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Tipping Points in a Dynamic Stochastic IAM

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Abstract

We use a dynamic stochastic general equilibrium model of integrated climate and economy (DSICE) to account for abrupt and irreversible climate change. We model a climate shock in the form of a stochastic tipping point. We investigate the impact of the tipping point externality on optimal mitigation policy.

We conclude that the optimal mitigation policy depends on the dynamic pattern of the impact. In the case of abrupt and irreversible climate change with a permanent impact, the optimal policy implies a constant anti-tipping effort to prevent the catastrophe, calling for immediate limitations on emissions.

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Keywords: Climate change policy and uncertainty, discounting abrupt and irreversible climate change, tipping points, stochastic IAM.

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1 Introduction

The climate system is complex and poorly understood, creating substantial uncertainty about future climate trends and the response to anthropogenic GHGs. This uncertainty is increasingly important in the analysis of economics of climate change; for reviews, see Heal and Kriström (2002) and Pindyck (2007). Most integrated assessment models that have attempted to incorporate uncertainty (e.g., Nordhaus (1994), Nordhaus (2008), and Pizer (1999)) have not modeled uncertainty explicitly, relying instead on certainty equivalent formulations. It is unclear if they can produce reliable analyses of how risk-averse agents respond to these uncertainties.

An important feature of the climate system is the nature and extent of damages arising from climate change. Most IAMs follow DICE2007 (Nordhaus, 2008) and assume that damages are a function of contemporaneous temperature. However, possible climate change externalities are more complex. Many scientists are worried about climate change triggering large and irreversible events leading to significant and long-lasting damages.

According to Lenton et al. (2008) abrupt climate change can occur if perturbations of earth system components (e.g. atmosphere, oceans, land surface) are significant enough to trigger the earth system to move beyond a certain threshold, i.e., a tipping point. Lenton et al. (2008) characterizes tipping points for some major elements of the climate system (e.g., collapse of thermohaline circulation (THC), changes in El Niño southern oscillation (ENSO) and permafrost melting). While the likelihood of a tipping point occurring may be a function of contemporaneous temperature, their effects are long lasting and independent of future temperatures.

The IAM community, despite numerous attempts, has so far not been able to produce a stochastic IAM flexible enough to represent uncertainty in a quantitatively realistic manner. In particular, the representation of time should be compatible with the natural frequencies of both the natural and social processes related to climate change. Models that assume long time periods, such as ten years, represent neither social nor physical processes because nontrivial dynamics and feedbacks may occur in either system during a single decade.

Furthermore, the literature on IAM considers several approaches to studying optimal economic responses to catastrophic events in the climate system. We argue that none of these approaches deals appropriately with the specific nature of the problem. A faithful representation would include (i) a fully stochastic formulation of abrupt changes, and (ii) a representation of the irreversibility of the catastrophe.

In order to assess the impact of stochastic tipping points on climate policies we use DSICE (Cai, Judd and Lontzek, 2012), a dynamic stochastic general equilibrium extension of the full DICE2007 (Nordhaus, 2008) model. We have chosen the DICE2007 model as a starting point, because (i) it is open code, (ii) extensively documented, (iii) understood by the community, (iv) known to most economists and (v) it has a relatively low dimensionality and can be solved easily and fast. The last point however doesn't limit DSICE. In fact, in the eight-dimensional DSICE model we use one-year time periods over a 600-year horizon instead of the ten-year time periods used in DICE2007. DSICE solves in less than an hour on a laptop. Furthermore, the numerical solution algorithm behind DSICE is able to deal with models of higher dimensions than DSICE. In particular, far larger models can be solved when using parallel computing methods. Therefore, contrary to common belief (EPA, 2010), it is well within the scientific frontier to integrate large, long-run dynamic IAM's with DSGE models. DSICE is an example of that.

The general version of DSICE (Cai, Judd and Lontzek, 2012) allows for both business cycle uncertainty and abrupt climate change. In this study we consider exclusively the latter form of uncertainty. The risk of abrupt climate change increases as the atmospheric temperature increases, making emission control policies increasingly important.

We use multidimensional dynamic programming methods to solve the stochastic DSICE model and study dynamically optimal policy responses to tipping point risks.

The one policy question we explore is the optimal mitigation policy. DICE2007 implies a "ramp" structure for mitigation policy, with mitigation policy initially being mild but rising substantially over time. Nordhaus (2008) used a certainty-equivalent approach to tipping points and argued that the marginal mitigation policy also had a ramp structure. DSICE comes to a substantially different conclusion, arguing that the marginal mitigation policy is roughly a constant. This initial result clearly shows the importance of modeling stochastic climate change as a stochastic process.

