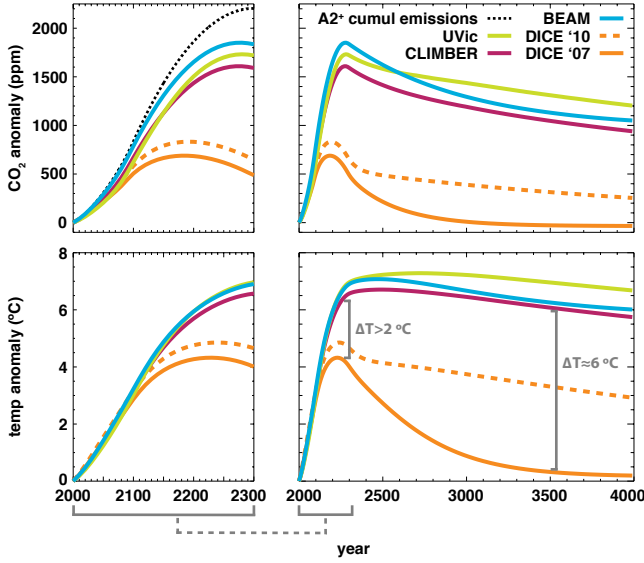


Fig. 1 CO₂ (top) and temperature (bottom) anomalies for BEAM and DICE compared to the intermediate complexity models CLIMBER-2 and UVic for the A2⁺ scenario (all described in Section 5). BEAM CO₂ matches output of the more complex models well for the duration of the simulation. DICE performs well only for the first several decades but then diverges rapidly. Dotted black line shows cumulative emissions (the CO₂ anomaly if no ocean uptake occurred). DICE removes nearly all emitted CO₂ after several hundred years; in more realistic models, half persists for millennia.

2 DICE carbon model

The DICE carbon cycle is a simple box diffusion model of the atmosphere, upper ocean, and lower ocean that assumes constant fractional transfer of CO₂ from each of the three reservoirs. After any CO₂ perturbation, concentrations reach equilibrium when the reservoir concentrations have adjusted so that their transfers are equal. Transfers are described in a system of three linear first-order ordinary differential equations (Eqs. 1-3), which in DICE are specified discretely assuming 10-year timesteps. All versions of DICE use this same functional form, but parameter values have changed across versions.

$$M_{AT}(t) = CE(t) + \phi_{11} \cdot M_{AT}(t-1) + \phi_{21} \cdot M_{UP}(t-1) \quad (1)$$

$$M_{UP}(t) = \phi_{12} \cdot M_{AT}(t-1) + \phi_{22} \cdot M_{UP}(t-1) + \phi_{32} \cdot M_{LO}(t-1) \quad (2)$$

$$M_{LO}(t) = \phi_{23} \cdot M_{UP}(t-1) + \phi_{33} \cdot M_{LO}(t-1) \quad (3)$$

where the ϕ_{ij} s are transfer coefficients, $CE(t)$ is cumulative CO₂ emissions over timestep t , and M_{AT} , M_{UP} , and M_{LO} are the mass of inorganic carbon (in gigatons) in the atmosphere, upper, and lower ocean, respectively. DICE does not explicitly describe the speciation of inorganic carbon, but the ocean reservoirs M_{UP} and M_{LO} can be thought of as comprising dissolved CO₂, HCO₃⁻, and CO₃²⁻.

Because the model has only four distinct fluxes between reservoirs, only four of the seven transfer coefficients are independent. (For example, $(1 - \phi_{11}) \cdot M_{AT}$ is the loss of carbon from the atmosphere to the upper ocean in a given timestep, and $\phi_{12} \cdot M_{AT}$ is that same carbon arriving in the ocean.) To conserve mass, coefficients must be related by:

$$\begin{aligned} \text{atmosphere to upper ocean:} & \quad \phi_{12} &= & (1 - \phi_{11}) \\ \text{upper ocean to atmosphere and lower ocean:} & \quad \phi_{21} + \phi_{23} &= & (1 - \phi_{22}) \\ \text{lower ocean to upper ocean:} & \quad \phi_{32} &= & (1 - \phi_{33}) \end{aligned}$$

3 BEAM carbon model

The 3-reservoir carbon cycle model was first outlined by Bolin and Eriksson (1959), who used a 2-layer ocean specification from Craig (1957) and derived transfer coefficients for CO₂ anomalies by considering carbonate chemistry. Bolin and Eriksson (1959) used fixed parameter values, but their model can be readily extended to allow coefficients to change with CO₂ uptake. The three differential equations of BEAM carbon transfer are essentially identical to those of DICE:

$$\frac{dM_{AT}}{dt} = E(t) - k_a \cdot (M_{AT} - A \cdot B \cdot M_{UP}) \quad (4)$$

$$\frac{dM_{UP}}{dt} = k_a \cdot (M_{AT} - A \cdot B \cdot M_{UP}) - k_d \cdot (M_{UP} - \frac{M_{LO}}{\delta}) \quad (5)$$

$$\frac{dM_{LO}}{dt} = k_d \cdot (M_{UP} - \frac{M_{LO}}{\delta}) \quad (6)$$

where $E(t)$ is the emissions rate. The four independent parameters describing the fluxes between reservoirs are now the two k_i s, which are inverse exchange timescales between atmosphere-ocean (k_a) and upper-lower ocean (k_d), and two dimensionless parameters: δ , the ratio of lower to upper ocean volume (~ 50), and $A \cdot B$, the equilibrium ratio of atmospheric to upper ocean inorganic carbon. The term $M_{AT} - A \cdot B \cdot M_{UP}$ is the *disequilibrium* between atmospheric and ocean inorganic carbon, eroded with time constant $1/k_a$; and $M_{UP} - M_{LO}/\delta$ is the disequilibrium between upper and lower ocean inorganic carbon, eroded with time constant $1/k_d$.

The only variation in coefficients over time occurs in $A \cdot B$, which we separate to distinguish factors with different dependences on environmental conditions. A is the ratio of atmosphere to ocean CO₂ concentration at equilibrium, which is weakly dependent on temperature: a warmer ocean holds less dissolved CO₂. B is the ratio of dissolved CO₂ to total ocean inorganic carbon at equilibrium (see Section 4), a strong function of acidity: more acidic seawater stores less inorganic carbon. Variation in B in particular alters uptake rates dramatically. In the business-as-usual scenario of Figure 1, atmospheric CO₂ rises 5.5 times over present-day concentrations by year 2200, but acidification and warming simultaneously raise $A \cdot B$ by a factor of 5.2. That is, the ocean's ability to hold inorganic carbon relative to atmospheric CO₂ drops nearly as fast as atmospheric CO₂ rises. This near-cancellation lowers average carbon uptake ~ 10 times below that expected from initial coefficient values.

