ANL/MCS-TM-307

CIM-EARTH: Community Integrated Model of Economic and Resource Trajectories for Humankind

Version 0.1

Mathematics and Computer Science Division
Technical Memorandum ANL/MCS-TM-307

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by
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July 2009, Revised January 2010

This work was supported by grants from the MacArthur Foundation and the University of Chicago Energy Initiative, and by the Office of Advanced Scientic Computing Research, Office of Science, U.S. Department of Energy, under Contract DE-AC02-06CH11357.

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Community Integrated Model of Economic and Resource Trajectories for Humankind
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Abstract

Climate change is a global problem with local climatic and economic impacts. Mitigation policies can be applied on large geographic scales, such as a carbon cap-and-trade program for the entire U.S., on medium geographic scales, such as the NO_x program for the northeastern U.S., or on smaller scales, such as statewide renewable portfolio standards and local gasoline taxes. To enable study of the environmental benefits, transition costs, capitalization effects, and other consequences of mitigation policies, we are developing dynamic general equilibrium models capable of incorporating important climate impacts. This report describes the economic framework we have developed and the current Community Integrated Model of Economic and Resource Trajectories for Humankind (CIM-EARTH) instance.

1 Introduction

A general equilibrium model determines prices and quantities over time for commodities such that supply equals demand for each good [1, 9, 14]. Such models have the following features:

- Many *industries* that hire labor, rent capital, and buy inputs to produce outputs. Each industry chooses a feasible production schedule to maximize its profit, the revenue received by selling its outputs minus the costs of producing them. Each demands commodities, such as labor and material, to create a product, such as automobiles. The number of automobiles that can be produced is a function of the amount of labor and material demanded.

- Many *consumers* who choose what to buy and how much to work subject to the constraint that purchases cannot exceed income. Each consumer chooses a feasible consumption schedule to maximize his happiness as measured by a utility function. Each consumer derives happiness from buying and using commodities but is budget limited; the consumer cannot spend more on commodities than is received in income. Income is obtained by selling and renting endowed commodities, such as labor and capital, and (in a model with imperfect competition) from producer profits.

- Many *markets* where producers and consumers trade that set wage rates and commodity prices. Given commodity prices, each industry and each consumer make their own decision without regard to the decisions made by others. The total sum of the decisions made, however, must be consistent in the sense that the consumer demand for goods cannot exceed the output produced by industries. This

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consistency is imposed in “markets” where economic actors meet and the prices are set to “clear” the markets. If the price of a commodity is positive, then supply must equal demand.

In this paper, we document our economic framework and the Community Integrated Model of Economic and Resource Trajectories for Humankind (CIM-EARTH) instance used in our studies. Section 2 describes a basic, static general equilibrium model and provides a small example with two industries, one consumer, and four markets to illustrate how these models are specified. Section 3 introduces the calibrated form of the static general equilibrium model and the tax instruments available in our framework and introduces the myopic general equilibrium model. Section 4 details the full CIM-EARTH v0.1 instance, including our dynamic trajectories for capital, labor productivity, and resource usage.

2 Background

The primary modeling challenge is estimating the production and utility functions that characterize the physical and economic processes constraining the supply and demand decisions of industries and consumers. For this purpose, we use constant elasticity of substitution (CES) production functions with the form

$$y = \left(\sum_i \alpha_i \left(\gamma_i x_i \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where y is the output, x_i are the input factors ranging from labor and capital to seeds and fertilizer, γ_i are efficiency units that determine how effectively these factors are used, α_i are share parameters (with $\alpha_i > 0$ and $\sum_i \alpha_i = 1$ constraints imposed; otherwise they can be more generally referred to as distribution parameters), and σ controls the degree to which the inputs can be substituted for one another. When $\sigma = 0$, we obtain the Leontief production function

$$y = \min_i \left\{ \gamma_i x_i \right\},$$

in which the inputs are perfectly complementary; an increase in output requires an increase in all inputs. When $\sigma = 1$, we obtain the Cobb-Douglas production function

$$y = \prod_i \left(\gamma_i x_i \right)^{\alpha_i}.$$

When $\sigma = \infty$, we obtain the linear production function

$$y = \sum_i \alpha_i \gamma_i x_i$$

in which the inputs are perfectly substitutable. These functions are typically combined in a nested fashion where each nest describes the substitutability among commodity bundles.

We now develop a simple instance with two industries, one consumer, and four markets to illustrate how these models are specified. The industries produce *material* and *energy*, respectively. The consumer supplies *capital* and *labor* and demands material and energy. The four markets are material, energy, capital, and labor. We consistently use y for *supply* variables and x for *demand* variables, use subscripts to label the commodity or factor being supplied or demanded, and use superscripts to label the agent (material or energy industries, or consumer) supplying/demanding the commodity/factor. The variables for this instance are described in Table 2.1 and the parameters in Table 2.2.

2.1 Industries

Industries maximize profit, revenue minus expenditures, subject to production constraints. The material industry in the simple instance demands material, energy, capital, and labor, while the energy industry

Table 2.1: Variables referenced in the simple instance.

p_m	price of material
p_e	price of energy
p_K	price of capital
p_L	price of labor
y_m	quantity of material supplied by material industry
y_e	quantity of energy supplied by energy industry
y_K	quantity of capital supplied by consumer
y_L	quantity of labor supplied by consumer
x_m^m	quantity of material demanded by material industry
x_e^m	quantity of energy demanded by material industry
x_K^m	quantity of capital demanded by material industry
x_L^m	quantity of labor demanded by material industry
x_K^e	quantity of capital demanded by energy industry
x_L^e	quantity of labor demanded by energy industry
x_m^c	quantity of material demanded by consumer
x_e^c	quantity of energy demanded by consumer

Table 2.2: Parameters referenced in the simple example.

σ_{me}^m	elasticity of substitution among material and energy for material industry
σ_{KL}^m	elasticity of substitution among capital and labor for material industry
σ^m	elasticity of substitution among (material, energy) and (capital, labor) bundles
σ_{KL}^e	elasticity of substitution among capital and labor for energy industry
σ_{me}^c	elasticity of substitution among material and energy for consumer
σ^c	elasticity of substitution among (material, energy) bundle and savings for consumer
α_m^m	share parameter for material demanded by material industry
α_e^m	share parameter for energy demanded by material industry
α_{me}^m	share parameter for (material, energy) bundle demanded by material industry
α_K^m	share parameter for capital demanded by material industry
α_L^m	share parameter for labor demanded by material industry
α_{KL}^m	share parameter for (capital, labor) bundle demanded by material industry
α_K^e	share parameter for capital demanded by energy industry
α_L^e	share parameter for labor demanded by energy industry
α_m^c	share parameter for material demanded by consumer
α_e^c	share parameter for energy demanded by consumer
α_{me}^c	share parameter for (material, energy) bundle demanded by consumer
α_S^c	share parameter for savings demanded by consumer
$\bar{y}_K^c$	endowment of capital for the consumer
$\bar{y}_L^c$	endowment of labor for the consumer

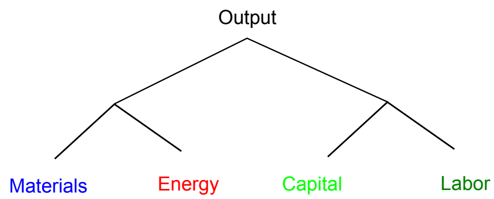

Figure 2.1: Basic nest for production function.