Furthermore, discussions of how to apply cost-benefit analysis to climate change policies usually focus on the appropriate discount rate. This is appropriate only for a limited set of externalities where the stock of GHG's affect economic output. However, the implementation of an abrupt and irreversible climate catastrophe to an IAM such as DICE2007 creates a two-dimensional nature of the global warming externality. This is due to the shock structure of the catastrophe. Our findings suggest that the expected optimal mitigation policy in the face of abrupt and irreversible climate change exhibits a flat pattern. Thus, our model results imply a substantially lower hurdle required to justify significant mitigation policies.

The remainder of this paper is organized as follows. The focus of section 2 is on specifying our approach to model uncertainty and irreversibility of abrupt climate change. Section 3 discusses the calculation of the hazard rate for abrupt climate change. Section 4 presents DSICE. In Section 5 we present the results of some simulations based on the optimal solution of the model. We also describe the policy implications derived from DSICE. Section 6 concludes.

2 Uncertainty & Irreversibility of Climate Change

What truly sets apart DSICE from all models integrating climate and the economy, is how uncertainty and irreversibility of abrupt climate change are treated. In our view, a stochastic model's main characteristic is intrinsic random events *within* that specific model. For the case of integrated assessment models, this implies solving a multi-dimensional, non-linear, dynamic stochastic optimization model for multiple periods.

For those problems, dynamic programming is currently the only tractable method to obtain accurate results. Most recently, Lontzek and Narita (2011) have applied dynamic programming methods to study models of climate and the economy. These models, however are substantially lower-dimensional. In contrast, all IAMs are considerably higher-dimensional but do not apply dynamic programming methods. Thus, the implications of a fully stochastic abrupt climate change on optimal mitigation has not yet been studied appropriately in models of integrated assessment.

There are many different types of uncertainty that are discussed in the IAM literature. First, many people examine parametric uncertainty, because we do not know with precision the value of key parameters. Second, economic models have substantial amounts of intrinsic uncertainty, meaning that even if one knew the parameter, there would still be uncertainty due to random exogenous events. DSICE is a model that focuses on intrinsic uncertainty, as is done in the DSGE literature in economics. However, the speed of our DSICE algorithm is fast enough that we could also do wide-ranging parameter sweeps to address parameter uncertainty.

Some studies model abrupt climate change as a deterministic process. For example, Mastrandrea and Schneider (2001) couple an older version of the DICE model with a simple climate model which simulates the functioning of the North Atlantic thermohaline circulation (THC). Information obtained from the additional climate model is used to enhance the exponential component in the DICE damage function. This specification gives rise to a steeper damage function (in terms of global warming) but is inconsistent with the nature of the externality of an irreversible catastrophe. Another example in which abrupt climate change is modeled deterministically is Nordhaus (2010). The author models sea level rise by increasing the curvature of the damage function.

Another branch of the literature on catastrophic climate change in IAMs (e.g.: Yohe et al. 2004) models tipping point events as happening when a threshold temperature is passed (or some other condition is met). In our view this approach (or at least the simplest version of it) is not appealing because it implies that if we have been at that threshold temperature level, then we can immediately infer (learn) that we are safe as long as we stay at or below that level.

Modeling abrupt climate change with a known threshold location is a special case of our hazard function approach. In that special case (e.g.: Keller et al., 2004) the hazard rate equals zero up to the threshold temperature level and is equal to one beyond that level. In contrast, knowing that there is a critical point but not knowing where it is would not imply a hazard rate. The latter formulation implies that for each temperature level at which a tipping point has not occurred we would know for sure that a tipping point can never occur at that temperature level. This is not in the statistical nature of a hazard rate. Instead we assume that even if a tipping point has not occurred yet, it could still occur after a cooling down of the climate.

The optimal mitigation policy in DICE2007 is increasing with global warming. This "ramp" structure of mitigation policy in the DICE2007 model arises because of the nature of the stock externality, i.e., a rising temperature in the atmosphere damages output today, but if temperature falls later on, damages fall as well. However, by including the possibility of an abrupt and irreversible climate change, one actually changes the nature of the climate externality. Figure 1 highlights the two aspects of the global warming externality. Curve A represents the pre-tipping damage factor as