DICE, with fixed parameters, could not match present-day atmospheric CO₂ uptake rates were it to use realistic values for reservoir sizes and exchange timescales. To approximate uptake slowed by changing acidity, DICE raises exchange timescales substantially above plausible physical values (Table 1). Real-world coefficients continue to evolve in an acidifying ocean, however, so DICE is unable to reproduce uptake rates over all time. In practice, DICE parameter values are set by fitting against relatively short simulations.¹

¹ DICE carbon cycle parameters are tuned to match those of another simple model (MAGICC) using an emissions trajectory from 1750-2100 that is a combination of historical emissions and the Intergovernmental Panel on Climate Change (IPCC) A1FI scenario (Nordhaus, 2008, 2007). The MAGICC carbon cycle includes ocean uptake described with a multi-exponential function and a four-box model representing the terrestrial carbon cycle. MAGICC parameters are themselves calibrated to output from the C⁴MIP carbon-cycle intercomparison project. (Meinshausen et al, 2011; Wigley, 2008)

equiv. coefficients		timescales (years)		
DICE	BEAM	$\tau_{\text{DICE '07 ('10)}}$	$\tau_{\text{BEAM,1994}}$	$\tau_{\text{BEAM,2200}}$
ϕ_{12}	k_a	50 (80)	5	5
ϕ_{21}	$k_a \cdot A \cdot B$	100 (200)	4.5	1.1
ϕ_{23}	k_d	200 (2,000)	20	20
ϕ_{32}	k_d / δ	3,000 (13,000)	1000	1000
<i>BEAM params from model year</i>		<i>1800</i>	<i>1994</i>	<i>2200</i>
	<i>A</i>	220	150	180
	<i>1/B</i>	220	130	31
	<i>A · B</i>	1.0	1.1	5.7

Table 1 Equivalence of coefficients in DICE and BEAM and corresponding timescales (inverses of transfer coefficients, x10 for DICE because the model uses 10-year timesteps, consistent to 1 significant figure with continuous representation). Because its parameters cannot evolve in time, DICE in both 2007 and 2010 versions approximates real-world CO₂ uptake primarily by lengthening exchange timescales. All parameters are shown to 1 or 2 significant figures; see Appendix A.2 to derive BEAM parameters more precisely.

Note that BEAM represents ocean CO₂ uptake only, and does not seek to capture the uncertain effects of the terrestrial biosphere. The terrestrial biosphere can impact the carbon cycle in competing ways: CO₂ fertilization generally increases terrestrial CO₂ uptake (e.g. Arora et al, 2013; Joos et al, 2001), but anthropogenic land use change is a source of CO₂ emissions (e.g. Brovkin et al, 2013). The net effect is at present uncertain even in sign (e.g. Jones et al, 2013; Sitch et al, 2005). If the user wants to include the terrestrial carbon cycle, one possibility is to use the parameterization suggested by Joos et al (2013).

4 Carbonate chemistry in BEAM

Ocean carbonate chemistry. Aqueous carbonate chemistry is well understood; we review it briefly only to explain its treatment in BEAM. See any aqueous chemistry textbook (e.g. Sarmiento and Gruber, 2006) for more thorough review. Uptake of CO₂ proceeds when concentrations in the atmosphere and upper ocean are out of equilibrium ($M_{AT} \neq A \cdot B \cdot M_{UP}$). As CO₂ dissolves, we assume instantaneous repartitioning of inorganic carbon species:



Partitioning is set by the dissociation coefficients k_1 and k_2 and the concentration of hydrogen ions $[\text{H}^+]$, i.e. the acidity (pH) of seawater.² The ‘carbon storage factor’ $1/B$, the equilibrium ocean total inorganic carbon relative to dissolved CO₂, is then:

$$\frac{1}{B} = \frac{[\text{CO}_2] + [\text{HCO}_3^-] + [\text{CO}_3^{2-}]}{[\text{CO}_2]} = 1 + \frac{k_1}{[\text{H}^+]} + \frac{k_1 k_2}{[\text{H}^+]^2} \quad (8)$$

Higher acidity (higher $[\text{H}^+]$, lower pH) reduces the ocean’s ability to store carbon (Fig. 2). As equation 7 indicates, dissolved CO₂ itself acts as a weak acid. Any ocean uptake of CO₂ therefore intrinsically reduces the efficiency of future uptake.

Solving for $[\text{H}^+]$ is complicated by the fact that ocean acidity is not simply governed by atmospheric CO₂. Seawater contains strong bases (and acids) whose combined effect raises pH above that for a pure water/CO₂ system. The carbonate system then acts as a

² The pH scale is $-\log_{10}([\text{H}^+])$, i.e. the ocean’s current pH of ~ 8.1 means that $[\text{H}^+] = 10^{-8.1}$ mol/kg.

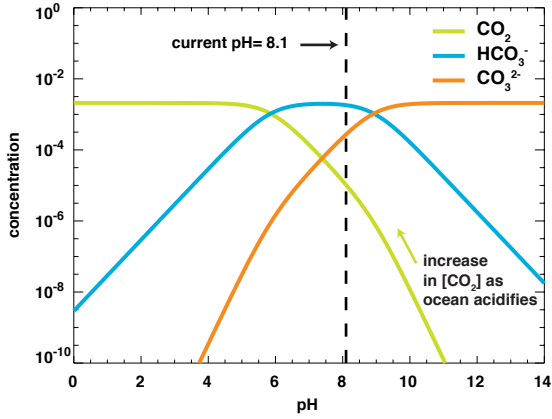


Fig. 2 Partitioning of dissolved inorganic carbon species in seawater (DIC) as a function of pH. For illustrative purposes, DIC is held constant at $2100 \mu\text{mol/kg}$ (Zeebe and Wolf-Gladrow, 2001). At present, bicarbonate dominates DIC and the carbon storage factor is ~ 170 . In a more acidic ocean (lower pH), CO_2 becomes more significant and the carbon storage factor drops. At $\text{pH} < 5$ (1000x increase in $[\text{H}^+]$ over present-day conditions), CO_2 dominates and the carbon storage factor approaches 1. Present-day seawater contains strong bases that raise pH; pure water interacting with current atmospheric CO_2 would have $\text{pH} \sim 5.6$.

buffer against further changes in acidity. To estimate those changes, we assume constant acid-neutralizing capability, or “alkalinity”, approximated as the amount of H^+ that would have to be added to convert all bicarbonate and carbonate to CO_2 :

$$\text{Alk} = [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}] = \left(\frac{k_1}{[\text{H}^+]} + \frac{2 \cdot k_1 \cdot k_2}{[\text{H}^+]^2} \right) \cdot M_{UP} \cdot B \quad (9)$$

where $M_{UP} \cdot B$ is the concentration of upper ocean CO_2 . We determine *Alk* by assuming equilibrium in the pre-industrial ocean with $\text{pH}=8.29$ (see Appendix A.3). BEAM solves for $[\text{H}^+]$ at each timestep using eqs. 8-9 and assuming constant alkalinity.³ Constant alkalinity is a reasonable assumption for several thousand years (Archer et al, 2009; Zeebe and Wolf-Gladrow, 2001). On long timescales, dissolution of calcium carbonate would help return pH to its original value, increasing drawdown of atmospheric CO_2 . BEAM will therefore underpredict CO_2 uptake on $\sim 10,000$ -year timescales.