demands only capital and labor. In particular, the material industry solves the optimization problem

$$\begin{array}{lll}\max\limits_{y_m \geq 0, x_i^m \geq 0} & p_m y_m - p_m x_m^m - p_e x_e^m - p_K x_K^m - p_L x_L^m & \\ \text{s.t.} & y_m & \leq \left(\alpha_{KL}^m (x_{KL}^m)^{\rho^m} + \alpha_{me}^m (x_{me}^m)^{\rho^m}\right)^{\frac{1}{\rho^m}} \\ & x_{KL}^m & \leq \left(\alpha_K^m (x_K^m)^{\rho_{KL}^m} + \alpha_L^m (x_L^m)^{\rho_{KL}^m}\right)^{\frac{1}{\rho_{KL}^m}} \\ & x_{me}^m & \leq \left(\alpha_m^m (x_m^m)^{\rho_{me}^m} + \alpha_e^m (x_e^m)^{\rho_{me}^m}\right)^{\frac{1}{\rho_{me}^m}}\ , \end{array} \tag{2.1}$$

where $\rho^m = \frac{\sigma^m - 1}{\sigma^m}$, $\rho_{KL}^m = \frac{\sigma_{KL}^m - 1}{\sigma_{KL}^m}$, and $\rho_{me}^m = \frac{\sigma_{me}^m - 1}{\sigma_{me}^m}$. The production function constraints in (2.1) are depicted graphically by the tree structure shown in Figure 2.1. Each node of the tree represents a production function with its own elasticity of substitution parameter that aggregates the inputs from below into a commodity bundle. The root node then aggregates the two intermediate commodity bundles into the total material output.

The energy industry solves a similar, but much simpler, optimization problem because it demands only capital and labor:

$$\begin{array}{ll}\max\limits_{y_e \geq 0, x_i^e \geq 0} & p_e y_e - p_K x_K^e - p_L x_L^e \\ \text{s.t.} & y_e \leq \left(\alpha_K^e (x_K^e)^{\rho_{KL}^e} + \alpha_L^e (x_L^e)^{\rho_{KL}^e}\right)^{\frac{1}{\rho_{KL}^e}}\ , \end{array} \tag{2.2}$$

where $\rho_{KL}^e = \frac{\sigma_{KL}^e - 1}{\sigma_{KL}^e}$.

2.2 Consumers

Consumers maximize their individual utility subject to a budget constraint; expenditures cannot exceed income. The consumer in the simple instance demands material and energy, while supplying capital and labor. The supply of capital and labor is an endowed commodity; the consumer begins the period with a certain quantity (endowment) of labor and capital (accumulated from past savings). In particular, the consumer solves the optimization problem

$$\begin{array}{lll}\max\limits_{0 \leq y_j^c \leq \bar{y}_j^c, x_i^c \geq 0} & x^c & \\ \text{s.t.} & x^c & \leq \left(\alpha_S^c (x_S^c)^{\rho^c} + \alpha_{me}^c (x_{me}^c)^{\rho^c}\right)^{\frac{1}{\rho^c}} \\ & x_{me}^c & \leq \left(\alpha_m^c (x_m^c)^{\rho_{me}^c} + \alpha_e^c (x_e^c)^{\rho_{me}^c}\right)^{\frac{1}{\rho_{me}^c}} \\ & \multicolumn{2}{l}{p_m x_m^c + p_e x_e^c + x_S^c \leq p_L y_L^c + p_K y_K^c + \Pi_m + \Pi_e\ ,} \end{array} \tag{2.3}$$

where $\rho^c = \frac{\sigma^c - 1}{\sigma^c}$, $\rho_{me}^c = \frac{\sigma_{me}^c - 1}{\sigma_{me}^c}$, and Π_m and Π_e are the material and energy industry profits returned to the consumer as a dividend, respectively. The savings demanded by the consumer, x_S^c, is necessary for myopic dynamic models to approximate the future utility of consumption. In dynamic models, these savings are invested and enter the economy as malleable capital in the next time step. In practice, we choose σ^c to be one, so that the CES function aggregating savings and the (material, energy) bundle reduces to the Cobb-Douglas function, which implies that a fixed share of consumer income goes to savings each year. The utility function constraints in (2.3) are depicted graphically by the tree structure shown in Figure 2.2.

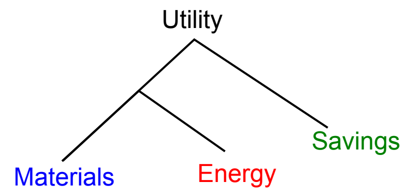

Figure 2.2: Basic nest for utility function.

2.3 Markets

The market clearing conditions for the current working model are as follows.

$$\begin{array}{lcl} 0 \leq p_m & \perp & y_m \geq x_m^m + x_m^c \\ 0 \leq p_e & \perp & y_e \geq x_e^m + x_e^c \\ 0 \leq p_K & \perp & y_K \geq x_K^m + x_K^e \\ 0 \leq p_L & \perp & y_L \geq x_L^m + x_L^e \end{array}$$

The complementarity condition signified by $\perp$ implies that one of the two inequalities in each expression must be saturated. That is, either supply equals demand and the price is nonnegative, or supply exceeds demand and the price is zero. In particular, a zero price means that the market for the good or factor collapses.

2.4 Complementarity Problem

Because the optimization problems solved by the industries and consumers are convex in their own variables and satisfy a constraint qualification, we can replace each with an equivalent complementarity problem obtained from the first-order optimality conditions by adding Lagrange multipliers. These optimality conditions in combination with the market clearing conditions form a square complementarity problems that can be solved by applying a generalized Newton method, such as PATH [5–7]. The construction of the first-order optimality conditions and the overall complementarity problem is automated in the framework and not discussed further.

3 General Equilibrium Model

The available data for general equilibrium models is typically the revenues and expenditures in a base year, rather than the quantities. To use this data, we formulate the general equilibrium problem using the calibrated form of the constant elasticity of substitution functions [2],

$$\frac{y}{\bar{y}} = \left(\sum_i \theta_i \left(\gamma_i \frac{x_i}{\bar{x}_i} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $\frac{y}{\bar{y}}$ is the ratio between the output of the industry to a base-year value, $\frac{x_i}{\bar{x}_i}$ are the ratios of the input commodities to their base-year values, γ_i are the efficiency units that determine how effectively these factors can be used, θ_i are the share parameters with $\theta_i > 0$ and $\sum_i \theta_i = 1$, and σ controls the degree to which the inputs can be substituted for one another. The Leontief production function ($\sigma = 0$) is

$$\frac{y}{\bar{y}} = \min_i \left\{ \gamma_i \frac{x_i}{\bar{x}_i} \right\},$$

the Cobb-Douglas production function ($\sigma = 1$) is

$$\frac{y}{\bar{y}} = \prod_i \left(\gamma_i \frac{x_i}{\bar{x}_i} \right)^{\theta_i},$$

Table 3.1: Variables in the simple dimensionless example.

$\mathbf{p}_m$	change in material price
$\mathbf{p}_e$	change in energy price
$\mathbf{p}_K$	change in energy price
$\mathbf{p}_L$	change in energy price
$\mathbf{y}_m$	change in quantity of material supplied by material industry
$\mathbf{y}_e$	change in quantity of energy supplied by energy industry
$\mathbf{y}_K$	change in quantity of capital supplied by consumer
$\mathbf{y}_L$	change in quantity of labor supplied by consumer
$\mathbf{x}_m^m$	change in quantity of material demanded by material industry
$\mathbf{x}_e^m$	change in quantity of energy demanded by material industry
$\mathbf{x}_K^m$	change in quantity of capital demanded by material industry
$\mathbf{x}_L^m$	change in quantity of labor demanded by material industry
$\mathbf{x}_K^e$	change in quantity of capital demanded by energy industry
$\mathbf{x}_L^e$	change in quantity of labor demanded by energy industry
$\mathbf{x}_m^c$	change in quantity of material demanded by consumer
$\mathbf{x}_e^c$	change in quantity of energy demanded by consumer

and the linear production function ($\sigma = \infty$) is

$$\frac{y}{\bar{y}} = \sum_i \theta_i \gamma_i \frac{x_i}{\bar{x}_i} \, .$$

As before, these functions are typically combined in a nested fashion where each nest describes the substitutability among commodity bundles.