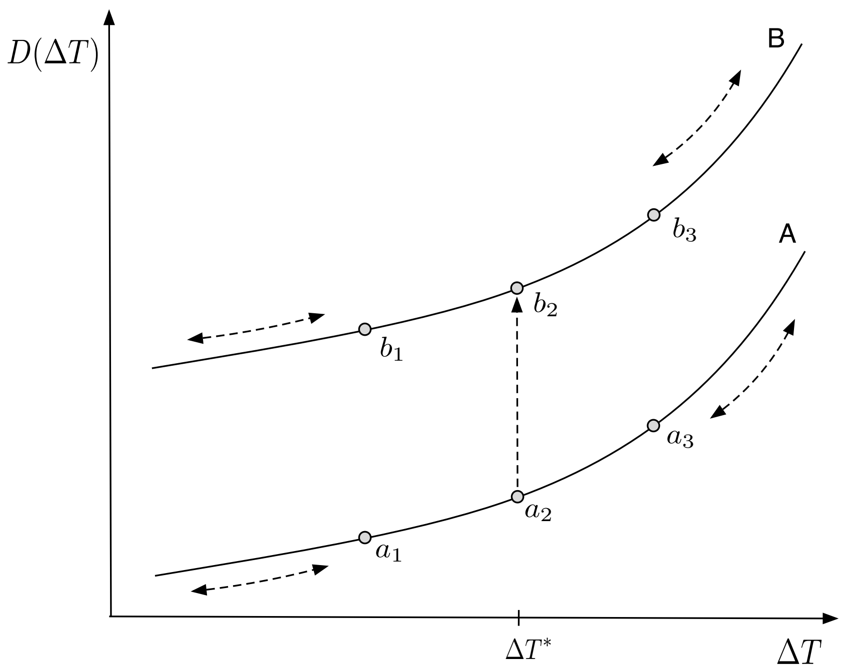

Figure 1: The nature of externality from abrupt and irreversible climate change

a function of temperature change (ΔT). It is monotonically increasing and convex. In a pre-tipping regime any movement along A (i.e., $a_1 \leftrightarrow a_2 \leftrightarrow a_3$) can occur, depending on the change in global average temperature. As a consequence, the resulting damage rate $D(\Delta T)$ will exhibit a smooth pattern. The same logic applies in a post-tipping regime, which is denoted by B. Any movement along B (i.e., $b_1 \leftrightarrow b_2 \leftrightarrow b_3$) can occur, depending on the change in global average temperature after a tipping point has occurred. Now, assume that a tipping point has occurred at a_2, which correspond to a global warming level of ΔT^* [1]. In that case, the damage rate resulting from the change in the climate system will be represented by B (i.e., $a_2 \rightarrow b_2$). However, the irreversible nature of abrupt climate change prevents a movement back to A in case of global cooling and temperature levels below the realized tipping point level. Instead the damage factor will remain to move along B. This stochastic aspect of a tipping point can only be captured by specifying a hazard rate and modeling abrupt climate change as a jump process where the Markovian hazard rate only depends on contemporaneous conditions. It cannot be captured by changing the shape (e.g., increasing the exponents) of the damage function of global warming.

[1] Note that our specification allows for a tipping point to occur even in a phase of global cooling. This feature results from the stochastic nature of a hazard rate formulation of abrupt climate change.

3 Hazard Rate Calibration of Tipping Point Events

The analysis of a stochastic tipping point requires the specification of a hazard rate expressed as a function of temperature. Unfortunately, as Kriegler et al. (2009) points out, the lack of data and limited understanding of the underlying processes make it difficult to assess the likelihood of changes in the earth system due to global warming. Lenton et al. (2008) provides estimates for a tipping of THC, West Antarctic ice sheet (WAIS) and ENSO. Table 1 summarizes some properties of these tipping elements. The authors suggest a global warming of $> 3°C$ and an estimated timescale of passing a tipping point for these elements of > 100 years.

Tipping Element	Global Warming	Timescale	key Impacts
THC	$+3-5°C$	$\approx 100yr.$	reg. cooling, sea level+
WAIS	$+3-5°C$	$> 300yr.$	sea level +5 m.
ENSO	$+3-6°C$	$\approx 100yr.$	Drought (e.g: SE Asia)
Permafrost	–	$< 100yr.$	CH_4 and CO_2 release

Table 1: Tipping Elements in the climate System: based on Lenton et al. (PNAS, 2008)

As of today, most probability assessments of abrupt climate change come from expert elicitation surveys (e.g. Kriegler et al., 2009 and Zickfeld et al. 2007). In these elicitations, the experts are asked about their beliefs about probabilities of a tipping point occurring at, or before some time (typically the year 2100) based on an emission scenario. Kriegler et al. (2009) conducts an extensive expert elicitation on some major tipping elements and their likelihood of abrupt change. The tipping points under investigation are: the THC, the Greenland ice sheet (GIS), the WAIS, the Amazon rainforest and the ENSO. The likelihood of at least one of these elements to reach its tipping point is estimated to be $> 50\%$ for a global mean temperature change of $> 4°C$ and 16% for a global mean temperature change of $2°C-4°C$. More estimates of a WAIS collapse are presented in Vaughan and Spouge (2002). The authors obtain a low probability ($< 2\%$) of a WAIS tipping by 2100. However, the probability of a significant sea level rise by 2100 resulting from changes in WAIS is estimated at $> 10\%$. Gregory et al. (2004) estimate the GIS to tip in the range of $> 3°C$ with a persistent sea-level rise by 7 meters over a period of more that 1,000 years. The authors claim that a $> 3°C$ warming is likely to occur before the year 2100. Similar results are obtained by Oppenheimer (2005) and Oppenheimer and Alley (2004) for GIS and WAIS.