Temperature dependence of coefficients: The discussion above concerned only changes in ocean carbon storage capacity due to changing acidity. To a lesser degree, temperature affects both the carbon storage factor $1/B$ and CO_2 solubility $1/A$. The magnitudes of temperature-induced changes are absolutely smaller than acidity effects, and temperature-dependent changes in *A* and *B* partially counteract each other: the solubility of CO_2 decreases in a warmer ocean, but the dissociation constants k_1 and k_2 grow with temperature, raising the carbon storage factor. (See Appendix A.2 for equations.)

constant	10°C	12°C	15°C	percentage change 10-15°C
$1/A$	$7.02 \cdot 10^{-3}$	$6.58 \cdot 10^{-3}$	$6.00 \cdot 10^{-3}$	-15%
k_1	$8.00 \cdot 10^{-7}$	$8.30 \cdot 10^{-7}$	$8.75 \cdot 10^{-7}$	9%
k_2	$4.63 \cdot 10^{-10}$	$4.94 \cdot 10^{-10}$	$5.47 \cdot 10^{-10}$	18%

Table 2 Representative values for the temperature-dependent BEAM parameters $1/A$, k_1 , and k_2 at selected temperatures. The dissociation and solubility effects act in opposite directions on ocean carbon storage.

³ The assumption of a static equilibrium is not strictly true, since the ocean’s “biological pump” can produce short-term variations in alkalinity (e.g. Gangstø et al, 2011), but their effect on CO_2 uptake is small.

5 Model validation and comparison to DICE

Several published experiments comparing carbon cycle representations in Earth system models of intermediate complexity (EMICs) allow us to validate BEAM. The recent EMIC intercomparison (Eby et al, 2013) designed for the IPCC Fifth Assessment Report (IPCC, 2013) allows comparison over the historical period.⁴ BEAM output is consistent with the range of CO₂ anomalies from seven EMICs over this period. (See Online Resources Section 1 and Fig. OR1.) The most rigorous test of a carbon cycle is however a comparison of long-term behavior after significant anthropogenic CO₂ emissions.

For such a long-term validation, we use the study of Montenegro et al (2007), who compared the University of Victoria Earth system climate model (UVic) (Eby et al, 2009) and the CLIMate and BiosphERe 2 model (CLIMBER-2) (Petoukhov et al, 2000). (See Online Resources Section 1 for discussion of limitations of other intercomparisons.) The UVic model consists of a full three-dimensional, 19-layer ocean model and an energy-moisture balanced model of the atmosphere (Weaver et al, 2001). CLIMBER-2 consists of a simpler three-reservoir ocean that includes biogeochemistry (Brovkin et al, 2002) and sedimentation (Archer et al, 1998) coupled to a two-dimensional atmospheric model. In comparisons shown here, models were forced with an A2⁺ CO₂ emissions scenario, which reproduces the business-as-usual SRES A2 scenario for 100 years (Nakicenovic et al, 2000) followed by linearly declining emissions that reach zero after another 200 years, for a cumulative emissions total of 5134 GtC.⁵ Both models were run for 10,000 years (starting at pre-industrial using historical emissions), although because BEAM does not include long-term adjustment of alkalinity, we compare here only the first 2000 years.

CO₂ evolution in BEAM is consistent with that in UVic and CLIMBER-2 throughout the simulation period (Fig. 1). CO₂ evolution in DICE is consistent for only the first few decades. By 100 years, DICE atmospheric CO₂ is markedly lower than that of the other models (Fig. 1, left). After 300 years, when emissions cease, rapid drawdown of atmospheric carbon in DICE 2007 returns climate to near pre-industrial levels, while BEAM and the intermediate complexity models retain persistent high CO₂ and temperatures elevated by ~6–7 °C for millennia (Fig. 1, right). (DICE 2010 retains a moderate anomaly.) By the end of the simulation period, BEAM does begin to diverge in behavior from the more realistic models: BEAM CO₂ concentrations begin to asymptote while slow CO₂ uptake continues in the intermediate complexity models. For very long simulations, the assumption of constant alkalinity in BEAM would have to be relaxed to allow restoration of pH and continued uptake. (One possible parametrization is that suggested by Kheshgi and Archer (2004).)

The dominant driver of difference between DICE and the more realistic carbon cycle models, including BEAM, is the changing carbon partitioning as the ocean acidifies. Figure 3 shows atmospheric CO₂ evolution for 2007 and 2010 DICE compared to BEAM with and without temperature-dependent coefficients. Omitting the temperature dependence of CO₂ solubility and dissociation constants produces negligible changes for the first few hundred years and only small changes even at millennial timescales.

For perspective, Figure 3 also shows the original Bolin and Eriksson (1959) linearized model, which is similar to 2010 DICE. While 2007 DICE quickly draws down any CO₂ perturbation to near pre-industrial levels, both 2010 DICE and the Bolin and Eriksson representation retain some anthropogenic CO₂ in the atmosphere for the duration of the simulation.

⁴ The historical period is defined as 850–2005 C.E., but emissions are significant only in the last 200 years.

⁵ Montenegro et al (2007) used an older version of historical emissions; BEAM calibration is based on more recent emissions estimates. See Online Resources Section 2 for discussion.

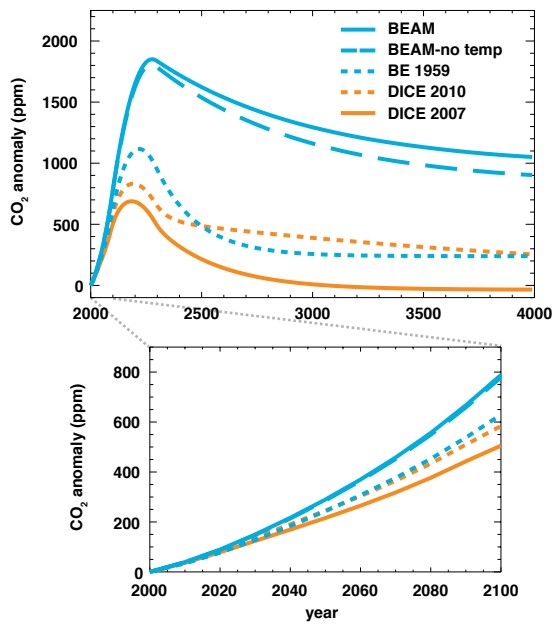


Fig. 3 Atmospheric CO₂ anomaly for different versions of BEAM and DICE carbon cycle models for the A2⁺ scenario. *BEAM* (solid blue) is the full model described here; *BEAM-no temp* (dashed blue) omits temperature dependence; and *BE 1959* (dotted blue) is our coding of the original linear Bolin and Eriksson model. Temperature dependence in BEAM coefficients has a relatively minor effect and some users may wish to neglect it. Long-term, 2010 DICE retains more CO₂ in the atmosphere than does 2007 DICE because it has a longer ocean equilibration timescale and effectively a smaller ocean. Atmospheric CO₂ in the Bolin and Eriksson model is fully equilibrated by the end of the simulation; in 2010 DICE it is still evolving toward a lower equilibrium level with a time constant of $\sim 13,000$ years.