The optimization problems solved by the industries and consumers and the market clearing conditions are then expressed in terms of the dimensionless variables

$$\mathbf{p} = \frac{p}{\bar{p}} \, , \quad \mathbf{x} = \frac{x}{\bar{x}} \, , \quad \mathbf{y} = \frac{y}{\bar{y}} \, .$$

These dimensionless variables represent the change in prices and quantities from their base-year values. The share parameters are then calibrated so that in the base year $\mathbf{p} = 1$, $\mathbf{x} = 1$, and $\mathbf{y} = 1$. That is, we choose shares that replicate the base-year data. These share parameters are conveniently expressed strictly in terms of expenditures. Thus, the calibrated share form of the production and utility functions alludes to a desirable linear change of variables for the model that leaves all endogenous variables and calibration parameters dimensionless.

3.1 Calibrated Form of Simple Instance

We now apply this change of variables to the simple instance. In particular, we make the substitutions $p = \mathbf{p}\bar{p}$, $y = \mathbf{y}\bar{y}$, and $x = \mathbf{x}\bar{x}$. We then define $\bar{r} = \bar{p}\bar{y}$ and $\bar{e} = \bar{p}\bar{x}$, the base-year revenues and expenditures, respectively. The variables in the calibrated form are described in Table 3.1 and the parameters in Table 3.2.

3.1.1 Industries

Recall that the material industry is solving the profit maximization problem,

$$\begin{aligned}
&\max_{y_m \geq 0, x_i^m \geq 0} \; p_m y_m - p_m x_m^m - p_e x_e^m - p_K x_K^m - p_L x_L^m \\
&\text{s.t.} \quad y_m \leq \left(\alpha_{KL}^m (x_{KL}^m)^{\rho^m} + \alpha_{me}^m (x_{me}^m)^{\rho^m} \right)^{\frac{1}{\rho^m}} \\
&\qquad x_{KL}^m \leq \left(\alpha_K^m (x_K^m)^{\rho_{KL}^m} + \alpha_L^m (x_L^m)^{\rho_{KL}^m} \right)^{\frac{1}{\rho_{KL}^m}} \\
&\qquad x_{me}^m \leq \left(\alpha_m^m (x_m^m)^{\rho_{me}^m} + \alpha_e^m (x_e^m)^{\rho_{me}^m} \right)^{\frac{1}{\rho_{me}^m}} \, .
\end{aligned}$$

Table 3.2: Parameters in the simple dimensionless example.

σ^m_{me}	elasticity of substitution among material and energy for material industry
σ^m_{KL}	elasticity of substitution among capital and labor for material industry
σ^m	elasticity of substitution among (material, energy) and (capital, labor) bundles
σ^e_{KL}	elasticity of substitution among capital and labor for energy industry
σ^c_{me}	elasticity of substitution among material and energy for consumer
σ^c	elasticity of substitution among (material, energy) bundle and savings for consumer
$\bar{e}^m_m$	base-year expenditure on material demanded by material industry
$\bar{e}^m_e$	base-year expenditure on energy demanded by material industry
$\bar{e}^m_K$	base-year expenditure on capital demanded by material industry
$\bar{e}^m_L$	base-year expenditure on labor demanded by material industry
$\bar{e}^e_K$	base-year expenditure on capital demanded by energy industry
$\bar{e}^e_L$	base-year expenditure on labor demanded by energy industry
$\bar{e}^c_m$	base-year expenditure on material demanded by consumer
$\bar{e}^c_e$	base-year expenditure on energy demanded by consumer
$\bar{e}^c_s$	base-year expenditure on share parameter for savings demanded by consumer
$\bar{r}_m$	base-year revenue from sales of material
$\bar{r}_e$	base-year revenue from sales of energy
$\bar{r}_K$	base-year revenue from sales of capital
$\bar{r}_L$	base-year revenue from sales of labor

Performing the change of variables, we obtain the equivalent optimization problem,

$$\begin{aligned}
&\max_{\mathbf{y}_m \bar{y}_m \geq 0, \mathbf{x}^m_i \bar{x}^m_i \geq 0} \; \mathbf{p}_m \bar{p}_m \mathbf{y}_m \bar{y}_m - \mathbf{p}_m \bar{p}_m \mathbf{x}^m_m \bar{x}^m_m - \mathbf{p}_e \bar{p}_e \mathbf{x}^m_e \bar{x}^m_e - \mathbf{p}_K \bar{p}_K \mathbf{x}^m_K \bar{x}^m_K - \mathbf{p}_L \bar{p}_L \mathbf{x}^m_L \bar{x}^m_L \\
&\text{s.t.} \quad \mathbf{y}_m \bar{y}_m \leq \left(\alpha^m_{KL} (\mathbf{x}^m_{KL} \bar{x}^m_{KL})^{\rho^m} + \alpha^m_{me} (\mathbf{x}^m_{me} \bar{x}^m_{me})^{\rho^m} \right)^{\frac{1}{\rho^m}} \\
&\qquad \mathbf{x}^m_{KL} \bar{x}^m_{KL} \leq \left(\alpha^m_K (\mathbf{x}^m_K \bar{x}^m_K)^{\rho^m_{KL}} + \alpha^m_L (\mathbf{x}^m_L \bar{x}^m_L)^{\rho^m_{KL}} \right)^{\frac{1}{\rho^m_{KL}}} \\
&\qquad \mathbf{x}^m_{me} \bar{x}^m_{me} \leq \left(\alpha^m_m (\mathbf{x}^m_m \bar{x}^m_m)^{\rho^m_{me}} + \alpha^m_e (\mathbf{x}^m_e \bar{x}^m_e)^{\rho^m_{me}} \right)^{\frac{1}{\rho^m_{me}}} .
\end{aligned}$$

After simplification, we arrive at the calibrated form of the optimization problem,

$$\begin{aligned}
&\max_{\mathbf{y}_m \geq 0, \mathbf{x}^m_i \geq 0} \; \bar{r}_m \mathbf{p}_m \mathbf{y}_m - \bar{e}^m_m \mathbf{p}_m \mathbf{x}^m_m - \bar{e}^m_e \mathbf{p}_e \mathbf{x}^m_e - \bar{e}^m_K \mathbf{p}_K \mathbf{x}^m_K - \bar{e}^m_L \mathbf{p}_L \mathbf{x}^m_L \\
&\text{s.t.} \quad \mathbf{y}_m \leq \left(\theta^m_{KL} (\mathbf{x}^m_{KL})^{\rho^m} + \theta^m_{me} (\mathbf{x}^m_{me})^{\rho^m} \right)^{\frac{1}{\rho^m}} \\
&\qquad \mathbf{x}^m_{KL} \leq \left(\theta^m_K (\mathbf{x}^m_K)^{\rho^m_{KL}} + \theta^m_L (\mathbf{x}^m_L)^{\rho^m_{KL}} \right)^{\frac{1}{\rho^m_{KL}}} \\
&\qquad \mathbf{x}^m_{me} \leq \left(\theta^m_m (\mathbf{x}^m_m)^{\rho^m_{me}} + \theta^m_e (\mathbf{x}^m_e)^{\rho^m_{me}} \right)^{\frac{1}{\rho^m_{me}}} ,
\end{aligned}$$

where, for example, $\theta^m_K = \alpha^m_K \left(\frac{\bar{x}^m_K}{\bar{x}^m_{KL}} \right)^{\rho^m_{KL}}$ and $\theta^m_L = \alpha^m_L \left(\frac{\bar{x}^m_L}{\bar{x}^m_{KL}} \right)^{\rho^m_{KL}}$.