3.1 Hazard Rate Calibration - A General Formula

In order to derive a hazard rate of a tipping we use information obtained from expert elicitation studies. In general these experts' opinions reveal their probabilistic beliefs for a tipping point occurring within a time frame under alternative assumptions about temperature changes within that specific time frame. Our goal is to extract the hazard rate from these cumulative probabilities. We use the information specified by the experts and reverse engineer the hazard rate for the tipping element.

First, we specify a time path for a temperature scenario. We assume that it is quadratic[2].

$$T_t = a_0 + a_1 t + a_2 t^2 \tag{1}$$

The coefficients a_0, a_1 and a_2 can be calculated by specifying three temperature levels at different time periods[3]. In the next step, we specify the relationship between the temperature path and its derivative, and the rate of probability of a tipping point at a time when temperature is $temp_t$. We consider a specification that is quadratic in the temperature level and linear in the first order change of temperature. This results in a hazard rate as a function of time.

$$h_t = b_0 + b_1 T_t + b_2 {T_t}^2 + c_1 \frac{\partial T_t}{\partial t} \tag{2}$$

Inserting (1) in (2) and integrating up to time t we obtain H_t the cumulative hazard[4].

$$H_t = \int_0^t h_s \,\mathrm{d}s. \tag{3}$$

Using the formula in (3) we calculate the probability of a tipping point having *not* occurred[5] until time t to be e^{-H_t}. Consequently, the probability of a tipping point having occurred until t is $1 - e^{-H_t}$. Using idiosyncratic information for each tipping point, the coefficients b_0, b_1, b_2 and c_1 can be determined and the hazard rate can be calibrated.

3.2 Calibrating the Hazard Rate of a THC Collapse

Based on an expert elicitation survey, Zickfeld et al. (2007) presents subjective probabilities that a collapse of THC will occur or be irreversibly triggered by 2100. In order to calculate a hazard rate for abrupt climate change, we use the intrinsic information provided in the expert elicitation studies by Zickfeld et al., 2007 and Kriegler et al., 2009. Figure 2 shows the results from an expert elicitation conducted by Zickfeld et al., 2007. It illustrates the experts' subjective cumulative probabilities that a collapse of THC will occur or be irreversibly triggered by 2100.

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[2] A quadratic time path allows us to model convex as well as concave temperature paths.

[3] E.g., $T_{t_0} = 1$, $T_{t_1} = 1.5$ and $T_{t_2} = 2$.

[4] The hazard rate H_t is now a function of time t, the three specified temperature levels T_{t_0}, T_{t_1}, T_{t_2} and t_0, t_1 and t_2, the corresponding periods in which these temperature levels are believed to occur.

[5] The general formula for the probability of observing n tipping point events of the same kind over the interval $[t_0, t]$ is $({H_t}^n - {H_{t_0}}^n) \cdot n!^{-1} \cdot e^{H_{t_0} - H_t}$

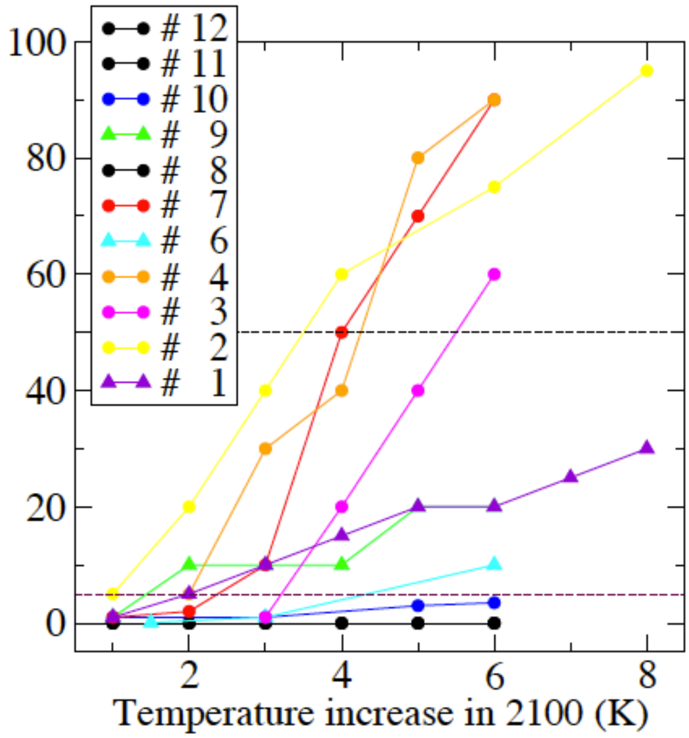

Figure 2: Zickfeld et al. (2007, Climatic Change): Expert's subjective probability (in %) that a collapse of THC will occur or be irreversibly triggered by 2100

The survey data obtained in Zickfeld et al. (2007) on THC collapse show a huge range of different opinions, which underlines the uncertainty about abrupt climate change from a decision maker's point of view.