Both achieve this in part by effectively reducing the total ocean volume. The equilibrium ratio of total ocean to atmospheric carbon, $(\delta + 1) / A \cdot B$, is ~ 46 in present-day BEAM and ~ 32 in 2007 DICE, but only ~ 20 in 2010 DICE and ~ 5 in Bolin and Eriksson (1959).⁶

6 BEAM in economic and policy analyses

Replacing the DICE carbon cycle with BEAM can significantly alter economic projections of the cost of climate change, especially for analyses involving longer timescales. We estimate the importance of BEAM to one measure of the economic impact of climate change, the social cost of carbon (SCC), used in cost-benefit analysis of federal regulations that affect greenhouse gases. The SCC is the present value of the harms (over some integration time) of an additional ton of emitted CO₂. It is calculated by running a business-as-usual case, repeating the run with an additional ton of CO₂ emissions in one year, calculating the difference in consumption over the modeled period, and discounting those harms back to present value. We estimate SCC in DICE with and without substitution of BEAM⁷, using the DICE emissions assumptions over the modeled 600-year period rather than the A2+ emissions scenario used in previous examples. As before, BEAM leaves more anthropogenic CO₂ in the atmosphere, producing temperature anomalies higher by ~ 2 - 3°C in the year 2300 (Fig. OR4, compare to Figure 1). Those higher temperatures produce larger climate harms, raising the year-2005 SCC by 60% in 2007 DICE and 40% in 2010 DICE (Table OR1).

The accuracy of the long-term carbon cycle is still more important in analyses that weight future harms more heavily. In standard DICE, continued economic growth under

⁶ Although 2010 DICE and Bolin and Eriksson (1959) are mathematically equivalent, they are not exactly equivalent in intent: Bolin and Eriksson described evolution of CO₂ anomalies rather than total reservoirs. The reduction in ocean carbon content in 2010 DICE exceeds that produced by any plausible choice of pH.

⁷ For simplicity, we use the version of BEAM with no temperature dependence. (See Fig. 3.)

climate change leads to an implied discount rate that is relatively high (mean 2.5% and 2.3%/year over 600 years for 2007/2010 DICE). If discount rates are lower, either because of the choice of discounting parameters or because future climate harms are larger, substituting BEAM for less-accurate carbon cycle models alters results more strongly. We give several examples in Table OR1. With alternative discount parameters that produce mean discount rates of 0.5 and 0.6%/year in 2007/2010 DICE⁸, the use of BEAM raises SCC estimates not by 60 and 40% but by 160 and 80%. A similar increase results if damages are assumed to follow a cubic rather than quadratic dependence on temperature increase (following Ackerman et al, 2010). If climate damages are allowed to significantly reduce economic growth, use of BEAM can have order of magnitude effects on SCC estimates (Moyer et al, 2014).

7 Conclusions

Many authors have pointed out that the carbon cycle representations in DICE and other simple IAMs do not accurately reproduce the response of more physical models on centennial timescales (e.g. Hof et al, 2012; van Vuuren et al, 2011). We show here that DICE performance worsens still further over time. While recent updates of the DICE carbon cycle reduce discrepancies somewhat, all versions of DICE diverge from predicted real-world behavior within decades. Our analysis here confirms that discrepancies occur because linearized models cannot capture the changing ocean carbon storage potential due to changing ocean acidity. We also show that these discrepancies can be largely eliminated by adding to the DICE framework a single equation describing acidity evolution. While accurate representation of the long-term carbon cycle is most critical in economic analyses with long time horizons and low discount rates, carbon cycle errors can affect policy recommendations even in standard modeling frameworks. The BEAM model offers a simple, computationally tractable carbon cycle representation that retains fidelity over millennial timescales.

A BEAM model equations, parameter values, and initial conditions

A.1 Equations

BEAM consists of four independent equations: three that track total carbon in each layer (atmosphere, upper, and lower ocean- Eq. 10), and one that tracks acidity (Eq. 13). Carbon transfers in BEAM are described by:

$$\frac{d}{dt} \begin{pmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{pmatrix} = \begin{pmatrix} -k_a & k_a \cdot A \cdot B & 0 \\ k_a & -(k_a \cdot A \cdot B) - k_d & \frac{k_d}{\delta} \\ 0 & k_d & -\frac{k_d}{\delta} \end{pmatrix} \begin{pmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{pmatrix} + E(t) \quad (10)$$

where the M_i s represent the mass of carbon (in CO₂ or dissolved inorganic carbon) in the atmosphere (AT), upper ocean (UP), and lower ocean (LO); and $E(t)$ is rate of anthropogenic CO₂ emissions. (Emissions units must match those of concentrations and are therefore specified in mass of carbon in CO₂.)

The parameter A is the ratio of mass of CO₂ in atmospheric to upper ocean dissolved CO₂, i.e. A is inversely proportional to CO₂ solubility. Solubility is set by ‘Henry’s law’, which prescribes that in equilibrium, the concentrations of CO₂ in the atmosphere and ocean are related by a coefficient dependent only on temperature. Henry’s Law may be written in various forms; for convenience we define the coefficient k_H as a

⁸ DICE uses Ramsey discounting (discount rate $r = \eta \cdot g + \rho$, where g is the growth rate), with ρ (the pure rate of time preference) at 1.5%/year and η (the elasticity of the marginal utility of consumption) at 2 and 1.5 for 2007/2010 DICE, respectively. In our alternate example we assume $\rho=0$ and $\eta=1$, similar to Stern (2008).

dimensionless ratio of the molar concentrations of CO_2 in atmosphere and ocean. The parameter A is then

$$A = k_H \cdot \frac{AM}{OM/(\delta + 1)} \quad (11)$$

where AM are OM are the number of moles in the atmosphere and ocean, respectively, and $OM/(\delta + 1)$ signifies the upper ocean only. B is the ratio of dissolved CO_2 to total oceanic carbon, a function of acidity:

$$B = \frac{1}{1 + \frac{k_1}{[\text{H}^+]} + \frac{k_1 k_2}{[\text{H}^+]^2}} \quad (12)$$

where k_1 and k_2 are dissociation constants. Alkalinity, Alk , is used to determine $[\text{H}^+]$ by solving the quadratic:

$$\frac{M_{UP}}{Alk} = \frac{1 + \frac{k_1}{[\text{H}^+]} + \frac{k_1 k_2}{[\text{H}^+]^2}}{\frac{k_1}{[\text{H}^+]} + \frac{2k_1 k_2}{[\text{H}^+]^2}} \quad (13)$$

Three parameters are temperature-dependent (k_1 , k_2 , and k_H , Eqs. 14-16). Because the temperature dependence has a relatively minor aggregate effect, however, some users may wish to omit it.