The same procedure is used to obtain the energy industry optimization problem,

$$\begin{aligned}
&\max_{\mathbf{y}_e \geq 0, \mathbf{x}^e_i \geq 0} \; \bar{r}_e \mathbf{p}_e \mathbf{y}_e - \bar{e}^e_K \mathbf{p}_K \mathbf{x}^e_K - \bar{e}^e_L \mathbf{p}_L \mathbf{x}^e_L \\
&\text{s.t.} \quad \mathbf{y}_e \leq \left(\theta^e_K (\mathbf{x}^e_K)^{\rho^e_{KL}} + \theta^e_L (\mathbf{x}^m_L)^{\rho^m_{KL}} \right)^{\frac{1}{\rho^m_{KL}}} .
\end{aligned}$$

3.1.2 Consumers

After substitution and simplification, the utility maximization problem solved by each consumer becomes

$$
\begin{aligned}
&\max_{0\le \mathbf{y}_j^c\le 1, \mathbf{x}_i^c\ge 0} \quad \mathbf{x}^c \\
&\text{s.t.} \quad \mathbf{x}^c \le \left(\theta_S^c(\mathbf{x}_S^c)^{\rho^c} + \theta_{me}^c(\mathbf{x}_{me}^c)^{\rho^c}\right)^{\frac{1}{\rho^c}} \\
&\qquad \mathbf{x}_{me}^c \le \left(\theta_m^c(\mathbf{x}_m^c)^{\rho_{me}^c} + \theta_e^c(\mathbf{x}_e^c)^{\rho_{me}^c}\right)^{\frac{1}{\rho_{me}^c}} \\
&\qquad \bar{e}_m^c\mathbf{p}_m\mathbf{x}_m^c + \bar{e}_e^c\mathbf{p}_e\mathbf{x}_e^c + \bar{e}_S^c\mathbf{x}_S^c \le \bar{r}_L^c\mathbf{p}_L\mathbf{y}_L^c + \bar{r}_K^c\mathbf{p}_K\mathbf{y}_K^c + \Pi_m + \Pi_e\,,
\end{aligned}
$$

where Π_m and Π_e are the material and energy industry profits returned to the consumer as a dividend, respectively.

3.1.3 Markets

A direct substitution in the market clearing conditions yields the following complementarity constraints.

$$
\begin{array}{lcl}
0 \le \mathbf{p}_m\bar{p}_m & \perp & \bar{y}_m\mathbf{y}_m \ge \bar{x}_m^m\mathbf{x}_m^m + \bar{x}_m^c\mathbf{x}_m^c \\
0 \le \mathbf{p}_e\bar{p}_e & \perp & \bar{y}_e\mathbf{y}_e \ge \bar{x}_e^m\mathbf{x}_e^m + \bar{x}_e^c\mathbf{x}_e^c \\
0 \le \mathbf{p}_L\bar{p}_L & \perp & \bar{y}_L\mathbf{y}_L \ge \bar{x}_L^m\mathbf{x}_L^m + \bar{x}_L^e\mathbf{x}_L^e \\
0 \le \mathbf{p}_K\bar{p}_K & \perp & \bar{y}_K\mathbf{y}_K \ge \bar{x}_K^m\mathbf{x}_K^m + \bar{x}_K^e\mathbf{x}_K^e
\end{array}
$$

We then divide the left inequality by $\bar{p}$ and multiply the right inequality by $\bar{p}$ so that all the prefactors can be expressed as revenues and expenditures. This multiplication is possible because we know in the base year that all prices are positive. The market conditions are then

$$
\begin{array}{lcl}
0 \le \mathbf{p}_m & \perp & \bar{r}_m\mathbf{y}_m \ge \bar{e}_m^m\mathbf{x}_m^m + \bar{e}_m^c\mathbf{x}_m^c \\
0 \le \mathbf{p}_e & \perp & \bar{r}_e\mathbf{y}_e \ge \bar{e}_e^m\mathbf{x}_e^m + \bar{e}_e^c\mathbf{x}_e^c \\
0 \le \mathbf{p}_L & \perp & \bar{r}_L\mathbf{y}_L \ge \bar{e}_L^m\mathbf{x}_L^m + \bar{e}_L^e\mathbf{x}_L^e \\
0 \le \mathbf{p}_K & \perp & \bar{r}_K\mathbf{y}_K \ge \bar{e}_K^m\mathbf{x}_K^m + \bar{e}_K^e\mathbf{x}_K^e\,.
\end{array}
$$

This modification allows us to construct a model without any reference to base-year prices.

3.1.4 Calibration

Assuming revenues equal expenditures in all industry objective functions, consumer budget constraints, and market clearing conditions, we can choose values for the share parameters so that $\mathbf{p} = 1$, $\mathbf{y} = 1$, and $\mathbf{x} = 1$ solve the general equilibrium problem. That is, the prices and quantities do not deviate from their base-year levels. This process of choosing the share parameters based on base-year data is referred to as calibration to a base year.

In particular, by choosing

$$
\begin{array}{lcl@{\qquad}lcl}
\theta_m^m & = & \frac{\bar{e}_m^m}{\bar{e}_m^m+\bar{e}_e^m} & \theta_K^e & = & \frac{\bar{e}_K^e}{\bar{e}_K^e+\bar{e}_L^e} \\
\theta_e^m & = & \frac{\bar{e}_e^m}{\bar{e}_m^m+\bar{e}_e^m} & \theta_L^e & = & \frac{\bar{e}_L^e}{\bar{e}_K^e+\bar{e}_L^e} \\
\theta_K^m & = & \frac{\bar{e}_K^m}{\bar{e}_K^m+\bar{e}_L^m} & \theta_K^c & = & \frac{\bar{e}_K^c}{\bar{e}_K^c+\bar{e}_L^c} \\
\theta_L^m & = & \frac{\bar{e}_L^m}{\bar{e}_K^m+\bar{e}_L^m} & \theta_L^c & = & \frac{\bar{e}_L^c}{\bar{e}_K^c+\bar{e}_L^c} \\
\theta_{KL}^m & = & \frac{\bar{e}_{KL}^m}{\bar{e}_{KL}^m+\bar{e}_{me}^m} & \theta_{me}^c & = & \frac{\bar{e}_{me}^c}{\bar{e}_{me}^c+\bar{e}_S^c} \\
\theta_{me}^m & = & \frac{\bar{e}_{me}^m}{\bar{e}_{me}^m+\bar{e}_{me}^m} & \theta_S^c & = & \frac{\bar{e}_S^c}{\bar{e}_{KL}^c+\bar{e}_S^c}\,,
\end{array}
$$

where $\bar{e}_{KL}^m = \bar{e}_K^m + \bar{e}_L^m$, $\bar{e}_{me}^m = \bar{e}_m^m + \bar{e}_e^m$, and $\bar{e}_{KL}^c = \bar{e}_K^c + \bar{e}_L^c$, one can show that $\mathbf{p} = 1$, $\mathbf{y} = 1$, and $\mathbf{x} = 1$ is a solution to the resulting general equilibrium problem. This calibration procedure is automated in our framework.

3.2 Taxes and Subsidies

Taxes are an important part of an economy and any general equilibrium model. Import and export taxes play an important role in determining the sizes of bilateral trade flows; domestic taxes can reallocate economic activity into more socially advantageous efforts; environmental taxes can be levied to encourage carbon-neutral behaviors and slow the emission of CO_2 and other harmful pollutants into the atmosphere. Here we detail how taxes are included in the production, consumption, and investment equations. Subsidies are simply a negative tax rate. Tax revenues are aggregated into region-specific tax accounts and are rebated to consumers.