The experts' opinions reveal their probabilistic beliefs for a THC collapse by 2100 under alternative assumptions about temperature at 2100. Our goal is to extract the hazard rate from these cumulative probabilities. With the path of temperature in the scenario and the probability of a tipping point occurring by 2100, we reverse engineer the hazard rate for that tipping element. More precisely, we compute the probability of of a major tipping point event as a function of the current atmospheric temperature (measured as the deviation from preindustrial temperature), assuming that this tipping point had not yet happened. We calibrate the beliefs of the average optimistic and pessimistic expert[6]. The resulting hazard rates are illustrated in Figure 3. Our hazard rates implies that at any point in time, given a global warming of e.g. $2°C$, the average pessimistic expert estimates the probability of a THC collapse in that year to be about 4%, while the average optimistic expert's estimate is 1%. As already mentioned, our hazard rate calculations are compatible with the experts'

[6]We use the formulas from the previous section and assume $a_2 = b_2 = c_1 = 0$ for simplicity.

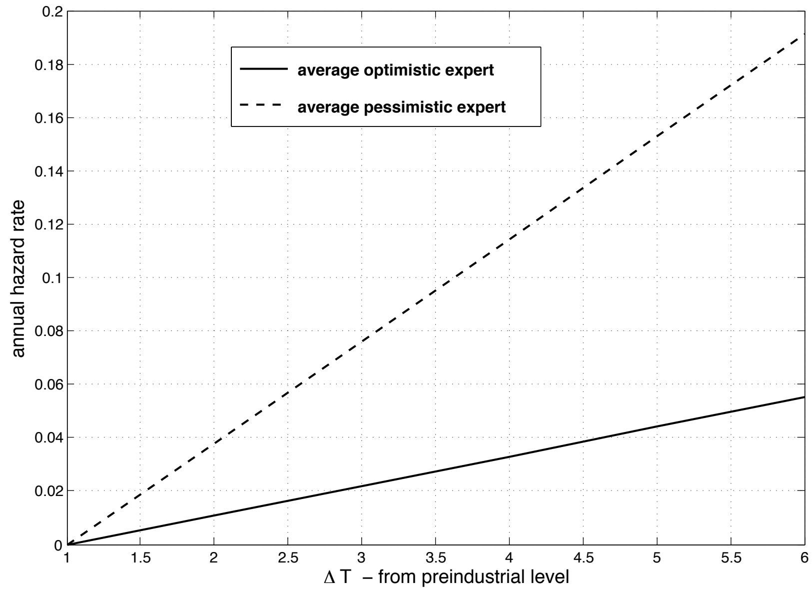

Figure 3: Annual Hazard rate for THC collapse - based on expert's subjective probability that a collapse of THC will occur or be irreversibly triggered by 2100 (Zickfeld et al., 2007)

believed evolution of temperature which is expected by 2100. Figure 3 shows that the probability of a catastrophe is an increasing function of global warming. It is controllable because temperature is controllable.

The specification of the hazard rate in our model is close to the average optimistic expert's assessment. However, we would like to point out that the analysis of a tipping element in this study should not be seen as exclusively restricted to the collapse of the THC. Instead, it can be thought of as representative of any tipping element with similar characteristics. With this specification, even experts with very optimistic views might agree that there is a non-trivial probability of abrupt climate change.

In general, our assumption that the tipping event is a probabilistic function of current temperature might look inappropriate since it seems that we say that the physics beyond it are random. However, our claim is that we doubt that a single statistic such as global average temperature describes the complex climate system. Instead, we assume that even a deterministic climate system is a highly complex system, and best modeled by a stochastic process. Therefore, the stochastic model represents *our* uncertainty about a system which may be deterministic.