A.2 Parameter values

Recommended BEAM parameter values are given in Table 3. Most are well established. Time constants k_a , k_d , and the ratio of upper to lower ocean (δ) are not well constrained; we use reasonable values from Bolin and Eriksson (1959). We determine alkalinity by assuming equilibrium in the pre-industrial ocean at pH=8.29 (see Appendix A.3). It is also possible to specify Alk and adjust pre-industrial pH, k_a , k_d , and δ for best fit to more complex models.

parameter	representative value	source
k_a (years ⁻¹)	.2	Bolin and Eriksson (1959)
k_d (years ⁻¹)	.05	Bolin and Eriksson (1959)
δ	50	Bolin and Eriksson (1959)
k_H	$1.23 \cdot 10^3$	Weiss (1974)
k_1 (mol/kg)	$8.00 \cdot 10^{-7}$	Mehrbach et al (1973)
k_2 (mol/kg)	$4.53 \cdot 10^{-10}$	Mehrbach et al (1973)
AM (mol)	$1.77 \cdot 10^{20}$	Warneck (1999)
OM (mol)	$7.8 \cdot 10^{22}$	Garrison (2009) – (for conversion see ⁹)
Alk	767.0 Gt C	(for conversion to $\mu\text{mol}/\text{kg}$ see ¹⁰)

Table 3 Recommended BEAM parameters. Temperature-dependent parameters k_H , k_1 , and k_2 are calculated here for $T=10^\circ\text{C}$ (283.15 K), roughly the volume mean ocean temperature. For convenience we state alkalinity in units of Gt C; our value is equivalent to $\sim 2317 \mu\text{mol}/\text{kg}$, consistent with standard literature values, e.g. $\sim 2364 \mu\text{mol}/\text{kg}$ (Sarmiento and Gruber, 2006).

To include temperature-dependent effects, replace k_1 , k_2 , and k_H with their temperature-dependent forms of Eqs. 14-16. Following Archer et al (2004), we assume the global ocean temperature anomaly is equal to anomaly in mean surface temperature, so this anomaly is added to the baseline ocean temperature (i.e. $T=283.15 \text{ K} + \Delta T$). This assumption is likely an upper limit for ocean temperature change.

⁹ $OM \approx 1.37 \cdot 10^9 \text{ km}^3 \text{ ocean} \cdot \frac{10^{15} \text{ cm}^3}{\text{km}^3} \cdot \frac{1.027 \text{ g seawater}}{\text{cm}^3 \text{ seawater}} \cdot \frac{1 \text{ mol water}}{18 \text{ g}} \approx 7.8 \cdot 10^{22} \text{ moles}$

¹⁰ $Alk = 767.0 \text{ Gt C} \cdot \frac{1 \text{ mol}}{12 \text{ g}} \cdot \frac{10^{15} \text{ g}}{\text{Gt}} \cdot \frac{1}{OM/(\delta+1)} \cdot \frac{1 \text{ mol water}}{18 \text{ g}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \approx 2317 \mu\text{mol}/\text{kg}$

CO₂ solubility (Henry's law) (Weiss, 1974)

$$k_H = \frac{1}{k_0} \cdot \frac{\text{liter seawater}}{1.027 \text{ kg}} \cdot \left(\frac{55.57 \text{ mol}}{\text{liter}} \right) \quad \text{with} \quad (14)$$

$$k_0 = \exp \left[\frac{9345.17}{T} - 60.2409 + 23.3585 \cdot \ln \left(\frac{T}{100} \right) + S \cdot \left(.023517 - .00023656 \cdot T + .0047036 \cdot \left(\frac{T}{100} \right)^2 \right) \right]$$

First and second dissociation constants (Mehrbach et al, 1973)

$$k_1 = 10^{-pK_1} \quad \text{and} \quad k_2 = 10^{-pK_2}$$

$$\text{with} \quad pK_1 = -13.721 + (0.031334 \cdot T) + \frac{3235.76}{T} + 1.3 \cdot 10^{-5} \cdot S \cdot T - (0.1031 \cdot S^{0.5}) \quad (15)$$

$$\text{and} \quad pK_2 = 5371.96 + (1.671221 \cdot T) + (0.22913 \cdot S) + (18.3802 \cdot \log(S)) - \frac{128375.28}{T} \quad (16)$$

$$- (2194.30 \cdot \log(T)) - (8.0944 \cdot 10^{-4} \cdot S \cdot T) - (5617.11 \cdot \frac{\log(S)}{T}) + 2.136 \cdot \frac{S}{T}$$

where k_0 has units $\frac{\text{mol C}}{\text{kg seawater} \cdot \text{atm}}$, k_H is dimensionless, k_1 and k_2 have units mol/kg seawater , T is temperature (K), and S is salinity ($\sim 35 \text{g/kg seawater}$).

A.3 Initial conditions and model implementation

We set pre-industrial ocean carbon content by assuming equilibrium with atmospheric CO₂ at 280 ppm (IPCC, 2007) and pH of 8.29. Pre-industrial pH is chosen so that after running forward with historical emissions (Boden et al, 2010), BEAM present-day pH matches that of UVic and CLIMBER-2 (~ 8.16 , Montenegro et al (2007)). Matching pH is needed for a valid comparison because pH values markedly affect CO₂ uptake. Both pre-industrial and present-day pH levels are uncertain by ± 0.05 (Feely et al, 2009). We define 'present-day' as that point where atmospheric CO₂ matches 380 ppm, the 2007 DICE initial condition, which occurs near year 1994 for BEAM. The exact timing is not significant for subsequent CO₂ anomaly evolution.

	<i>atm. CO₂ (ppm)</i>	<i>M_{AT} (GtC)</i>	<i>M_{UP} (GtC)</i>	<i>M_{LO} (GtC)</i>	<i>pH</i>
BEAM pre-industrial	280	596	713	35,625	8.29
BEAM present-day	380	809	725	35,641	8.18
DICE '07 present-day	380	809	1,255	18,365	(NA)
DICE '10 present-day	370	787	1,600	10,010	(NA)

Table 4 BEAM, DICE initial conditions. 'Present-day' is year 2005 for DICE 2007 and 2000 for DICE 2010. Total ocean carbon content in BEAM is consistent with literature values, e.g. $\sim 38,000 \text{ GtC}$ (Ciais et al, 2013).

Because the atmosphere/upper ocean exchange timescale is short in BEAM, the present-day upper ocean is nearly in equilibrium with the atmosphere ($M_{AT}/M_{UP} \sim A \cdot B \sim 1.1$), though the lower ocean is slightly out of equilibrium ($M_{LO}/M_{UP} \sim 49$ while $\delta = 50$). Both DICE versions begin with excess atmospheric CO₂ relative to the upper ocean. (In 2007 DICE, $M_{AT}/M_{UP} \sim 0.6$ but $A \cdot B \sim 0.5$.) The larger upper ocean carbon reservoir in DICE (2007 and 2010) than in BEAM reflects a larger equilibrium upper ocean/atmosphere carbon ratio and can be thought of as a deeper mixed ocean layer. DICE total ocean carbon is $\sim 1/2$ the real-world value; this difference can be thought of as a smaller total ocean volume.