3.2.1 Ad Valorem Taxes

Each agent in the model pays a tax on value of the goods and factors demanded; the tax rate on the good merely rescales the expenditure terms in the optimization problems. Using the material industry from the simple example, the expenditure on energy becomes

$$(1 + s_e^m)p_e x_e^m \,,$$

where s_e^m is the tax rate on energy for the material industry. When we switch to the dimensionless model, we obtain

$$(\bar{e}_e^m + \bar{s}_e^m)\mathbf{p}_e \mathbf{x}_e^m \,,$$

where $\bar{s}_e^m = s_e^m \bar{p}_e \bar{x}_e^m$ is the ad valorem tax expenditure for the material producer on energy inputs. Notice that this formulation is especially simple if we consider a constant tax rate, since the expenditure prefactor is then the sum of all the base-year expenditures, including tax expenditures, on the input.

3.2.2 Excise Taxes

Some taxes, such as the federal gasoline tax, are applied to volumes rather than value. The treatment of these excise taxes is slightly different from that of ad valorem taxes. Using the material industry from the simple example, we calculate the expenditure on energy with an excise tax as

$$(p_e + t_e^m)x_e^m \,,$$

where t_e^m is the tax rate per unit of energy for the material industry. When we switch to the dimensionless model, we obtain

$$(\bar{e}_e^m \mathbf{p}_e + \bar{t}_e^m)\mathbf{x}_e^m \,,$$

where $\bar{t}_e^m = t_e^m \bar{x}_e^m$ is the excise tax expenditure for the material producer for energy inputs. The distinction between ad valorem and excise taxes matters only as the commodity prices stray from unity, and the difference is strongly dependent on the excise tax rate. For example, if the price of a commodity taxed in the base year at 10% doubles, then the tax revenues would be off by approximately 5% using the incorrect tax representation.

3.2.3 Production Taxes

Production taxes are taxes paid by industries on the goods they produce. Using the material industry from the simple example, we calculate the revenue from material production as

$$((1 - s_m^m)p_m - t_m^m)y_m \,,$$

where s_m^m and t_m^m are the ad valorem and excise tax rates, respectively. When we switch to the dimensionless model, we obtain

$$((\bar{r}_m^m - \bar{s}_m^m)\mathbf{p}_m - \bar{t}_m^m)\mathbf{y}_m \,,$$

where $\bar{s}_m^m$ and $\bar{t}_m^m$ are the ad valorem and excise tax expenditure for the material producer on their outputs, respectively. Excise production taxes may be needed to study the effects of a producer-level carbon tax. For example, if were to charge an emissions tax based on the amount of coal mined rather than the amount of coal burned to generate energy, then we would need an excise production tax on mined coal. We do not expect carbon to be priced in this way, but the analysis extends to this case.

3.2.4 Income Taxes

Income taxes are subtracted from the consumer incomes at the point of payment. These taxes have the same form as production taxes but are levied on the consumer revenue terms. Using consumer capital from the simple example, we calculate the modified revenue term as

$$((1 - s^c_K)p_K - t^c_K)y_K \,,$$

where s^c_K and t^c_K are the ad valorem and excise tax rates, respectively. When we switch to the dimensionless model, we obtain

$$((\bar{r}^c_K - \bar{s}^c_K)\mathbf{p}_K - \bar{t}^c_K)\mathbf{y}_K \,,$$

where $\bar{s}^c_K$ and $\bar{t}^c_K$ are the ad valorem and excise tax expenditures, respectively. We note that while excise taxes on labor and capital are not likely to be imposed, the modeling framework allows their inclusion.

3.2.5 Carbon Taxes

Carbon taxes are excise taxes placed on the inputs and outputs of producers and consumers. Since in general no data is available for industry expenditures on carbon emissions in the base year because emissions are free in most of the world, we use parameters based on energy use volume data.

Using the material industry from the simple example, we calculate the expenditure on energy with a carbon tax as

$$(p_e + t^m_e f^m_e)x^m_e \,,$$

where t^m_e is the tax rate per emissions unit and f^m_e is an emissions factor that converts from energy units to emissions units. When we switch to the dimensionless version, we obtain

$$(\bar{e}^m_e \mathbf{p}_e + t^m_e \bar{f}^m_e)\mathbf{x}^m_e \,,$$

where $\bar{f}^m_e = f^m_e \bar{x}^m_e$ is the base-year emissions generated by the material industry from the use of energy.

3.2.6 Import and Export Duties

For international trade, we treat domestic and imported goods as distinct products. Each region contains an importer for each commodity that buys goods internationally and sells them domestically. Import and export duties are paid by the importers in each of the regions. Because the importer inputs commodities from many regions, we need to distinguish between import and export duties because the duties are paid at the destination or origination points, respectively. That is, we must distribute the revenue to the correct region. Using the material commodity from the simple example, we calculate the expenditures for the material importer in region r' as

$$(1 + s^{i,r'}_{m,r} + s^{e,r'}_{m,r})p_{m,r}x^{i,r'}_{m,r} \,,$$

where $s^{i,r'}_{m,r}$ is the ad valorem import duty rate for material imported from region r and $s^{e,r'}_{m,r}$ is the ad valorem export duty rate for material exported by region r. When we switch to the dimensionless model, we obtain

$$(\bar{e}^{i,r'}_{m,r} + \bar{s}^{i,r'}_{m,r} + \bar{s}^{e,r'}_{m,r})\mathbf{p}_{m,r}\mathbf{x}^{i,r'}_{m,r} \,,$$

where $\bar{s}^{i,r'}_{m,r}$ is the ad valorem import duty amount for material imported from region r and $\bar{s}^{e,r'}_{m,r}$ is the ad valorem export duty amount for material exported by region r.

Excise import and export duties can also be modeled in this framework and used for border taxes on emissions. Using the same importer, we calculate the border-tax adjustment as

$$(p_{m,r} + t^{i,r'}_{m,r}f^{i,r'}_{m,r} + t^{e,r'}_{m,r}f^{e,r'}_{m,r})x^{i,r'}_{m,r} \,,$$

where $t^{i,r'}_{m,r}$ and $t^{e,r'}_{m,r}$ are the import and export duties per emissions unit, respectively, and $f^i_{m,r}$ and $f^e_{m,r}$ are emission factors that convert from material units to emissions units for imports and exports, respectively. There is considerable discrepancy in how one measures these emissions factors, given the difficult carbon accounting because the commodity in question may not be produced in the taxing region. When we switch to the dimensionless model, we obtain

$$(\bar{e}^{i,r'}_{m,r}\mathbf{p}_{m,r} + t^{i,r'}_{m,r}\bar{f}^{i,r'}_{m,r} + t^{e,r'}_{m,r}\bar{f}^{e,r'}_{m,r})\mathbf{x}^{i,r'}_{m,r}\,,$$

where $\bar{f}^{i,r'}_{m,r} = f^{i,r'}_{m,r}\bar{x}^{i,r'}_{m,r}$ and $\bar{f}^{e,r'}_{m,r} = f^{e,r'}_{m,r}\bar{x}^{i,r'}_{m,r}$ are the base-year emissions for the material importer.

3.2.7 Endogenous Tax Rates

Endogenous taxes rates are required to implement cap-and-trade policies. In this case, the tax rate is determined by the model so that the cap is not violated. We will consider an endogenous carbon emissions tax, but other endogenous taxes can be added to the model. The mechanism for setting the rate is to create a market for emissions with a fixed supply. The price of emissions is then determined so that the demand for emissions does not exceed the supply.

In the simple example, an endogenous tax on the emissions from energy consumption introduces the constraints

$$0 \le t_e \quad \perp \quad F_e \ge f^m_e x^m_e + f^c_e x^c_e\,,$$

where t_e is the endogenous tax rate on energy, F_e is the cap on emissions from energy, f^m_e and f^c_e are the emission factors for the material industry and the consumer, respectively, and x^m_e and x^c_e are their demands for energy. When we switch to the dimensionless model, we obtain

$$0 \le t_e \quad \perp \quad F_e \ge \bar{f}^m_e \mathbf{x}^m_e + \bar{f}^c_e \mathbf{x}^c_e\,,$$

where $\bar{f}^m_e = f^m_e \bar{x}^m_e$ and $\bar{f}^c_e \bar{x}^c_e$ are the base-year emissions generated by the material industry and consumer from the use of energy, respectively.