4 Tipping Point Analysis in DSICE

The DSICE model (see Cai, Judd and Lontzek 2012) is a dynamic stochastic general equilibrium model integrating climate and the economy. DSICE is basically a DGSEextension of the DICE-CJL model (see Cai, Judd and Lontzek 2012), which itself is a numerically stable 1-year time period version of DICE2007. This version of DSICE allows for shocks to the climate, e.g., abrupt and irreversible climate change due to stochastic tipping point events. The general structure of the DSICE model with one tipping point is represented by the following seven-dimensional stochastic optimization problem:

$$\max_{c_t,\mu_t} \ \mathbb{E}\left\{\sum_{t=0}^{T} \beta^t u(c_t, l_t)\right\}$$

$$\begin{array}{rrcl} \text{s.t.} & k_{t+1} & = & (1-\delta)k_t + \Omega_t(1-\Lambda_t)Y_t - c_t, \\ & \mathbf{M}_{t+1} & = & \mathbf{\Phi}^{\mathrm{M}}\mathbf{M}_t + (E_t, 0, 0)^\top, \\ & \mathbf{T}_{t+1} & = & \mathbf{\Phi}^{\mathrm{T}}\mathbf{T}_t + (\xi_1 F_t, 0)^\top, \\ & J_{t+1} & = & g(J_t, \omega_t^J) \end{array}$$

where the utility function is $u(c,l) = c^{1-\gamma}/(1-\gamma)$. Output is $Y_t \equiv A_t k_t^\alpha l_t^{1-\alpha}$, the damage rate is

$$\Omega_t \equiv \frac{J_t}{1 + \pi_1 T_t^{\mathrm{AT}} + \pi_2 (T_t^{\mathrm{AT}})^2}$$

The costs of mitigation are given by $\Lambda_t \equiv \psi_t^{1-\theta_2}\theta_{1,t}\mu_t^{\theta_2}$. The state transition matrices for the carbon and temperature stocks are given by $\mathbf{\Phi}^{\mathrm{M}}$ and $\mathbf{\Phi}^{\mathrm{T}}$, respectively. J_t is the tipping point state, modeled by a Markov process. $J_t = 1$ until a tipping point event occurs and $0 < J_t < 1$ thereafter.

We assume that $\mathbf{M}_t = (M_t^{\mathrm{AT}}, M_t^{\mathrm{LO}}, M_t^{\mathrm{UP}})^\top$ is a three-dimensional vector describing the masses of carbon concentrations in the atmosphere, and lower and upper levels of the ocean. To be more specific, these concentrations evolve over time according to:

$$\begin{array}{rcl} M_{t+1}^{\mathrm{AT}} & = & (1-\phi_{12})M_t^{\mathrm{AT}} + \phi_{21}M_t^{\mathrm{UP}} + E_t \\ M_{t+1}^{\mathrm{LO}} & = & (1-\phi_{32})M_t^{\mathrm{LO}} + \phi_{23}M_t^{\mathrm{UP}} \\ M_{t+1}^{\mathrm{UP}} & = & \phi_{12}M_t^{\mathrm{AT}} + \phi_{32}M_t^{\mathrm{LO}} + (1-\phi_{21}-\phi_{23})M_t^{\mathrm{UP}} \end{array}$$

Total carbon emission is $E_t = E_t^{\mathrm{IND}} + E_t^{\mathrm{LAND}}$. While E_t^{LAND} is some pre-specified emission of carbon from land use, and $E_t^{\mathrm{IND}} = \sigma_t(1-\mu_t)Y_t$ denotes industrial carbon emissions. Furthermore, temperature in the atmosphere and ocean is described by the two-dimensional vector $\mathbf{T}_t = (T_t^{\mathrm{AT}}, T_t^{\mathrm{LO}})^\top$. The temperatures dynamically evolve according to:

$$\begin{array}{rcl} T_{t+1}^{\mathrm{AT}} & = & T_t^{\mathrm{AT}} + \xi_1(F_{t+1} - \xi_2 T_t^{\mathrm{AT}} - \xi_3(T_t^{\mathrm{AT}} - T_t^{\mathrm{LO}})) \\ T_{t+1}^{\mathrm{LO}} & = & T_t^{\mathrm{LO}} + \xi_4(T_t^{\mathrm{AT}} - T_t^{\mathrm{LO}}) \end{array}$$

where $F_t = \eta \log_2(M_t^{\mathrm{AT}}/M_0^{\mathrm{AT}}) + F_t^{\mathrm{EX}}$ is total radiative forcing. In order to be consistent in the choice of a finite difference scheme for the numerical analysis, F_t depends on the stock if atmospheric carbon at t and on its inertia. A complete description of all equations of DSICE and modifications to DICE2007 can be found in Cai, Judd and Lontzek (2012).