The more realistic BEAM representation does have one drawback, that the sensitivity of coefficients mandates fine timesteps to avoid instability during numerical integration. The figures shown here were generated using 0.01 year timesteps. Timesteps as coarse as $1/10^h$ year can produce oscillation in pH and uptake/release of oceanic CO₂ when emissions change rapidly (e.g. Oeschger et al, 1975). Resulting error in atmospheric CO₂ anomaly for the emissions scenario used here would reach several percent. Note that this instability means that BEAM cannot be used to simulate the response to an abrupt addition of CO₂. For code of the full BEAM representation, see www.rdcep.org/carbon-cycle-model.

B Temperature model

For completeness, we describe the 2007 DICE temperature model, which appears to adequately capture temperature evolution (Fig. 4). Just as the ocean takes up CO_2 in response to atmospheric CO_2 perturbations, it also takes up heat in response to surface warming, with a long equilibration time because of the large thermal inertia of the ocean. Heat uptake in DICE is represented by a linear model similar to that used for carbon uptake. Because heat uptake is in reality largely linear, this representation adequately reproduces climate behavior. As with the carbon cycle, many of the coefficients in the DICE 2007 temperature model given below are calibrated to the MAGICC model (Wigley et al, 2007) or taken from the IPCC (2001) and IPCC (2007).

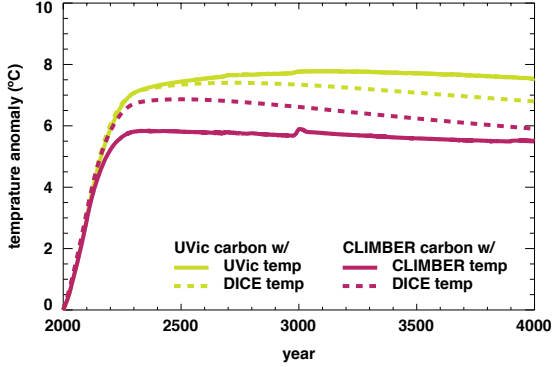


Fig. 4 The two-box DICE temperature model appears to adequately capture temperature evolution in more complex models. We drive the 2007 DICE temperature model with atmospheric CO_2 anomalies from UVic (green) and CLIMBER-2 (maroon) from the A2⁺ emissions scenario and compare atmospheric temperatures from DICE (dashed) to those models' own temperature representations (solid). DICE temperature evolutions differs somewhat in behavior but lies within the range of uncertainty.

The DICE temperature model uses only two layers, the atmosphere and lower ocean; the upper ocean is assumed to follow atmospheric temperature (Eqs. 17-18, but note that we have re-organized parameters to be more intuitive). Radiative forcing F due to increased atmospheric CO_2 warms the atmosphere (and upper ocean), producing a disequilibrium with the lower ocean that is eroded with timescales $1/\mu$:

$$T_{AT}(t) = T_{AT}(t-1) + \mu_{AT} \cdot [A \cdot (T_{eq}(t) - T_{AT}(t-1)) - \gamma \cdot (T_{AT}(t-1) - T_{LO}(t-1))] \quad (17)$$

$$T_{LO}(t) = T_{LO}(t-1) + \mu_{LO} \cdot \gamma \cdot (T_{AT}(t-1) - T_{LO}(t-1)) \quad (18)$$

where the T_i s are atmospheric and lower ocean temperature changes (in $^\circ\text{C}$) since pre-industrial times; γ relates atmosphere-ocean heat transfer to temperature anomaly ($\gamma = 0.3 \text{ W/m}^2/^\circ\text{C}$); A is the general climate sensitivity ($1.3 \text{ W/m}^2/^\circ\text{C}$, derived by dividing DICE assumptions of the forcing per doubling of CO_2 ($\alpha = 3.8 \text{ W/m}^2$) by the assumed equilibrium warming after doubling of CO_2 ($\beta = 3.0^\circ\text{C} / \text{doubling}$)); and $T_{eq}(t)$ is the equilibrium temperature that would be produced by the imposed forcing: $T_{eq}(t) = F(t)/A$. Forcing $F(t)$ is assumed to be linear with the logarithm of the fractional change in CO_2 since pre-industrial times, a standard assumption in climate science¹¹:

$$F(t) = \alpha \cdot \log_2(M_{AT}(t)/M_{AT}(PI)) \quad (19)$$

where $M_{AT}(PI)$ is the mass of pre-industrial atmospheric carbon (596.4 Gt C, equivalent to ~ 280 ppm CO_2).

Note that equilibration timescales for the atmosphere and lower ocean need not be equal since temperature is not a conserved quantity: $\mu_{AT} = 0.22/10$ years so $\tau_{AT} \sim 45$ years while $\mu_{LO} = (1/6)/10$ years so $\tau_{LO} \sim 60$ years. The 2010 DICE temperature model uses the same equations with small adjustments to three coefficients: $\gamma = 0.31 \text{ W/m}^2/^\circ\text{C}$, $\mu_{AT} = 0.208$ ($\tau_{AT} \sim 48$ years), and $\beta = 3.2^\circ\text{C} / \text{doubling}$. Because DICE 10-year timesteps are long relative to these timescales, DICE coefficient values differ from those of a continuum representation. If the model is rewritten to use 1-year timesteps, Marten and Newbold (2013) recommend using $\gamma = 0.5072 \text{ W/m}^2/^\circ\text{C}$, $\mu_{AT} = 0.0586$, $\mu_{LO} = 0.018336$ to replicate 2007 DICE.

¹¹ In 2007 DICE, radiative forcing is specified as a function of CO_2 at timesteps (t) and ($t+1$), but that assumption can be relaxed without significant difference. 2007 DICE also adds a constant of 10^{-6} to M_{AT} to preclude $F(t) = 0$, presumably for numerical reasons. We neglect forcing from other greenhouse gases to compare with models driven only by changing CO_2 , but additional forcings can be added to $F(t)$.