In a calibrated model having no endogenous carbon tax in the base year, we set $F_e = \bar{f}^m_e + \bar{f}^c_e$. Analysis of the entire general equilibrium problem shows that the tax rate in the base year is then zero. By using a fraction of the base-year emissions, a positive tax rate is obtained.

3.3 Myopic Dynamics

The simplest dynamic general equilibrium models are *myopic*, in which the agents look only at their current state and do not consider the future. In this case, we solve a sequence of static general equilibrium models with dynamic trajectories for the factor endowments and efficiency units. The primary drivers of economic development are capital accumulation, labor productivity, and resource usage. These dynamic phenomena can be represented in a variety of ways, and these add still more parameters to the model that must be ground in historical data. For the prototype model we have constructed exogenous time-series forecasts of important economic drivers by extrapolation from historical data, with forecasts constrained by physical restrictions such as expected availability of fossil-fuel reserves.

4 CIM-EARTH v0.1

We now describe in some detail the full CIM-EARTH v0.1 prototype we have built for testing and development. The model is written in the AMPL modeling language [8] and is available for download at `http://cim-earth.org`. Data from initial runs can be explored at the website by using simple interactive visualization tools such as Google Maps.

4.1 Regions and Sectors

The regional and sectoral resolution of the CIM-EARTH v0.1 is shown in Table 4.1. This particular aggregation was chosen to study carbon leakage, the impact of a unilateral carbon emissions policy on the global movement of industrial production capacity away from that region. Therefore, the model contains more detailed resolution in the energy-intensive industrial sectors and in the sectors that provide transport services to importers to move goods around the world. A map of the regional aggregation is shown in Figure 4.1.

Table 4.1: Aggregate regions and sectors for the 16×16 model used here.

16 Regions	**16 Sectors** (per region)
Oceania	Agriculture and forestry
Southeast Asia	Coal
Japan	Oil
Rest of East Asia	Natural gas
India	Iron and Steel
Rest of South Asia	Chemicals
Russia, Georgia, & Asiastan	Nonferrous metals
Middle East & N. Africa	Cement/mineral products
Sub-Saharan Africa	Other manufacturing
Western Europe	Refined petroleum
Rest of Europe	Electricity
Brazil	Land transport
Mexico	Air transport
Rest of Latin America	Sea transport
USA	Government services
Canada	Other services

This model does not contain a government consumer; it contains only a producer of government goods and services, which include defense, social security, and education. Industries and consumers demand these government goods and services. The government producer is treated like any other producer and can be subject to ad valorem and excise taxes. All taxes collected by a region are returned to consumers in that region.

Trade among regions is handled through importers of each commodity in each region. The importers buy commodities both domestically and internationally, transport these commodities, and sell the imports to producers and consumers. Domestic production and imports are treated as separate commodities that are traded in different markets. We use three types of transportation: land transportation including freight by trucks and pipelines, air transportation, and sea transportation. Each of these transport services is an industry/commodity in the model. Since the importers do not care about the origination of the transport services, however, we model international transportation as a homogeneous commodity with one global price, and each region supplies some transportation services.

4.2 Production Functions

The production functions in each region have the nested structure shown in Figure 4.2. As before, each node represents a CES function aggregating the production factor branches coming into it from below. The importers are modeled like other producers using the nested CES production function shown at the bottom right of Figure 4.2. We use a Leontief production function to aggregate between the imported good and the relevant total transport margin so that the amount of transport demanded scales with the amount of the good imported. We use a subnest to represent the importer use of air, land, and sea transport with a small elasticity of substitution, $\sigma = 0.2$.

The elasticities of substitution we use have been taken from the computable general equilibrium literature, in particular, from the MIT EPPA model [13] and the GTAP model [11]. Our general strategy for treating widely uncertain parameters such as substitution elasticities is to explore the full range of possible values in parameter sweeps and to work toward improved understanding of these parameters in an evolutionary fashion.

The base-year revenues and expenditures are all reported in the GTAP database. These values are used to calibrate the model to the base year. In particular, our share parameters are calibrated with the GTAP version 7 database of global expenditure values [10]. Emission amounts are obtained from the energy volume information in GTAP-E [3].

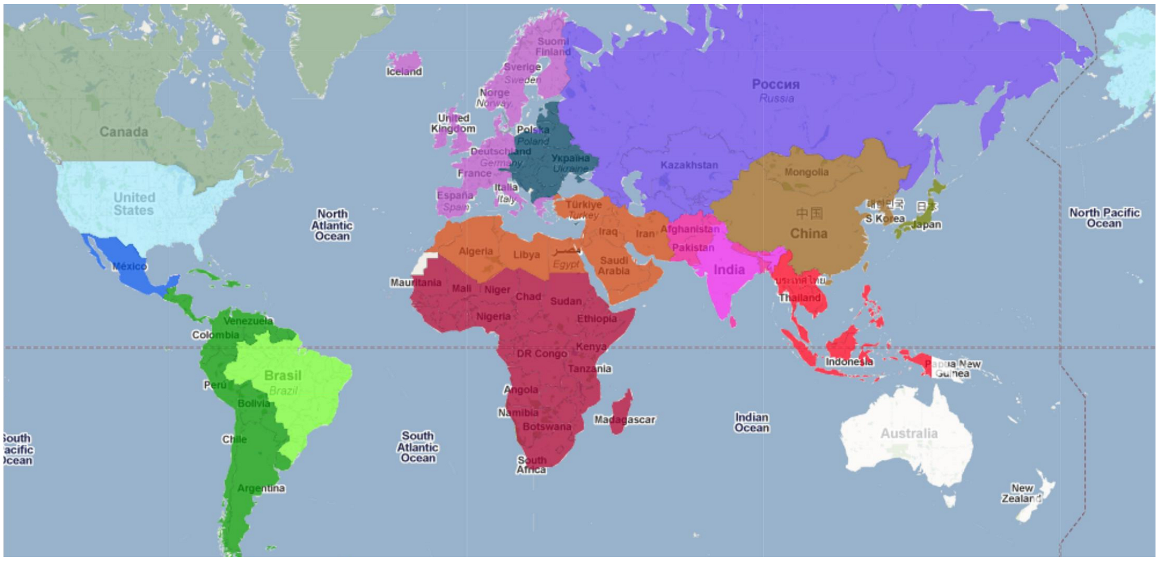

Figure 4.1: Regional aggregation of the CIM-EARTH v0.1 prototype.

4.3 Dynamic Trajectories

We currently solve a myopic general equilibrium model with dynamic trajectories for the factor endowments and efficiency units. The primary drivers of economic development are capital accumulation, labor productivity, and resource usage. For the CIM-EARTH v0.1 we have constructed exogenous time-series forecasts of important economic drivers by extrapolation from historical data, with forecasts constrained by physical restrictions such as expected availability of fossil-fuel reserves. Thus far we have prototyped exogenous statistical trend forecasts for labor endowment, labor productivity, energy efficiency, agricultural land endowment, land productivity (yield), and availability of fossil-fuel resources and extraction technology. These simple dynamics provide a stable basis for the current round of model testing. Several examples are described in more detail below.

4.3.1 Capital Accumulation

Our current model uses a basic, perfectly fluid model of capital with a 4% yearly depreciation rate. In this case, investment is modeled as a contribution to the present state consumer utility, with the investment amounts calibrated to historical data. Investment enters the consumer utility function in a Cobb-Douglas nest with the government services and consumption bundles, implying that a fixed share of consumer income in each year goes to government services, investment, and consumption.