We solve the stochastic optimization problem using dynamic programming. We first represent it as the seven-dimensional dynamic programming problem:

$$\begin{aligned} V_t(k, \mathbf{M}, \mathbf{T}, J) &= \max_{c,\mu} \quad u(c,l) + \beta\mathbb{E}[V_{t+1}(k^+, \mathbf{M}^+, \mathbf{T}^+, J^+)] \\ \text{s.t.} \quad k^+ &= (1-\delta)k + \Omega_t(1-\Lambda_t)f(k,t) - c, \\ \mathbf{M}^+ &= \mathbf{\Phi}^{\mathrm{M}}\mathbf{M} + (E_t, 0, 0)^\top, \\ \mathbf{T}^+ &= \mathbf{\Phi}^{\mathrm{T}}\mathbf{T} + (\xi_1 F_t, 0)^\top, \\ J^+ &= g(J, \omega^J) \end{aligned}$$

We then use numerical methods to compute approximations to the value function at each time period. See Judd(1998) for a description of the basic methods we use. We solve this problem with a one-year time horizon for 600 years. The dynamic programming algorithm runs from year 600 backwards. See Cai, Judd and Lontzek (2012) for a more detailed description of the numerical solution method.

5 Results and Policy Implications

Based on the optimal solution of the dynamic problem we simulate 1,000 optimal paths. Figure 4 shows the statistical results of the simulation runs. We report the expected mitigation policy (solid line), the 0% and 100% quantiles (dashed lines) and the median (dotted line). The upper envelope in Figure 4 represents the pre-tipping emission control rate, whereas the lower envelope represents the emission control rate in the post-tipping regime. The lower envelope represents the policy response to the DICE2007 model without the tipping point. Out of our 1,000 simulations, the fist tipping point occurs before 2030. Furthermore, by the year 2150 more than 50% of our simulation runs have produced a tipping point. Note that the expected mitigation policy in the face of a tipping point has a flattened profile.

Furthermore, Figure 4 illustrates the two distinct aspects of the abrupt climate change externality which have been discussed in Section 2. On the one hand, the DICE-like "ramp" structure of emission control implies a weak policy to cut emissions for low global warming but then becomes more stringent over time when global warming becomes more severe.

On the other hand, the threat of an irreversible and abrupt climate change results in a nearly constant extra effort to delay, if not prevent, the tipping point from occurring. The latter aspect leads to a higher emission control rate if the tipping point has not yet occurred, i.e., a nearly constant increase in the "ramp."

The drop in the emission control rate after the tipping represents that effort to delay the tipping event. Figure 4 implies an augmented mitigation markup in the range of 15% to 20%. Thus, in a pre-tipping regime, the additional optimal amount of mitigation is constant over time, even though the danger and costs of a catastrophe

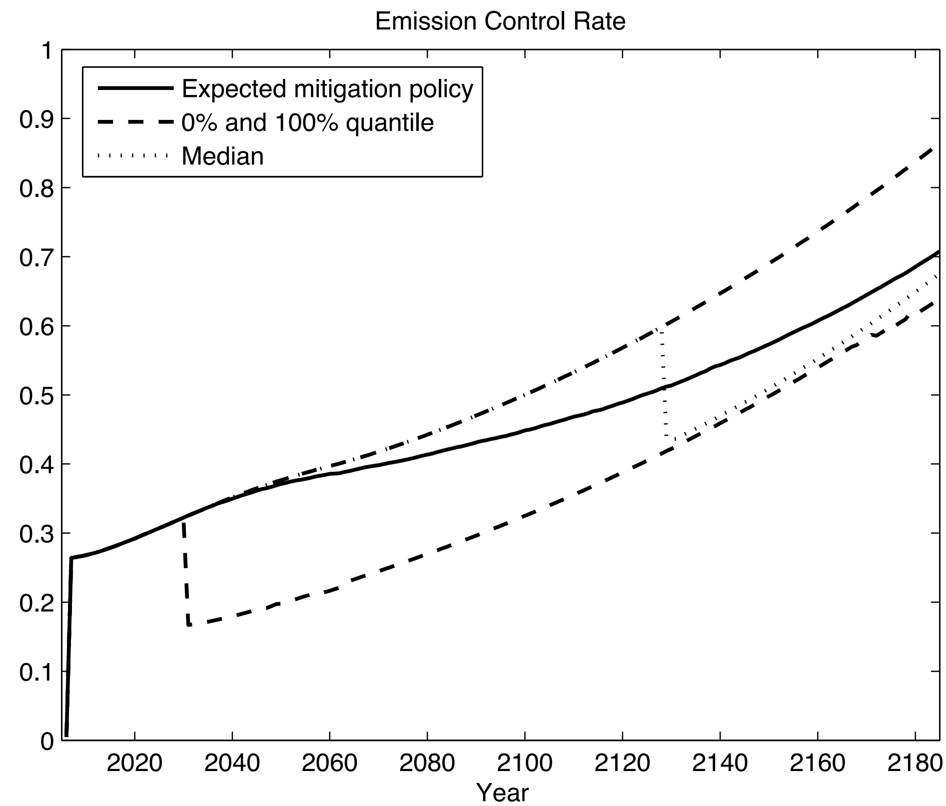

Figure 4: Optimal emission control policy - statistical results from 1,000 simulations of the optimal stochastic path.