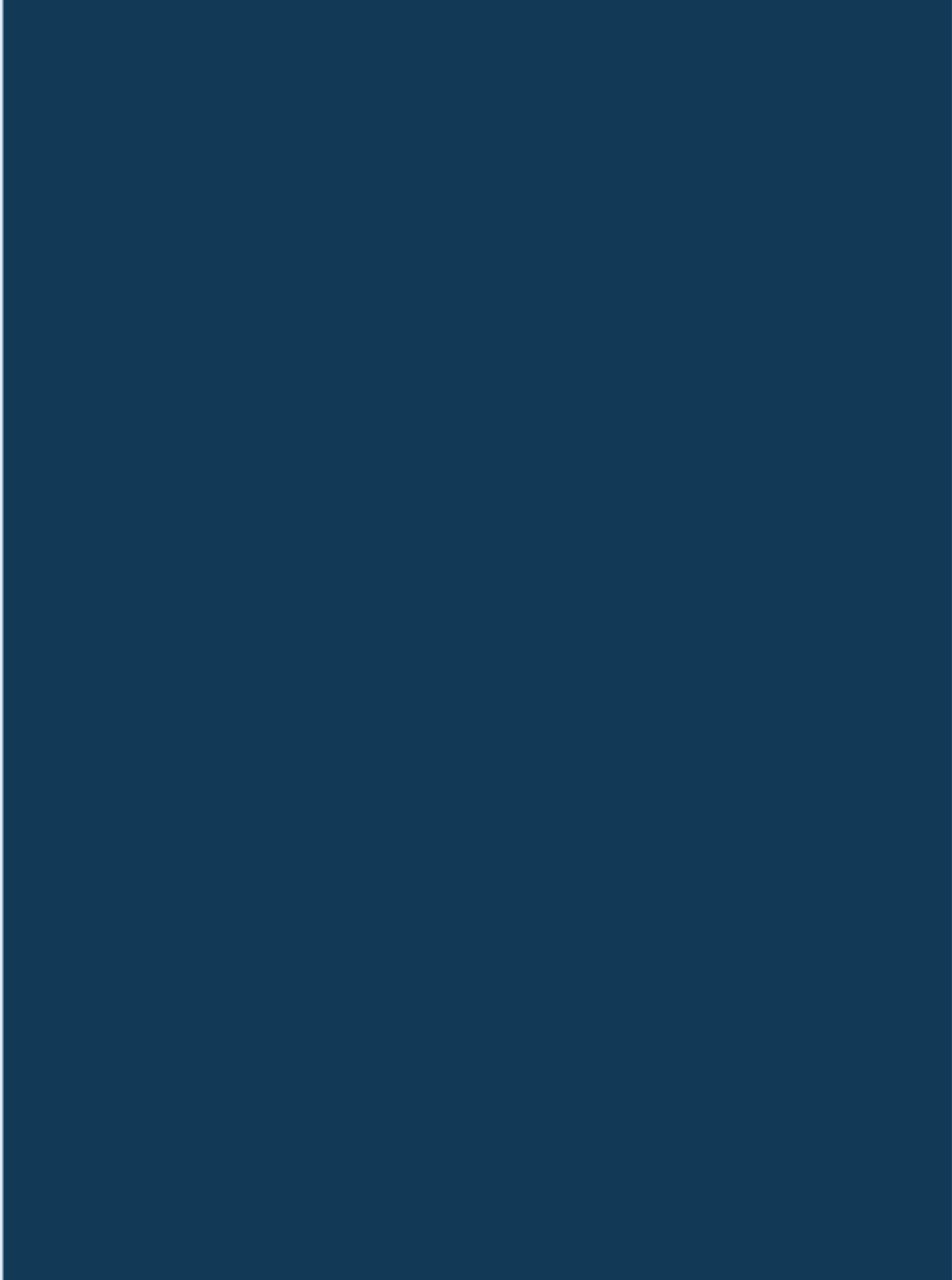
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References

- Ackerman F, Stanton E, Bueno R (2010) Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE. *Ecological Economics* 69(8):1657–1665
- Archer D, Kheshgi H, Maier-Reimer E (1998) Dynamics of fossil fuel CO₂ neutralization by marine CaCO₃. *Global Biogeochemical Cycles* 12(2):259–276
- Archer D, Martin P, Buffett B, Brovkin V, Rahmstorf S, Ganopolski A (2004) The importance of ocean temperature to global biogeochemistry. *Earth and Planetary Science Letters* 222(2):333–348
- Archer D, Eby M, Brovkin V, Ridgwell A, Cao L, Mikolajewicz U, Caldeira K, Matsumoto K, Munhoven G, Montenegro A, et al (2009) Atmospheric lifetime of fossil fuel carbon dioxide. *Annual Review of Earth and Planetary Sciences* 37:117–134
- Arora VK, Boer GJ, Friedlingstein P, Eby M, Jones CD, Christian JR, Bonan G, Bopp L, Brovkin V, Cadule P, et al (2013) Carbon-concentration and carbon-climate feedbacks in CMIP5 Earth system models. *Journal of Climate* 26(15)
- Boden TA, Marland G, Andres RJ (2010) Global, regional, and national fossil-fuel CO₂ emissions. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, US Department of Energy, Oak Ridge, Tenn, USA doi 10.3334/CDIAC/00001_V2010
- Bolin B, Eriksson E (1959) Distribution of matter in the sea and the atmosphere. *The Atmosphere and the Sea in Motion*
- Brovkin V, Bendtsen J, Claussen M, Ganopolski A, Kubatzki C, Petoukhov V, Andreev A (2002) Carbon cycle, vegetation and climate dynamics in the Holocene: Experiments with the CLIMBER-2 model. *Global Biogeochem Cycles* 16(4):1139
- Brovkin V, Boysen L, Arora V, Boisier J, Cadule P, Chini L, Claussen M, Friedlingstein P, Gayler V, Van Den Hurk B, et al (2013) Effect of anthropogenic land-use and land-cover changes on climate and land carbon storage in CMIP5 projections for the twenty-first century. *Journal of Climate* 26(18)
- Ciais P, Sabine C, Bala G, Bopp L, Brovkin V, Canadell J, Chhabra A, DeFries R, Galloway J, Heimann M, et al (2013) Carbon and other biogeochemical cycles. *Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*
- Craig H (1957) The natural distribution of radiocarbon and the exchange time of carbon dioxide between atmosphere and sea. *Tellus* 9(1):1–17
- Denman KL, Brasseur G, Chidthaisong A, Ciais P, Cox PM, Dickinson RE, Hauglustaine D, Heinze C, Holland E, Jacob D, et al (2007) Couplings between changes in the climate system and biogeochemistry. In *Climate change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the IPCC*, eds. Solomon S et al. Cambridge University Press
- Eby M, Zickfeld K, Montenegro A, Archer D, Meissner K, Weaver A (2009) Lifetime of anthropogenic climate change: millennial time scales of potential CO₂ and surface temperature perturbations. *Journal of Climate* 22(10):2501–2511
- Eby M, Weaver AJ, Alexander K, Zickfeld K, Abe-Ouchi A, Cimadoribus A, Crespin E, Drijfhout S, Edwards N, Eliseev A, et al (2013) Historical and idealized climate model experiments: an intercomparison of earth system models of intermediate complexity. *Climate of the Past* 9(3):1111–1140
- Feely RA, Doney SC, Cooley SR (2009) Ocean acidification: present conditions and future changes in a high-CO₂ world. *Oceanography* 22(4):36–47
- Gangstør R, Joos F, Gehlen M (2011) Sensitivity of pelagic calcification to ocean acidification. *Biogeosciences* 8(2):433–458
- Garrison T (2009) *Oceanography: an invitation to marine science*. Cengage Learning
- Goodess C, Hanson C, Hulme M, Osborn T (2003) Representing climate and extreme weather events in integrated assessment models: a review of existing methods and options for development. *Integrated Assessment* 4(3):145–171
- Hof AF, Hope CW, Lowe J, Mastrandrea MD, Meinshausen M, van Vuuren DP (2012) The benefits of climate change mitigation in integrated assessment models: the role of the carbon cycle and climate component. *Climatic change* 113(3-4):897–917