Consumer investment increases the capital stock through a sector that produces capital goods, which are demanded by the consumer. This sector behaves as any other, demanding material goods and services in order to produce the capital good. This sector does *not* demand capital, labor, or energy. By far the largest expenditure of the capital goods sector is on construction services, reflecting the fact that most capital is buildings, with sizable demands also from various industrial sectors, such as machinery, transport equipment, and computing equipment.

The capital endowment in the next period is obtained from the dynamic equation

$$\bar{y}^c_{K,t+1} = (1-\delta)\bar{y}^c_{K,t} + x^c_{I,t}\,,$$

where the capital depreciation rate, δ, is exogenously specified. The amount of capital in the base year is calibrated to data. When we switch to the dimensionless version using the base-year data, we have the dynamic equation

$$\bar{\mathbf{y}}^c_{K,t+1} = (1-\delta)\bar{\mathbf{y}}^c_{K,t} + \frac{\bar{x}^c_{I,0}}{\bar{y}^c_{K,0}}\mathbf{x}_{I,t}$$

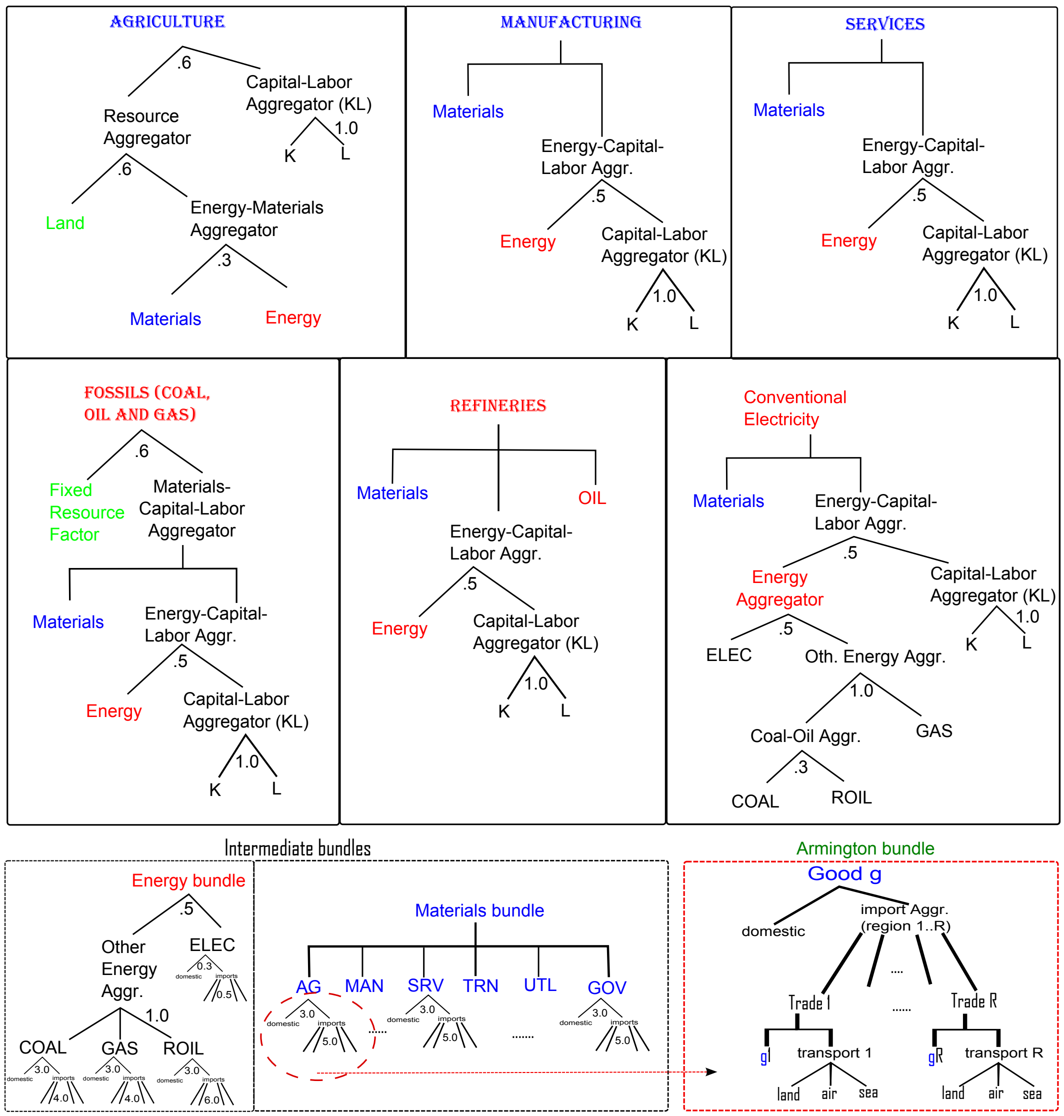

Figure 4.2: Full model production functions. Each node of the tree represents a CES production function with the given elasticity of substitution. Nodes with vertical line factor inputs use a zero elasticity CES function (Leontief function). Most other nodes are labeled with their elasticities.

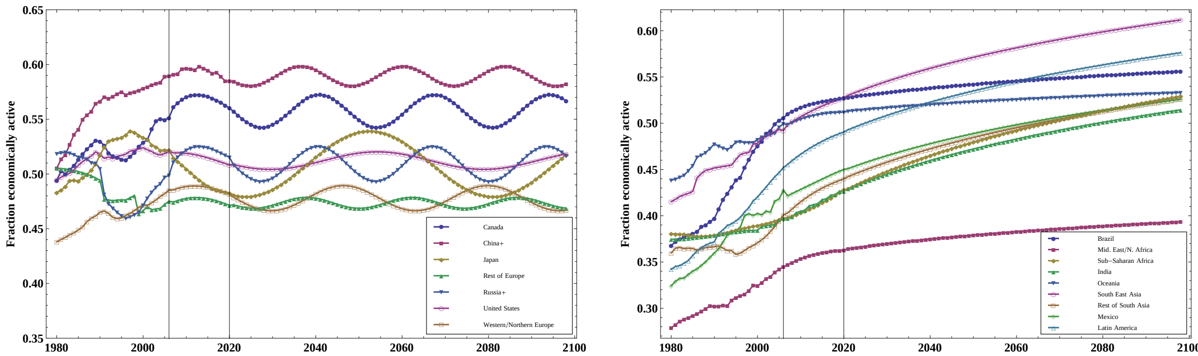

Figure 4.3: Regional economic activity rates; historical data 1980-2006 is complemented by expert ILO forecasts to 2020, and basic statistical extrapolations beyond

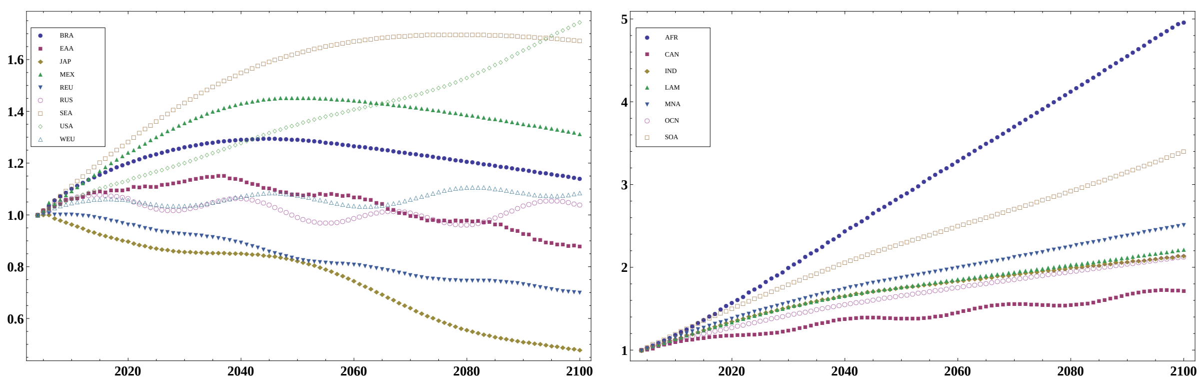

Figure 4.4: Regional labor endowment (population time participation rate).

with the boundary condition $\bar{\mathbf{y}}^c_{K,0} = 1$. The ratio in the dimensionless equation is available from data.