are rising. We conclude that the optimal mitigation policy response to the threat of abrupt and irreversible climate change in climate depends on the dynamic pattern of the adverse impacts. If these impacts are permanent, the optimal policy is one with immediately stringent limitations on emissions. The results for the optimal emission control rate in Figure 4 are based on a 10% damage in GDP (i.e., $J = 0.9$) in case of a tipping point event. Only a few studies provide an estimate of the loss in output from a tipping point catastrophe, such as the THC collapse. Tol (2009) and Keller et al. (2004) estimate the loss in GDP from a tipping event in the range of 1% - 3%. Other studies, such as Mastrandrea and Schneider (2001) and Nordhaus (1994) use much higher estimates. Figure 5 displays the difference in the emission control rate between a post-tipping regime and an pre-tipping regime.

The three lines represent three different tipping points which exhibit different impacts on the economy. The purpose of Figure 5 is to demonstrate that the constancy of the optimal anti-tipping effort is retained, even for tipping points with different impact. Therefore, the qualitative effects do not change if we use 1% (dotted line), 2% (dashed line) or 10% (solid line) damage. We conclude that the optimal policy does not depend on the magnitude of the damage but rather it is inherent in the stochastic structure of the jump process shock.

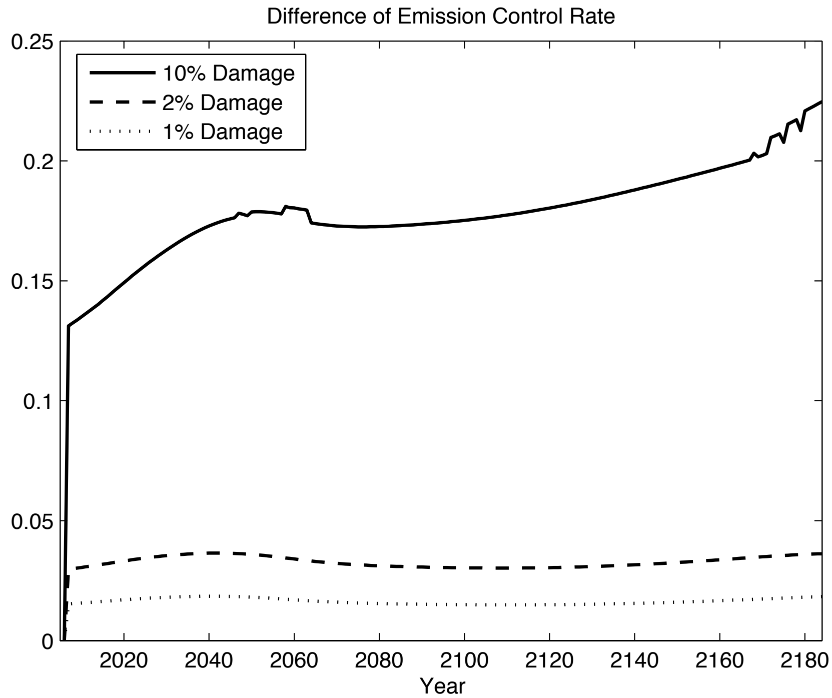

Figure 5: Nearly constant anti-tipping effort for tipping points with different damage

6 Conclusion

In our opinion a stochastic IAM is inevitable for an appropriate analysis of abrupt climate change with permanent and significant damage over a large time horizon.

The core features of abrupt climate change are uncertainty and irreversibility. It is uncertain when a tipping point will occur and whether it will occur at all. In addition, once a tipping point has occurred, the tipping element cannot tip back and the damage to the society from the new state of the tipping element cannot be undone.

In this study we present an application of DSICE to study the optimal mitigation policy in the face of abrupt and irreversible climate change. We specify the probability of a tipping point as a hazard rate which we calibrate according to experts' opinions.

We find that the resulting optimal policy profile is significantly flatter than the emission controls implied by DICE2007, a deterministic model. Therefore, the possibility of these low-probability but high-damage events imply an increased urgency to climate policy and show that studies which do not incorporate a true stochastic model cannot fully capture the impact of climate uncertainties.

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The Center brings together experts in economics, physical sciences, energy technologies, law, computational mathematics, statistics, and computer science to undertake a series of tightly connected research programs aimed at improving the computational models needed to evaluate climate and energy policies, and to make robust decisions based on outcomes.

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