- IPCC (2001) *Climate Change 2001: The Scientific Basis*. Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press
- IPCC (2007) *Climate Change 2007: The Physical Science Basis*. Contribution to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change, vol 4. Cambridge University Press
- IPCC (2013) *Climate Change 2013: The Physical Science Basis*. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, United Kingdom and NY, USA.
- Jones C, Robertson E, Arora V, Friedlingstein P, Shevliakova E, Bopp L, Brovkin V, Hajima T, Kato E, Kawamiya M, et al (2013) Twenty-first-century compatible CO₂ emissions and airborne fraction simulated by CMIP5 Earth system models under four representative concentration pathways. *Journal of Climate* 26(13)
- Joos F, Müller-Fürstenberger G, Stephan G (1999) Correcting the carbon cycle representation: How important is it for the economics of climate change? *Environmental Modeling & Assessment* 4(2-3):133–140
- Joos F, Prentice I, Sitch S, Meyer R, Hooss G, Plattner G, Gerber S, Hasselmann K (2001) Global warming feedbacks on terrestrial carbon uptake under the Intergovernmental Panel on Climate Change (IPCC) emission scenarios. *Global Biogeochemical Cycles* 15(4):891–908
- Joos F, Roth R, Fuglestedt J, Peters G, Enting I, Bloh Wv, Brovkin V, Burke E, Eby M, Edwards N, et al (2013) Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis. *Atmospheric Chemistry and Physics* 13(5):2793–2825
- Kheshgi HS, Archer DE (2004) A nonlinear convolution model for the evasion of CO₂ injected into the deep ocean. *Journal of Geophysical Research* 109(C2)
- Maier-Reimer E, Hasselmann K (1987) Transport and storage of CO₂ in the ocean— an inorganic ocean-circulation carbon cycle model. *Climate dynamics* 2(2):63–90
- Marten AL, Newbold SC (2013) Temporal resolution and DICE. *Nature Climate Change* 3(6):526–527
- Mehrbach C, Culbertson C, Hawley J, Pytkowicz R (1973) Measurement of the apparent dissociation constants of carbonic acid in seawater at atmospheric pressure. *Limnology and Oceanography* pp 897–907
- Meinshausen M, Raper S, Wigley T (2011) Emulating coupled atmosphere-ocean and carbon cycle models with a simpler model, MAGICC6—Part 1: Model description and calibration. *Atmospheric Chemistry and Physics* 11(4):1417–1456
- Montenegro A, Brovkin V, Eby M, Archer D, Weaver A (2007) Long term fate of anthropogenic carbon. *Geophysical Research Letters* 34(19):L19,707
- Moyer E, Woolley M, Glotter M, Matteson N, Weisbach D (2014) Climate impacts on economic growth as drivers of uncertainty in the social cost of carbon. *Journal of Legal Studies* 43(2), DOI 10.1086/678140
- Nakicenovic N, Alcamo J, Davis G, de Vries B, Fenhann J, Gaffin S, Gregory K, Grubler A, Jung TY, Kram T, et al (2000) *Special report on emissions scenarios: A special report of Working Group III of the Intergovernmental Panel on Climate Change*. Cambridge University Press
- Nordhaus W (1993) Rolling the ‘DICE’: an optimal transition path for controlling greenhouse gases. *Resource and Energy Economics* 15(1):27–50
- Nordhaus W (2008) *A question of balance: Weighing the options on global warming policies*. Yale Univ Pr
- Nordhaus WD (2007) *Accompanying notes and documentation on development of DICE-2007 model: Notes on DICE-2007*. v8 of September 21, 2007. Available online at: www.econ.yale.edu/~nordhaus/homepage/Accom_Notes_100507.pdf
- Nordhaus WD (2010) Economic aspects of global warming in a post-Copenhagen environment. *Proceedings of the National Academy of Sciences* 107(26):11,721–11,726
- Oeschger H, Siegenthaler U, Schotterer U, Gugelmann A (1975) A box diffusion model to study the carbon dioxide exchange in nature. *Tellus* 27(2):168–192
- Petoukhov V, Ganopolski A, Brovkin V, Claussen M, Eliseev A, Kubatzki C, Rahmstorf S (2000) CLIMBER-2: a climate system model of intermediate complexity. Part I: model description and performance for present climate. *Climate dynamics* 16(1):1–17
- Revelle R, Suess HE (1957) Carbon dioxide exchange between atmosphere and ocean and the question of an increase of atmospheric CO₂ during the past decades. *Tellus* 9(1)
- Sarmiento JL, Gruber N (2006) *Ocean biogeochemical dynamics*, vol 1015. Princeton University Press
- Schultz PA, Kasting JF (1997) Optimal reductions in CO₂ emissions. *Energy Policy* 25(5):491–500
- Sitch S, Brovkin V, von Bloh W, van Vuuren D, Eickhout B, Ganopolski A (2005) Impacts of future land cover changes on atmospheric CO₂ and climate. *Global Biogeochemical Cycles* 19(2):GB2013
- Stern N (2008) *The economics of climate change*. *The American Economic Review* pp 1–37
- van Vuuren D, Lowe J, Stehfest E, Gohar L, Hof A, Hope C, Warren R, Meinshausen M, Plattner G (2011) How well do integrated assessment models simulate climate change? *Climatic Change* 104(2):255–285
- Warneck P (1999) *Chemistry of the natural atmosphere*, vol 71, 2nd edn. Access Online via Elsevier

- Warren R, Mastrandrea M, Hope C, Hof A (2010) Variation in the climatic response to sres emissions scenarios in integrated assessment models. *Climatic change* 102(3-4):671–685
- Weaver A, Eby M, Wiebe E, Bitz C, Duffy P, Ewen T, Fanning A, Holland M, MacFadyen A, Matthews H, et al (2001) The UVic Earth System Climate Model: Model description, climatology, and applications to past, present and future climates. *Atmosphere Ocean* 39(4):361–428
- Weiss R (1974) Carbon dioxide in water and seawater: the solubility of a non-ideal gas. *Marine Chemistry* 2(3):203–215
- Wigley T, Raper S, Salmon M, Osborn T, developers (2007) MAGICC (Model for the Assessment of Greenhouse-gas Induced Climate Change). National Center for Atmospheric Research, Boulder, Colorado. Available online at www.cgd.ucar.edu/car/wigley/magicc/
- Wigley TM (2008) MAGICC/SCENGEN 5.3: User manual (version 2). NCAR, Boulder, CO p 80
- Zeebe R, Wolf-Gladrow D (2001) CO₂ in seawater: equilibrium, kinetics, isotopes, vol 65. Elsevier Science



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