4.3.2 Labor Productivity

Another primary driver of economic growth is the growth in population, and hence the supply of available labor, and growth in the productivity of labor. In the current model prototype, population data from 1950 to 2008, with forecasts to 2050, is taken from the 2008 UN population database [15], extrapolated to 2100 at full UN regional resolution, and aggregated to the model regions. Historical economic activity rates, the fraction of people that participate in the economy either with a job or looking for a job, from 1980 to 2006 are taken from the ILO [12], along with projections to 2020, aggregated to the model regions, and extrapolated to 2100. Figure 4.3 shows the activity rates data for the aggregated regions, along with the ILO forecasts to 2020 and the extrapolations to 2100.

To most easily continue the ILO activity rate forecasts with continuous and relatively smooth extrapolations, we have used a simple one-frequency oscillating function for countries in which the ILO forecast model predicts behavior that is not monotonically increasing. This choice is merely for convenience and should not be interpreted as sourced from any special knowledge of the rate of economic participation in these regions. Further, we note that the small fluctuations from the long-run mean have no substantial impact in simulation output. We intend to replace these deterministic fluctuations with stochastic variation calibrated to historical variance. We will also replace these highly aggregated activity rates with detailed forecasts of male and female participation rates in five-year age groups. These then will be combined to estimate the endowment forecasts in Figure 4.4.

Increasing productivity is modeled by inclusion of a productivity coefficient in the industrial production

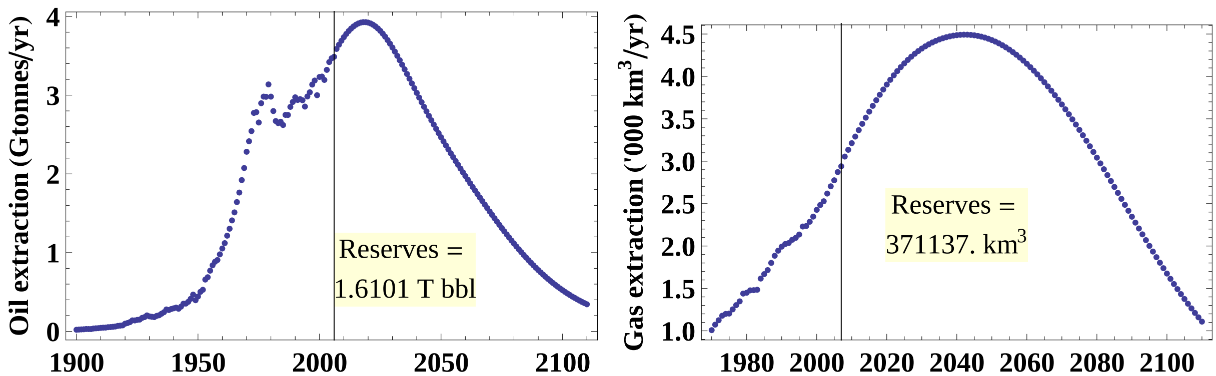

Figure 4.5: World conventional oil (left) and gas (right) depletion profile.

function. For labor productivity, a basic capital labor aggregate in the production process takes the form

$$\mathbf{x}_{KL} \leq (\theta_K(\mathbf{x}_K)^{\rho_{KL}} + \theta_L(\gamma_L(t)\mathbf{x}_L)^{\rho_{KL}})^{\frac{1}{\rho_{KL}}} ,$$

where $\gamma_L(t)$ is the labor productivity in year t and is tuned exogenously to match forecasts extrapolated from historical trends. For this preliminary description, data has been taken from the U.S. Bureau of Labor Statistics International Labor Comparisons Division and from the OECD stats division. The data covers only a subset of countries and economic regions, so that some assumptions have to be made about the approximate similarity among regions. We extrapolate the year-on-year growth rates in labor productivity to have constant mean and variance consistent with the historical record. For ease of use, we again use a simple one-frequency oscillation in place of stochastic fluctuations. The fluctuations again prove very small relative to long-term averages.

4.3.3 Resource Usage

In this section, we discuss the way we model crude fossil-fuel extraction, reserves, deplection, and backstops, which are vital in understanding how energy demand is met. Based on a simple fossil-fuel resource depletion model, we forecast Gaussian extraction curves fit to historical data for model regions independently, constrained to give future extraction equal to existing fossil-fuel reserves. Since the quantity of ultimately recoverable fossil-fuel resources is widely disputed, we treat this quantity as the primary point of uncertainty and we will explore the economic impacts of a wide range of reserve assumptions. This model combines forecasts of new reserve discoveries with advancing extraction technologies to predict extraction availability. This representation is easy for conventional fossil-fuel industries because the technologies are already well developed and detailed, global historical data already exists. For nascent energy technologies, however, such as shale, tar sands, wind, solar, and biofuels, modeling is a much greater challenge and requires an engineering-based approach. Figure 4.5 shows the sum of all regional extrapolations for oil and gas resources. The remaining global conventional crude oil in this trajectory is about 1.6 trillion barrels (Tbbl), which is near the median of expert estimates in the standard literature. The 2007 WEC Survey of Energy Resources [4] estimates global remaining resources of conventional crude plus proved reserves at about 1.8 Tbbl, though questions remain as to how much will be ultimately extractable. We have used simple, symmetric curves for these fits, implying a smooth fall-off of extraction rates as reserves are depleted. The remaining global conventional gas in this trajectory is about 371 trillion m^3, which is very near the 2007 WEC estimate of 386 trillion m^3.

Forecasts for coal depletion are more ambiguous, with a high ratio of estimated resources to proved reserves (5-5.5, much higher than that for gas or oil at 1.2 and 0.5, respectively), and serious questions about what percentage of this will be technologically recoverable and at what rates. Also essential, from the environmental and demand side, are representations of clean-coal technology developments and deployment (such as integrated gasification combined cycle), whether from government investment or from market forces

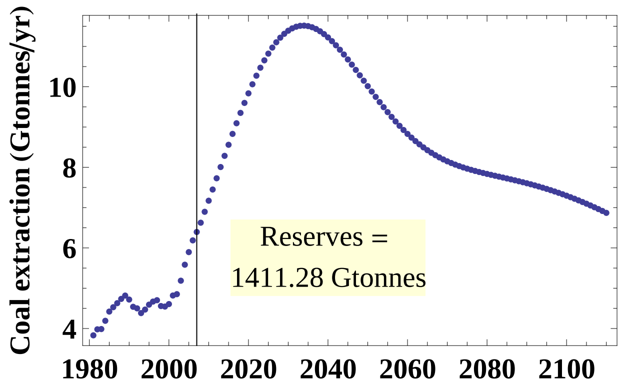

Figure 4.6: World coal depletion forecast.

stimulated by CO_2 and other pollutant pricing regimes. We leave these developments for future model versions.

Figure 4.6 shows the sum of regional coal resource extrapolations for the lowest level of ultimately recoverable reserve estimates used in this model version. This particular forecast is at the very low end of estimated recoverable resources at 1.4 trillion tonnes of ultimately extractable coal resource remaining in the ground. This amounts to an assumption of only about 25% of the existing coal resources being ultimately recoverable. This is at the very low end of any expert forecasts and should be considered a very unlikely, though not impossible, outcome.

Acknowledgments

This work was supported by grants from the MacArthur Foundation and the University of Chicago Energy Initiative, and by the Office of Advanced Scientific Computing Research, Office of Science, U.S. Department of Energy, under Contract DE-AC02-06CH11357.

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